

Review on Quantum Mechanics

1. Concepts of Quantum

* Photon has energy and the energy is quantized —

- { Blackbody Radiation
- { Photoelectric Effect
- { Hydrogen Spectra

* Photon has momentum and the momentum is quantized —

- { Compton Effect
- { Radiation Pressure

2. Wave-Particle Duality:

Both photon and matter particles have wave-particle

- Duality —
- { de Broglie matter wave
 - { Relation of E and P in relativity

* Single photon interference or Imagin

Single electron interference or diffraction

— How to explain these experiments ?

Try to understand the details of these experiments and useful relations, e.g., blackbody radiation:

$$\left\{ \begin{array}{l} \text{Planck Radiation Law: } P(\nu) = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{e^{h\nu/k_B T} - 1} \\ \text{Wien Displacement Law: } \lambda_m T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K} \\ \text{Stefan-Boltzmann Law: } P_{\text{total}} = \int_0^\infty P(\nu) d\nu = \frac{4}{c} \sigma T^4 \end{array} \right.$$

de Broglie Relation: $\left\{ \begin{array}{l} E = h\nu \\ \vec{p} = \hbar \vec{k}, \quad (p = \frac{h}{\lambda}) \end{array} \right.$

$$E = \sqrt{p^2 c^2 + m_0^2 c^4} \quad \text{in relativity theory.}$$

3. Fundamentals of Quantum Mechanics

* QM state : state vector $|\psi(t)\rangle$

* QM observables/variables/physical quantities : operator \hat{A}

* QM measurement : $\hat{A} |\psi(t)\rangle$

— only possible result is one of the eigenvalues of \hat{A} .

* Eigenvalue equation, eigen state, eigenvalues

$$\hat{A} |\psi\rangle = \lambda |\psi\rangle, \text{ where } \lambda \text{ is a complex number.}$$

* Superposition principle:

orthonormal basis $\{|u_n\rangle\}$, $\hat{A} |u_n\rangle = a_n |u_n\rangle$.

eigenvalues $\{a_n\}$ ($\langle u_m | u_n \rangle = \delta_{mn}$)

$$|\psi\rangle = \sum_n (c_n |u_n\rangle), \text{ where } c_n = \langle u_n | \psi \rangle.$$

Probability of obtaining a specific eigenvalue:

$$P(a_n) = \frac{|\langle u_n | \psi \rangle|^2}{\langle \psi | \psi \rangle} = \frac{|c_n|^2}{\langle \psi | \psi \rangle}$$

* Mean value:

$$\bar{A} = \langle \psi | \hat{A} | \psi \rangle$$

$$= \sum_n [P(a_n) a_n]$$

$$= \sum_n \left(\frac{|\langle u_n | \psi \rangle|^2}{\langle \psi | \psi \rangle} a_n \right)$$

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* Representation: When really calculate eigenvalues or mean values, it is common to project states and operators to certain representation, e.g., $\{x, y, z\}$ representation.

$$\hat{A} \psi(x) = a \psi(x)$$

$$1D: \int \psi^*(x) \hat{A}(x) \psi(x) dx$$

$$3D: \int \psi^*(\vec{r}) \hat{A}(\vec{r}) \psi(\vec{r}) d^3r = \iiint \psi^*(x, y, z) \hat{A} \psi(x, y, z) dx dy dz$$

* Uncertainty Principle and commutation relation.

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}, \quad [\hat{r}_i, \hat{p}_j] = i\hbar \delta_{ij},$$

($i, j = x, y, z$)

$$\Delta A \cdot \Delta B \geq \frac{1}{2} | \langle [\hat{A}, \hat{B}] \rangle |, \quad \text{e.g., } \Delta Q \cdot \Delta P \geq \frac{\hbar}{2}$$

* Evolution of state — Schrödinger equation:

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H}(t) |\psi(t)\rangle$$

$$\text{Where } \hat{H}(t) = \frac{\hat{p}^2(t)}{2m} + \hat{V}(t)$$