Chapter 2. Wave-Particle Duality

In a brief summary of Chapter 1 (Concept of Quantum):

(1) Blackbody Thermal Radiation $\Rightarrow$ linear oscillator's energy is discrete: $\varepsilon = nh\nu$, $n = 0, 1, 2, \cdots$

$(h = 6.626 \times 10^{-34}\ J\cdot s)$ (Planck Quanta Hypothesis ~~Assumption~~)

(2) Photoelectric Effect $\Rightarrow$ particle property of light: photon

Photon energy $\varepsilon = h\nu$. (Einstein Photon Hypothesis ~~Assumption~~)

(3) Compton Effect $\Rightarrow$ Photon has momentum

$$\vec{p} = \hbar\vec{K}, \quad |\vec{P}| = P = \frac{h}{\lambda}.$$

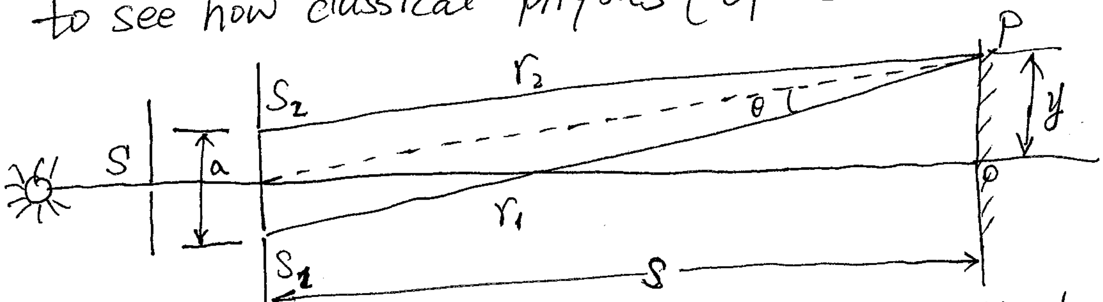
(4) Hydrogen Spectra $\Rightarrow$ Bound Atom has discrete energy levels

Transition occurs only when $h\nu = E_{n'} - E_n$.

§2.1. Wave Behavior of Light:

Wave behavior of light has been proven for many years through interference, diffraction, etc. experiments, and through the unification of Maxwell Equations of EM waves.

Let's use Young's double-slit experiment as an example, to see how classical physics (Optics and EM) explain it.



Two waves interfer at point P. Wave amplitudes are superposed at P, so the wave amplitude at P is given by

$$\tilde{U}(x',y') = \tilde{U}_1(x',y') + \tilde{U}_2(x',y'), \quad I(x',y') = \tilde{U}^*(x',y')\,\tilde{U}(x',y')$$

When $r_2 - r_1 = m\lambda$, constructive interference $\Rightarrow$ intensity maximum

$r_2 - r_1 = (m + \frac{1}{2})\lambda$, destructive " $\Rightarrow$ " minimum

Thus, the fringe interval (distance between two bright fringes) is given

$$\Delta y = \frac{s}{a}\lambda$$

For normal light, $a \sim 0.1-1\,mm$, $s \sim 1-10\,m$, $y \sim 1-10\,cm$

Wave Amplitude Superposition $\Longrightarrow$ Interference

§2.2 Single Photon Experiment:

If we decrease the light source intensity, until photons go through the experiment setup one-by-one. What's going to happen? Will we still see the interference fringes? And how?

— We can use film as a screen to record the experiment.

