ASEN-6265. Fundamentals of Spectroscopy for Optical Remote Sensing
Homework #6 (Atomic Structure)

1. (1) Please calculate the isotope (mass) shift of hydrogen atoms for ^1H, ^2H, and ^3H for n = 1 and 2, relative to the energy levels of R_∞. Use the unit of $A = R_\infty hc \frac{m_e}{M_p}$, and put the results into a table.

   (2) Draw a diagram to show the obtained isotope shifts. The main point is to show the shifts of each isotope relative to the energy levels with R_∞ and which isotope has higher or lower energy levels.

2. A sodium (Na) atom is placed in a static magnetic field **B**. 1) Derive the hyperfine splitting and shift of (relative to its fine structure) and the Zeeman splitting and shift of the hyperfine structure of its ground-state energy level, given the magnetic field is a weak field when compared to the hyperfine structure interaction and the Na nuclear angular momentum number I = 3/2. 2) Draw the ground-state energy levels including quantitative considerations of the fine and hyperfine structures, and Zeeman splitting. Mark the hyperfine and magnetic energy levels with F and m_F quantum numbers.

3. Assume an atom is in a very strong external magnetic field $\vec{B}$ that is larger than the internal magnetic field of the atom. Thus, the LS coupling of the atom is broken down, and the total orbital angular momentum $\hat{\vec{L}}$ and the total spin angular momentum $\hat{\vec{S}}$ precess around the external field $\vec{B}$ independently (Paschen-Back effect).
   (1) Write the equation of the atomic total magnetic moment;
   (2) Write the equation of the magnetic interaction energy in the field $\vec{B}$;
   (3) If this atom is a sodium atom (Na), please derive the energy splitting of its ground state and the first excited states, and then draw a diagram of the split energy levels;
   (4) Using selection rules $\Delta m_s = 0, \Delta m_l = 0, \pm 1$, please mark the allowed transitions between the first excited states and the ground state.

4. The combination energy of Helium atom is 24.5 eV. In order to ionize the two electrons of a Helium atom one by one, how much energy must be provided.

5. (1) A Helium atomic beam in ground state passes through an inhomogeneous magnetic field (Stern-Gerlach experiment). How many beams will be seen on the screen?
   (2) Under the same conditions, if the atomic beam consists of B atoms (Boron), how many beams will be seen? Why?
   (*Please analyze from the point of view of electron configurations to the atomic state of the ground state, and then to the magnetic field interaction with atomic magnetic moment.*)

6. Please write the electron configurations and figure out why the following atoms have the following ground states: (1) C ($^3P_0$); (2) O ($^3P_2$). The equivalent electrons and Hund's rules should be considered here. The allowable state of equivalent electrons and Hund's rules are attached at the end.

7. Laser Isotope Separation (LIS), Electron Paramagnetic Resonance (EPR), Nuclear Magnetic

Resonance (NMR) and Magnetic Resonance Imaging (MRI) are some important applications of atomic spectroscopy. You are encouraged to choose one of the topics to search on the web or library to find out (1) its working principle, (2) application field, and (3) how it is related to our class contents. Please write a 1-2 page study report for your research results.

Table 2.1. Terms of l^n configurations   *Equivalent electrons*   *allowable states*

Configuration		Terms		Statistical weight
s		^2S		2
s^2		^1S		1
p	p^5	$^2P^0$		6
p^2	p^4	^1SD	^3P	15
p^3		$^2PD^0$	$^4S^0$	20
d	d^9	^2D		10
d^2	d^8	^1SDG	^3PF	45
d^3	d^7	$^2PD_2FGH$	^4PF	120
d^4	d^6	$^1S_2D_2FG_2I$	$^3P_4DF_2GH\ ^5D$	210
d^5		$^2SPD_3F_2G_2HI$	$^4PDFG\ ^6S$	252
f	f^{13}	$^2F^0$		14
f^2	f^{12}	^1SDGI	^3PFH	91
f^3	f^{11}	$^2PD_2F_2G_2H_2IKL^0$	$^4SDFGI^0$	364
f^4	f^{10}	$^1S_2D_4FG_4H_2I_3KL_2N$	$^3P_3D_2F_4G_3H_4I_2K_2LM\ ^5SDFGI$	1001
f^5	f^9	$^2P_4D_5F_7G_6H_7I_5K_5L_3M_2NO^0$	$^4SP_2D_3F_4G_4H_3I_3K_2LM^0\ ^6PFH^0$	2002
f^6	f^8	$^1S_4PD_6F_4G_8H_4I_7K_3L_4M_2N_2Q$	$^3P_6D_5F_9G_7H_9I_6K_6L_3M_3NO\ ^7F$	3003
			$^5SPD_3F_2G_3HI_2K_2L$	
f^7		$^2S_2P_5D_7F_{10}G_{10}H_9I_9K_7L_5M_4N_2OQ^0$	$^4S_2P_2D_6F_5G_7H_5I_5K_3L_3MN^0\ ^8S^0$	3432
			$^6PDFGHI^0$	

\+ Hund's rules:
- Greatest S $(2S+1)$ → lowest energy
- For same S, greatest L → Lowest energy

\+
- Less than half-filled, minimum J → lowest energy
- More than half-filled, maximum J → lowest energy

HW #6 is due on Thursday, March 16th, 2017 in class.