Review on Quantum Mechanics

1. Concepts of Quantum

* Photon has energy and the energy is quantized —
  - Blackbody Radiation
  - Photoelectric Effect
  - Hydrogen Spectra

* Photon has momentum and the momentum is quantized —
  - Compton Effect
  - Radiation Pressure

2. Wave-Particle Duality:

Both photon and matter particles have wave-particle Duality —
- de Broglie matter wave
- Relation of E and P in relativity

* Single photon interference or Imaging

Single electron interference or diffraction

— How to explain these experiments?

Try to understand the details of these experiments and useful relations, e.g., blackbody radiation:

$$\begin{cases} \text{Planck Radiation Law}: \quad \rho(\nu) = \dfrac{8\pi h \nu^3}{c^3} \cdot \dfrac{1}{e^{h\nu/k_BT}-1} \\ \text{Wien Displacement Law}: \quad \lambda_m T = 2.898 \times 10^{-3}\ m \cdot K \\ \text{Stefan-Boltzmann Law}: \quad \rho_{total} = \int_0^{\nu} \rho(\nu)\, d\nu = \dfrac{4}{c} \sigma T^4. \end{cases}$$

de Broglie relation:
$$\begin{cases} E = h\nu \\ \vec{p} = \hbar \vec{k}, \quad \left(p = \dfrac{h}{\lambda}\right) \end{cases}$$

$$E = \sqrt{p^2c^2 + m_0^2c^4} \quad \text{in relativity theory.}$$

3. Fundamentals of Quantum Mechanics

* QM state : state vector $|\psi(t)\rangle$

* QM observables/variables/ physical quantities : operator $\hat{A}$

* QM measurement : $\hat{A}\,|\psi(t)\rangle$

— only possible result is one of the eigenvalues of $\hat{A}$.

* Eigenvalue equation, eigen state, eigenvalues

$$\hat{A}\,|\psi\rangle = \lambda\,|\psi\rangle,$$ where λ is a complex number.

* Superposition principle:

orthonormal basis $\{|u_n\rangle\}$, $\hat{A}\,|u_n\rangle = a_n\,|u_n\rangle$.

eigenvalues $\{a_n\}$ $\quad (\langle u_m|u_n\rangle = \delta_{mn})$

$$|\psi\rangle = \sum_n \left(c_n\,|u_n\rangle\right),$$ where $c_n = \langle u_n|\psi\rangle$.

Probability of obtaining a specific eigenvalue:

$$P(a_n) = \frac{|\langle u_n|\psi\rangle|^2}{\langle\psi|\psi\rangle} = \frac{|c_n|^2}{\langle\psi|\psi\rangle}$$

* Mean Value :

$$\bar{A} = \langle\psi|\hat{A}|\psi\rangle$$

$$= \sum_n \left[P(a_n)\,a_n\right]$$

$$= \sum_n \left(\frac{|\langle u_n|\psi\rangle|^2}{\langle\psi|\psi\rangle}\,a_n\right)$$

* Representation: when really calculate eigenvalues or mean values, it is common to project states and operators to certain representation, e.g., $\{x, y, z\}$ representation.

$$\hat{A}\,\psi(x) = a\,\psi(x)$$

1D: $\int \psi^*(x)\,\hat{A}(x)\,\psi(x)\,dx$

3D: $\int \psi^*(\vec{r})\,\hat{A}(\vec{r})\psi(\vec{r})\,d^3r = \iiint \psi^*(x,y,z)\,\hat{A}\,\psi(x,y,z)\,dx\,dy\,dz$

* Uncertainty Principle and commutation relation:

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}, \qquad [\hat{r}_i, \hat{P}_j] = i\hbar\,\delta_{ij}. \quad (i, j = x, y, z)$$

$$\Delta A \cdot \Delta B \geq \frac{1}{2}\left|\langle[\hat{A}, \hat{B}]\rangle\right|, \quad \text{e.g., } \Delta Q \cdot \Delta P \geq \frac{\hbar}{2}$$

* Evolution of state — Schrödinger equation:

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = \hat{H}(t)\,|\psi(t)\rangle.$$

where $\hat{H}(t) = \dfrac{\hat{\vec{P}}^2(t)}{2m} + \hat{V}(t)$