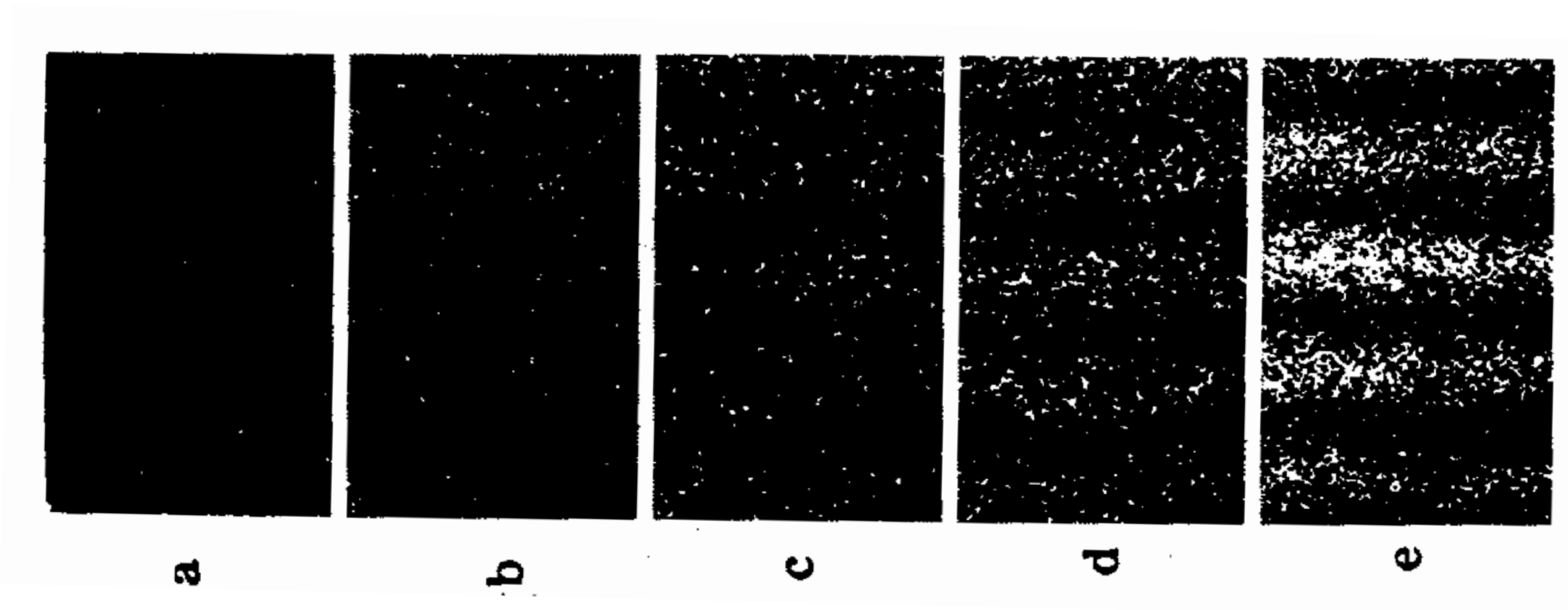
① If exposing the film for a long time to capture a large number of photons, the fringes do not disappear. Therefore, the pure particle interpretation that the fringes are due to interaction between different photons, must be rejected.

② If only exposing the film to a very short period so that only a few photons are received, observations show that each photon produces a localized impact on the film, but not a very weak interference pattern. Therefore, the purely wave interpretation must be rejected.

③ When more and more photons strike the photographic film, the distribution of photon impacts begins to have a continuous aspect. The density (i.e., the probability) of the impacts at each point of the **film** corresponds to the interference fringes. max on a bright fringe and **zero** on a dark fringe.

— Please look at the real experimental image of Single-photon interference.

Single Photon Double-Slit Interference Experiment



§2.3 Wave-Particle Duality of Light

How to explain the single-photon interference experiment?

- Since it is NOT the interference between photons, we must accept the fact that each single photon interfers with itself.

But how does this happen? Which slit does the photon go through?

- Once we try to use an instrument (e.g., PMT) to determine which slit the single photon goes through, the interference fringes disappear!!! Only diffraction pattern of the unblocked slit is shown. ⇒ Measurement does influence the results!

Conclusion: Light has wave-particle duality (i.e., the particle and wave aspects of light are inseparable), and each single photon interfers with itself!

