## ASEN-6265. Fundamentals of Spectroscopy for Optical Remote Sensing Homework \#10 (Molecular Spectroscopy)

1. For nitrogen $\mathrm{N}_{2}$ and oxygen $\mathrm{O}_{2}$ molecules
(a) Given that the equilibrium distances of the $\mathrm{N}-\mathrm{N}$ and $\mathrm{O}-\mathrm{O}$ are $\mathrm{r}_{\mathrm{e}}=1.0940 \AA$ and 1.2074 $\AA$ respectively, calculate their moments of inertia.
(b) Calculate their rotational constants $B_{e}$. The measured spectroscopic rotational constants of $\mathrm{N}_{2}$ and $\mathrm{O}_{2}$ are $\mathrm{B}_{0}=1.998 \mathrm{~cm}^{-1}$ and $1.438 \mathrm{~cm}^{-1}$ respectively. Why is there the difference between $B_{0}$ and $B_{e}$ ?
(c) Determine the transitions from $\mathrm{J}=0$ to $\mathrm{J}=1$ and from $\mathrm{J}=1$ to $\mathrm{J}=2$ for both molecules in wave numbers, in megahertz, and in eV .
2. The vibrational energy spacings for the hydrogen halides are

HF: $\quad 3958.4 \mathrm{~cm}^{-1}$
$\mathrm{HCl}: \quad 2989.7 \mathrm{~cm}^{-1}$
HBr: $\quad 2559.3 \mathrm{~cm}^{-1}$
HI: $\quad 2230.0 \mathrm{~cm}^{-1}$
Calculate the force constant of each hydrogen halide molecule and account for any trend in these force constants.
3. Spectroscopic constants of ${ }^{127} \mathrm{I}^{35} \mathrm{Cl}$ have been measured: $v_{\text {vib }}=384.18 \mathrm{~cm}^{-1}$ and $v_{\text {vib }} \mathrm{X}_{\mathrm{e}}=$ $1.465 \mathrm{~cm}^{-1}$. The bond dissociation energy of ${ }^{127} \mathrm{I}^{35} \mathrm{Cl}$ has also been measured to be 2.153 eV.
(a) Calculate the wave number of the transition between $\mathrm{v}=0$ and $\mathrm{v}=1$.
(b) Repeat part (a) for the transition between $v=1$ and $v=2$.
(c) What is the depth of the molecular potential curve of ${ }^{127} \mathrm{I}^{35} \mathrm{Cl}$ (that is, what is the value of $D_{e}$ )?
4. Consider the HCl molecule and use the following constants for the $\mathrm{X}^{1} \mathrm{~S}^{+}$ground state: $v_{\mathrm{vib}}$ $=2989.74 \mathrm{~cm}^{-1}, \quad V_{\text {vib }} X_{e}=52.05 \mathrm{~cm}^{-1}, \quad V_{\text {vib }} Y_{\mathrm{e}}=0.056 \mathrm{~cm}^{-1}, \quad \mathrm{~B}_{\mathrm{e}}=10.5909 \mathrm{~cm}^{-1}, \quad \mathrm{a}_{\mathrm{e}}=0.3019 \mathrm{~cm}^{-1}$, $\mathrm{r}_{\mathrm{e}}=1.27460 \AA$ (the first electronic excited state lies about $77000 \mathrm{~cm}^{-1}$ above the ground state).
(a) Calculate the rotational constant $\mathrm{D}_{\mathrm{e}}$.
(b) Calculate the first six rotational energy levels (consider the molecule as vibrating rotator).
(c) Calculate the first six vibrational energy levels.
(d) Sketch the vibration-rotation energy levels you have got.
5. Given the following constants for $\mathrm{N}_{2}$,

|  | $\tilde{v}_{e}$ | $\mathrm{~B}_{0}$ | $\mathrm{~B}_{1}$ | $\mathrm{D}_{0} \quad\left(\mathrm{~cm}^{-1}\right)$ |
| ---: | :--- | :--- | :---: | :---: | :---: |
| $\mathrm{N}_{2}$ | 2330.7 | 1.98957 | 1.97219 | $5.76 \mathrm{E}-6$ |

Where $\tilde{\boldsymbol{v}}_{e}$ is the vibrational frequency, $\mathrm{B}_{0}, \mathrm{~B}_{1}$ and $\mathrm{D}_{0}$ are the rotational constants. (All four quantities are in wavenumber $\left(\mathrm{cm}^{-1}\right)$ unit.) Shine a 532 nm laser beam through a Raman cell containing $\mathrm{N}_{2}$, and observe the scattering light to an angle perpendicular to the laser beam. Rayleigh, pure rotational Raman, and vibration-rotational Raman scattering lines will occur.
(1) What is the Rayleigh scattering light wavelength ( nm ) and corresponding wavenumber $\left(\mathrm{cm}^{-1}\right)$ ?
(2) Take this Rayleigh scattering light as the reference of wavenumber, figure out the relative wavenumber of pure rotation Raman ( $R R$ ) lines (originated from $v=0$ ). Mark the corresponding branch and consider the first 6 lines for each branch.
(3) Take this Rayleigh scattering light as the reference of wavenumber, figure out the relative wavenumber of vibration-rotational Raman (VRR) lines (only consider $v=0$ and $v=1$ vibrational levels and associated rotational levels). Mark the corresponding branch and consider the first 6 lines for each branch.
(4) For the vibration-rotational Raman lines, calculate the wavelength of the Q-Branch (0-0) transition lines. [( $0-0)$ means from $\mathrm{J}=0$ to $\mathrm{J}=0$ ]
(5) Assume the transition matrix elements (transition probability) are equal for the Stokes VRR and anti-Stokes VRR, estimate the intensity ratio between the Stokes and the anti-Stokes VRR lines from the Boltzmann factor.

