Fundamentals of Spectroscopy for Optical Remote Sensing
Homework #8 (Radiative Transitions)

1. Prove that the light emitted from a certain $J_a \rightarrow J_b$ transition is unpolarized and isotropic, provided the populations of all the upper M_a levels are the same.

2. Calculate the oscillator strength for the following transitions:
(1) Calculate the oscillator strengths for the absorption transitions from $3^2S_{1/2} \rightarrow 3^2P_{3/2}$ (D_2 line) and from $3^2S_{1/2} \rightarrow 3^2P_{1/2}$ (D_1 line) of a Na atom. Why is the D_1 oscillator strength smaller than D_2 line? Related Na parameters are the radiative lifetime (τ = 16.23 ns) of the excited state $3^2P_{3/2}$ and the D_2 wavelength λ = 589.1583 nm; the radiative lifetime (τ = 16.29 ns) of the excited state $3^2P_{1/2}$ and the D_1 wavelength λ = 589.7558 nm.
(2) Calculate the oscillator strength for the absorption transition from $a^5D_4 \rightarrow z^5F_5$ of an Fe atom. Compare this Fe oscillator strength to the Na oscillator strengths calculated above, what is the main reason for the Fe oscillator strength to be much smaller? Related Fe parameters are the radiative lifetime (τ = 61.0 ns) of the excited state z^5F_5 and the Fe transition wavelength λ = 372.0993 nm.

3. An excited atomic level $|E_K\rangle$ is connected with three lower levels E_n $|n\rangle$ by radiative transitions with spontaneous probabilities $A_{k1}=3\text{x}10^7$ s^{-1}, $A_{k2}=1\text{x}10^7$ s^{-1}, and $A_{k3}=5\text{x}10^7$ s^{-1}, and connected with the ground state $|0\rangle$ by $A_{k0} = 4\text{x}10^7$ s^{-1}.
   (1) Calculate the spontaneous lifetime τ_k and the relative populations N_n/N_k under cw excitation of $|E_K\rangle$, when τ_1 = 500 ns, τ_2 = 6 ns, and τ_3 = 10 ns (radiative lifetime of three lower levels).
   (2) Determine the Einstein coefficient B_{0k} for the excitation $|0\rangle \rightarrow |k\rangle$ from the ground state $|0\rangle$ with $\tau_0 = \infty$ at the wavelength λ = 400 nm and with the statistical weights g_0 = 1 and g_k = 3. At which spectral energy density ρ_ν is the induced absorption rate equal to the spontaneous decay rate of level $|E_K\rangle$? What is the intensity of a laser with a bandwidth of 10 MHz at this radiation density?
   (3) How large is the absorption cross section σ_{0k} if the absorption linewidth is solely determined by the lifetime of the upper level?

   Note: (2) and (3) are optional. You may work on them to get extra points.

4. Let us consider the energy level structure of a sodium (Na) atom only to the fine structure. Place this Na atom in a weak magnetic field, thus there will be Zeeman splitting to the ground state (3S) and the first excited state (3P). Refer to HW #6, problem No. 2. Draw the energy level diagram of the Na ground state and first excited state including fine structure and Zeeman splitting (using m_J to mark the magnetic energy levels). Please pay attention to the order of the magnetic energy levels.
(1) Given the selection rules for electric dipole transitions:
$$\Delta n \text{ is arbitrary, } \Delta S = 0,\ \Delta L = 0, \pm1,\ \Delta J = 0, \pm1, \text{ and } \Delta m_J = 0, \pm1,$$
Please determine how many transition lines are allowable from $^2P_{1/2}$ to the ground state $^2S_{1/2}$ and from $^2P_{3/2}$ to the ground state $^2S_{1/2}$, and draw the transition lines on the energy level diagram.
(2) Given the photon polarization rules in the electric dipole transitions: Δm = 0 gives linear polarization (π), and $\Delta m = \pm1$ gives circular polarization (σ), please use π and σ mark the polarization

properties for each allowable transition lines. How many linearly polarized lines and how many circularly polarized lines do you get for $^2P_{1/2}$ to $^2S_{1/2}$ and $^2P_{3/2}$ to $^2S_{1/2}$, respectively?

5. Please do a brief research on the Fabry-Perot Interferometer (FPI) measurements of MLT wind using airglow OI 5577 Å, 6300 Å, and 6363 Å lines. Try to address the following issues:
   (1) Draw energy diagram of OI and explain how the diagram is formed and what kind of transition each line belongs to.
   (2) Try to find approximate transition rate for each line.
   (3) Explain the principle of FPI to measure wind from these airglow signals.
   Note: (2) and (3) are optional. You may work on them to get extra points.

HW #8 is due on Thursday, April 6th, 2017 in class.