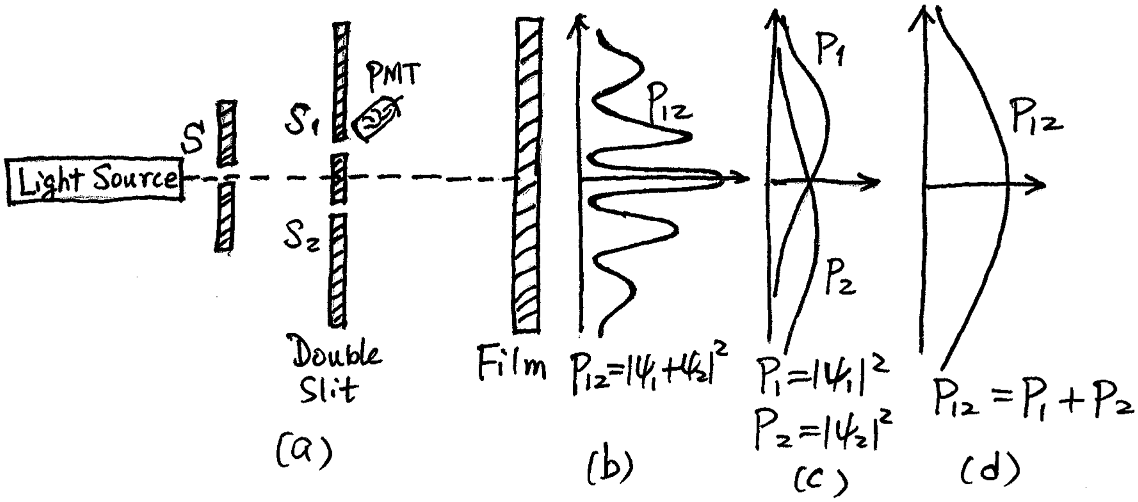


Figure. (a) Double-Slit interference experiment

(b) Interference Patten when both slits are open.

(c) Diffraction patten when one slit is open and another is covered or measured by PMT

(d) The sum of individual P_1 and P_2.

The classical wave theory of light provides the following interpretation of the fringes in non-single-photon experiment:

The light intensity at a point of the screen/film is proportional to the square of the amplitude of the electric field at this point. If $E_1(x)$ and $E_2(x)$ represent, in complex notation, the electric fields produced at x by slits 1 and 2 respectively, the total resultant field at this point, when slits 1 and 2 are both open, is $E(x) = E_1(x) + E_2(x)$.

Then we have the light intensity:

$$I(x) \propto |E(x)|^2 = |E_1(x) + E_2(x)|^2$$
$$= |E_1(x)|^2 + |E_2(x)|^2 + 2\vec{E}_1 \cdot \vec{E}_2$$

Since the intensity $I_1(x) \propto |E_1(x)|^2$,

$$I_2(x) \propto |E_2(x)|^2,$$

We have from the above equation that

$$I(x) \neq I_1(x) + I_2(x)$$

The difference is given by the interference term $2\vec{E}_1 \cdot \vec{E}_2$, which depends on the phase difference between $\vec{E}_1$ and $\vec{E}_2$.

The presence of this interference term explains the fringes.

Thus, the classical wave theory predicts that decreasing the intensity of the light source will simply cause the interference fringes to diminish in intensity but not to vanish.

— This is <u>not</u> consistent with the observed experimental Results.

How does quantum mechanics explain the double-slit interference?

- Replace the electric field $E(x)$ with probability amplitude $\psi(x)$, and then use the wave frame.
- Replace the intensity $I(x)$ with the probability $P(x)$ that describes the probability of a photon occurring at position x.

The probability $P(x) = \psi^*(x)\,\psi(x) = |\psi(x)|^2$.

