ASEN-6265. Fundamentals of Spectroscopy for Optical Remote Sensing
Homework #4 (Quantum Mechanics Basics)

1. Please summarize the postulates and principles of Quantum Mechanics in your own words, including QM state, operator, measurement, eigenvalue state, superposition of states, collapse of states, mean value, representations, principle of uncertainty, and principle of motion, etc.

2. In a one-dimensional problem, consider a particle whose wave function is

$$\psi(x) = N \frac{e^{ip_0 x/\hbar}}{\sqrt{x^2 + a^2}}$$

where a and p_0 are real constants and N is a normalization coefficient.
(1) Determine N so that $\psi(x)$ is normalized.
(2) The position of the particle is measured. What is the probability of finding a result between $-\frac{a}{\sqrt{3}}$ and $+\frac{a}{\sqrt{3}}$?
(3) Calculate the mean value of the momentum of a particle that has $\psi(x)$ for its wave function.

3. The energy operator for harmonic oscillator is

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}\kappa\hat{x}^2 = \frac{1}{2m}\left(\hat{p}^2 + m^2\omega_0^2\hat{x}^2\right),$$

where $\omega_0 = \sqrt{\kappa/m}$ is the intrinsic angular frequency of the harmonic oscillator. The normalized wave function for the ground state of the oscillator is given by

$$\psi_0(\xi) = \left(\frac{m\omega_0}{\pi\hbar}\right)^{1/4} e^{-\xi^2/2},$$

where $\xi = \left(\sqrt{m\omega_0/\hbar}\right)x$. (1) Please compute the mean values of the oscillator's momentum and energy. (2) Is this wave function an eigen function of the energy operator $\hat{H}$ (also called Hamilton operator)? If yes, how much is the eigen value?

4. Derive Heisenberg's principle of uncertainty $\Delta p \cdot \Delta q \geq \hbar/2$ from commutation relation, where q is position and p is momentum of a particle. Then derive $\Delta E \cdot \Delta t \geq \hbar/2$ and explain the natural linewidth of an atomic spectral line from the uncertainty principle.

5. Explain the quantum tunnel effect through solving the stationary-state Schrödinger equation of a finite potential well (like the example 2 for Schrödinger equation applications), and describe how a scanning tunneling microscope (STM) achieves a high resolution as of an atom dimension.

HW #4 is due in class on Thursday, February 16th, 2017.

Extra problems for QM basics homework (not required but welcome to try on)

6. For a particle moving within a range of x =0 to x = a, its normalized wave function is given by

Is this wave function an eigen function of momentum? Is it an eigen function of kinetic energy? If so, how much is the corresponding eigen value?

7. A particle's momentum and the position of itself have the commutation relation . However, the variables between different particles are commuted. In other words, the following relationship exists for particles 1 and 2:

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Try to prove that the operators and are commuted with each other.

8. Applications of the uncertainty principle in single-photon interference experiments: Taking a Michelson interferometer or a Fabry-Perot interferometer (FPI) as an example, the two arms in the two-beam interference or the many paths in the multiple-beam interference can apparently have different optical path lengths. Therefore, the time for a single photon to travel through different arms or paths will be different. As we discuss in the wave-particle duality chapter, we know interference patterns will be produced if we do NOT know which arm or path a single photon goes through. Then a natural question is by what time such a single photon will reach the photo detector. Can light speed exceed c – the light speed in vacuum? If not, how can we explain such single-photon interference with unequal arm length or unequal optical path length?