Similar to the complex amplitude in light wave, the probability amplitude $\psi(x)$ is also a complex:

$$\psi(x) = |\psi(x)|\, e^{i\varphi(x)}$$

When only one slit is open, the probability to find a photon at x is

$$P_1(x) = \psi_1^*(x)\,\psi_1(x) = |\psi_1(x)|^2$$

$$P_2(x) = \psi_2^*(x)\,\psi_2(x) = |\psi_2(x)|^2.$$

When both slits are open simultaneously, the probability amplitudes are superimposed:

$$\psi(x) = \psi_1(x) + \psi_2(x).$$

Then the probability

$$\begin{aligned} P_{12}(x) &= |\psi(x)|^2 = |\psi_1(x) + \psi_2(x)|^2 \\ &= |\psi_1(x)|^2 + |\psi_2(x)|^2 + 2|\psi_1(x)||\psi_2(x)|\cos[\varphi_1(x) - \varphi_2(x)] \\ &= P_1(x) + P_2(x) + \underline{2\sqrt{P_1(x)P_2(x)}\cos[\varphi_1(x) - \varphi_2(x)]} \end{aligned}$$

Thus, the interference term occurs, explaining the fringes.

Summarizing above discussion, the concept of wave-particle duality of light is given by the following:

① The particle and wave aspects of light are inseparable. Light behaves simultaneously like a wave and like a flux of particles (photons).

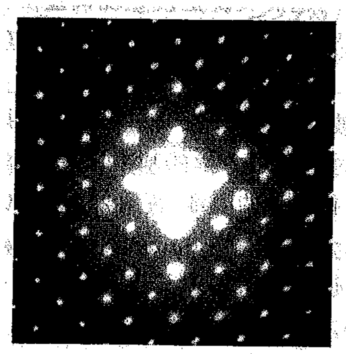
② The wave enables us to calculate the probability of the manifestation of a particle, and the particle defines a finite and quantized energy and momentum.

③ Predictions about the behavior of a photon can only be probabilistic. This is described by the probability amplitude $\psi(\vec{r},t)$ of a photon appearing at time t and at the position $\vec{r}$. Corresponding probability is proportional to $|\psi(\vec{r},t)|^2$.

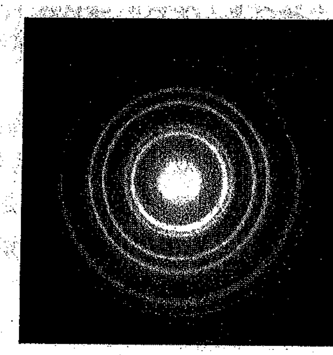
§2.4 Wave-Particle Duality of Material Particles

* In 1923, in analogies to the wave-particle duality of light, de Broglie put forth the following hypothesis: "Material particles, just like photons, can have a wavelike aspect." The wave associated with material particles is named as "de Broglie matter wave".

* In 1928, Davisson and Germer used a cubic nickel crystal and Thompson used thin films of celluloid, gold, platinum and aluminum, strikingly confirmed the existence of electron diffraction pattern. (See below: Electron diffraction pattern)
  — This was the first proof of de Broglie matter wave!



a. MoO_3 单晶的劳埃相



b. Au 多晶的德拜相

图 1－19 电子的衍射象 (Electron Diffraction)

* In 1970s, Merli et al., [Am. J. Phys., 44 (306), 1976] achieved the electron double-slit interference pattern, further proving the wave-particle duality of material particles.

(a) → (f): Electron interference fringe patterns filmed from a TV monitor at increasing current densities (from Merli et al., 1976).
(See attached article)

percent activity increase

$$= \frac{\text{final activity} - \text{initial activity}}{\text{initial activity}} \times 100. \qquad (2)$$

Two isotopes that are convenient for this demonstration, owing to their availability and relatively long half-lives, are ^{137}Cs ($t_{1/2} = 30$ yr, $E_\gamma = 0.66$ MeV, $E_\beta = 0.52$, 1.2 MeV) and ^{133}Ba ($t_{1/2} = 7.2$ yr, average $E_\gamma = 0.34$ MeV). These isotopes give about 50% and 140% activity increases, respectively. Any γ or β–γ emitter with γ energy between 0.1 and 1.0 MeV will show the effect.

The activity increase decreases with increasing γ-ray energy. After numerous correction factors were applied, the major one of which was the end-window factor for soft electrons, a relationship was obtained between scattered γ-ray energy and the activity increase. The details are beyond the scope of this note, but may be published elsewhere.

[1]H. Bradt *et al.*, Helv. Phys. Acta **19**, 77 (1946).

[2]C. M. Davisson and R. D. Evans, Rev. Mod. Phys. **24**, 85 (1952).

On the statistical aspect of electron interference phenomena

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In a recent paper,[1] hereafter called I, two of the present authors have described how to perform—for instructional purposes—an experiment on electron interference by using a standard electron microscope.

In this short note we wish to show in a very impressive way that the complete interference pattern we registered on the photographic plate is really the sum of many independent events, each due to the interaction between a single electron and the interference apparatus. This was deduced in I with a simple and realistic calculation based on the main assumption that electrons were emitted at a constant rate from the gun filament. In the present case this result is shown not from a calculation but from direct observation. In fact, the experiment performed in I has been repeated on a Siemens Elmiskop 101 equipped with a TV image intensifier.[2,3]

With regard to the formerly described setup, the Möllenstedt and Düker electron biprism[4] has been now inserted at the level of the selector aperture plane. The objective lens acts in this case as third condenser lens, thus increasing the coherence and versatility of the illuminating system, whereas the fringes are magnified on the final screen by means of the two projector lenses (cf. Fig. 4 in I). If the coherence condition is satisfied, it is possible to register on a photographic plate an intereference fringe pattern with spacing above 300 μm as shown in Fig. 1(f). The exposure time of the photographic plate lies in a range between 10 and 100 sec. By the same electron optical conditions, however, the TV image intensifier allows the observation of the interference pattern directly on the monitor by means of the electrons stored in the SEC target of the TV tube[2,3] in a time of about 0.1 sec.

Figure 1(f), together with Figs. 1(a)–1(e), was filmed directly from the TV monitor. We note that the image on the screen was clearly visible, as in normal TV transmission, and that by varying the biprism potential we could follow, without difficulty, all the diffraction and interference phenomena described in I.

However, the most interesting performance that such a device offers is connected to the direct observation of the statistical process of fringe formation. It can easily be seen that, at low current density, the image is built up from the statistically distributed light flashes of individual electrons, as is shown in the sequence of Figs. 1(a)–(f) registered at different current densities on the final screen.

The same result can be reached in another way that didactically is more illuminating in concept. In fact, we

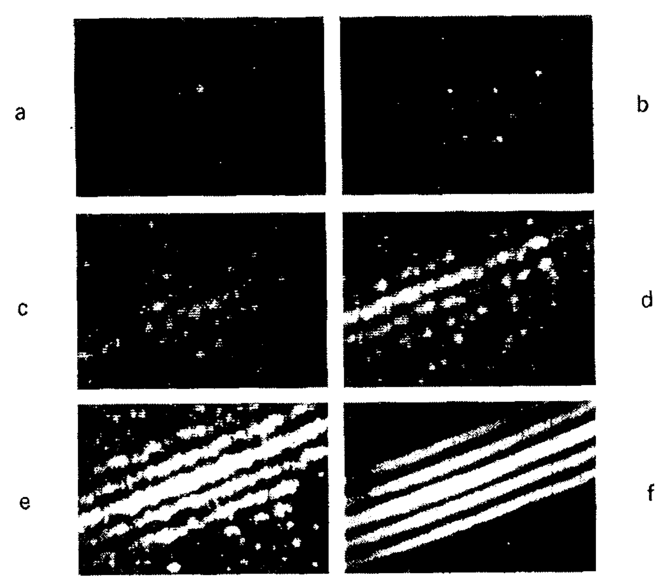

Fig. 1. (a–f) Electron interference fringe patterns filmed from a TV monitor at increasing current densities.

can operate with a very low electron current density which corresponds, on the average, to one or a few electrons arriving on the final screen in 0.04 sec [see Fig. 1(a)]. This is the lowest storage time available with the TV tube. While the electron optical conditions are kept constant, the storage time, which plays the same role as the exposure time of the photographic plate, can be increased step-by-step up to values of minutes. It can be verified that the image is gradually "filled" by the electrons until the shot noise vanishes completely.

We believe these results will be of great help to students by demonstrating to them, in an experimental form, the wave behavior of electrons and their statistical interpretation. Moreover, the whole apparatus is particularly valuable for student demonstrations in that the image can be directly seen by a large number of viewers and can possibly be recorded on video tape.

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[1]O. Donati, G. F. Missiroli, and G. Pozzi, Am. J. Phys. **41**, 639 (1973).

[2]K. H. Herrmann, D. Krahl, A. Kübler, and V. Rindfleisch, Siemens Rev. **36**, 6 (1969).

[3]K. H. Hermann, D. Krahl, A. Kübler, R. H. Müller, and V. Rindfleisch, in *Electron Microscopy in Material Science*, edited by U. Valdrè (Academic, New York, 1971), pp. 237–272.

[4]G. Möllenstedt and H. Düker, Naturwissenschaften **42**, 41 (1954).

THIRD NATIONAL CONFERENCE ON PERSONALIZED INSTRUCTION

The third national conference on personalized instruction will be held at the Mayflower Hotel in Washington, DC, 6-8 May 1976. The conference theme is "Accent on Materials." Papers will deal with this theme as well as with research on particular components of PSI, PSI in novel environments, and cost analysis and other administrative considerations. Miniworkshops will be held on introduction to PSI and adapting PSI to writing assignments. Participants may attend a PSI clinic to discuss their particular problems in discussion with colleagues and a panel of experts. For more information write to the Conference Coordinator, Center for Personalized Instruction, Georgetown University, Washington, DC 20057

§2.5. de Broglie Relationship

The expression of de Broglie matter wave is that all material particles posses a wave-particle duality nature. A material particle with energy E and momentum $\vec{p}$ is associated with a matter wave whose angular frequency $\omega = 2\pi\nu$ and wave vector $\vec{k}$ are given by the similar relations as for photons:

$$\begin{cases} E = h\nu, & \text{where } |\vec{k}| = \frac{2\pi}{\lambda} \\ \vec{p} = \hbar\vec{k} \end{cases}$$

Here, E and p are related through the following relationship:

Relativity domain: $E = c\sqrt{p^2 + m_0^2c^2} = \sqrt{p^2c^2 + m_0^2c^4}$.

Non-relativity domain: $E = \frac{p^2}{2m}$

The corresponding wavelength of the de Broglie matter wave is given by

$$\lambda = \frac{h}{|\vec{p}|} = \frac{h}{p} \qquad \text{(de Broglie relation)}$$

or $\lambda = \frac{2\pi}{|\vec{k}|}$

de Broglie wave $\sim e^{-i(Et - \vec{p}\cdot\vec{r})/\hbar}$.

The very small value of the Planck constant h (6.626×10^{-34} J.S) explains why the wavelike nature of matter is very difficult to demonstrate on a macroscopic scale. Complement A_I discusses the orders of magnitude of the de Broglie wavelengths associated with various material particles.

Complement A_I [From Quantum Mechanics, Vol. 1, by Cohen-Tannoudji]

ORDER OF MAGNITUDE OF THE WAVELENGTHS ASSOCIATED WITH MATERIAL PARTICLES

De Broglie's relation:

$$\lambda = \frac{h}{p} \tag{1}$$

shows that, for a particle of mass m and speed v, the smaller m and v, the longer the corresponding wavelength.

To show that the wave properties of matter are impossible to detect in the macroscopic domain, take as an example a dust particle, of diameter $1\,\mu$ and mass $m \simeq 10^{-15}$ kg. Even for such a small mass and a speed of $v \simeq 1$ mm/s, formula (1) gives:

$$\lambda \simeq \frac{6.6 \times 10^{-34}}{10^{-15} \times 10^{-3}} \text{ meter} = 6.6 \times 10^{-16} \text{ meter} = 6.6 \times 10^{-6}\,\text{Å} \tag{2}$$

Such a wavelength is completely negligible on the scale of the dust particle.

Consider, on the other hand, a thermal neutron, that is, a neutron ($m_n \simeq 1.67 \times 10^{-27}$ kg) with a speed v corresponding to the average thermal energy at the (absolute) temperature T. v is given by the relation:

$$\frac{1}{2} m_n v^2 = \frac{p^2}{2m_n} \simeq \frac{3}{2} kT \tag{3}$$

where k is the Boltzmann constant ($k \simeq 1.38 \times 10^{-23}$ joule/degree). The wavelength which corresponds to such a speed is:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{3m_n kT}} \tag{4}$$

For $T \simeq 300$ °K, we find :

$$\lambda \simeq 1.4\,\text{Å} \tag{5}$$

that is, a wavelength which is of the order of the distance between atoms in a crystal lattice. A beam of thermal neutrons falling on a crystal therefore gives rise to diffraction phenomena analogous to those observed with X-rays.

Let us now examine the order of magnitude of the de Broglie wavelengths associated with electrons ($m_e \simeq 0.9 \times 10^{-30}$ kg). If one accelerates an electron beam through a potential difference V (expressed in volts), one gives the electrons a kinetic energy:

$$E = qV = 1.6 \times 10^{-19}\, V \text{ joule} \tag{6}$$

($q = 1.6 \times 10^{-19}$ coulomb is the electron charge.) Since $E = \dfrac{p^2}{2m_e}$, the associated wavelength is equal to :

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e E}} \tag{7}$$

that is, numerically:

$$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 0.9 \times 10^{-30} \times 1.6 \times 10^{-19} V}} \text{ meter}$$

$$\simeq \frac{12.3}{\sqrt{V}} \text{ Å} \tag{8}$$

With potential differences of several hundreds of volts, one again obtains wavelengths comparable to those of X-rays, and electron diffraction phenomena can be observed with crystals or crystalline powders.

The large accelerators which are currently available are able to impart considerable energy to particles. This takes us out of the non-relativistic domain to which we have thus far confined ourselves. For example, electron beams are easily obtained for which the energy exceeds 1 GeV★ = 10^9 eV (1 eV = 1 electron-volt = 1.6×10^{-19} joule), while the electron rest mass is equal to $m_e c^2 \simeq 0.5 \times 10^6$ eV. This means that the corresponding speed is very close to the speed of light c. Consequently, the non-relativistic quantum mechanics which we are studying here does not apply. However, the relations:

$$E = h\nu \tag{9-a}$$

$$\lambda = \frac{h}{p} \tag{9-b}$$

remain valid in the relativistic domain. On the other hand, relation (7) must be modified since, relativistically, the energy E of a particle of rest mass m_0 is no longer $p^2/2m_0$, but instead:

$$E = \sqrt{p^2c^2 + m_0^2c^4} \tag{10}$$

In the example considered above (an electron of energy 1 GeV), $m_e c^2$ is negligible compared to E, and we obtain:

$$\lambda \simeq \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-10}} \text{ m} = 1.2 \times 10^{-15} \text{ m} = 1.2 \text{ fermi} \tag{11}$$

(1 fermi = 10^{-15} m). With electrons accelerated in this way, one can explore the structure of atomic nuclei and, in particular, the structure of the proton; nuclear dimensions are of the order of a fermi.

COMMENTS:

(*i*) We want to point out a common error in the calculation of the wavelength of a material particle of mass $m_0 \neq 0$, whose energy E is known. This error consists of calculating the frequency ν using (9-a) and, then, by analogy with electromagnetic waves, of taking c/ν for the de Broglie wavelength. Obviously, the correct reasoning consists of calculating, for example from (10) $\left(\text{or, in the non-relativistic domain, from the relation } E = \frac{p^2}{2m}\right)$ the momentum p associated with the energy E and then using (9-b) to find λ.

(*ii*) According to (9-a), the frequency ν depends on the origin chosen for the energies. The same is true for the phase velocity $V_\varphi = \frac{\omega}{k} = \nu\lambda$. Note, on the other hand, that the group velocity $V_G = \frac{d\omega}{dk} = 2\pi\frac{d\nu}{dk}$ does not depend on the choice of the energy origin. This is important in the physical interpretation of V_G.