Fundamentals of Spectroscopy for Optical Remote Sensing
Homework #7 (Spectral Linewidth and Lineshape)

1. The beam of a monochromatic laser passes through an absorbing atomic vapor with path length L = 5 cm. If the laser frequency is tuned to the center of an absorbing transition $|i\rangle \rightarrow |k\rangle$ with absorption cross-section $\sigma_0 = 10^{-14} cm^2$, the attenuation of the transmitted laser intensity is 10%. Calculate the atomic density N_i in the absorbing level $|i\rangle$.

2. For a spontaneous transition from the upper level $|k\rangle$ to the lower level $|i\rangle$,

   (1) If the radiative lifetime of $|k\rangle$ is 61.0 ns and the $|i\rangle$ is the ground state, derive the natural linewidth of this transition $\Delta\nu_n$.

   (2) If the radiative lifetime of $|k\rangle$ is 10.0 ns and the $|i\rangle$ is not the ground state but has a radiative lifetime of 50.0 ns, derive the natural linewidth of this transition $\Delta\nu_n$.

3. Derive the Doppler linewidth for the following spectral lines:
   (1) Vacuum ultraviolet (UV): for the Lyman α line (2p → 1s transition in the hydrogen atom) in a discharge with temperature T = 1500 K, molecular weight M = 1, wavelength λ = 121.6 nm, derive the Doppler linewidth $\Delta\nu_D$ in frequency domain.
   (2) Visible spectral region: for the Na D line (3p → 3s transition of the Na atom) in the mesopause region at temperature T = 200 K, λ = 589.1583 nm, derive the Doppler linewidth $\Delta\nu_D$.
   (3) Infrared region: for a vibrational transition between two rovibronic levels of the CO_2 molecule in a CO_2 cell at temperature T = 500 K, λ = 10 μm, derive the Doppler linewidth $\Delta\nu_D$.

4. A beam of NO_2 molecules with velocity $\bar{\upsilon}$ = 1000 m/s passes through a focused laser beam with a waist w = 0.1 mm. Please derive the transit time broadening $\Delta\nu_{tt}$.

5. An excited atom with spontaneous lifetime τ suffers quenching collisions.
   (1) Show that the line profile stays Lorentzian and doubles its linewidth if the mean time between two collisions is $\bar{t}_c = \tau$.
   (2) Calculate the pressure of N_2 molecules at T = 400 K for which $\bar{t}_c = \tau$ for collisions Na*+N_2 with the quenching cross section $\sigma_a = 4 \times 10^{-15}$ cm^2. (Note: Na* means the Na in an excited state.)

6. **Na Faraday filter vs. Na-DEMOF** -- A Na Faraday filter is an atomic resonance filter based on the resonant Faraday effect. A Na vapor cell is placed within a permanent magnetic field between two crossed polarizers (see Fig. 1). The magnetic field is oriented parallel to the optical axis. It will pass the light with frequency on the resonance of Na transition line (e.g., D_2), while block the light with

frequency outside the resonance line. Note: the Na Faraday filter is the device that made the daytime measurements of Na lidar possible since it cuts down the solar background so well (*see Laser Remote Sensing book Chapter 5 by Chu and Papen [2005]*).

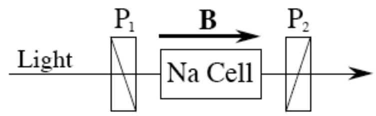

Fig. 1. Faraday filter setup

The Na double-edge magneto-optic filter (Na-DEMOF) setup is similar to the Faraday filter used for Na lidar daytime observations: a Na vapor cell is placed within a longitudinal magnetic field B between two polarizers. The difference is a quarter-wave plate added between the Na cell and the second polarizer for the Na-DEMOF (see Fig. 2). The Na-DEMOF is used as a double-edge filter to enable simultaneous wind and temperature measurements with 3-frequency Na Doppler lidar in the lower atmosphere using Rayleigh or Mie scattering (*see [Huang et al., 2009a,b] reference papers*).

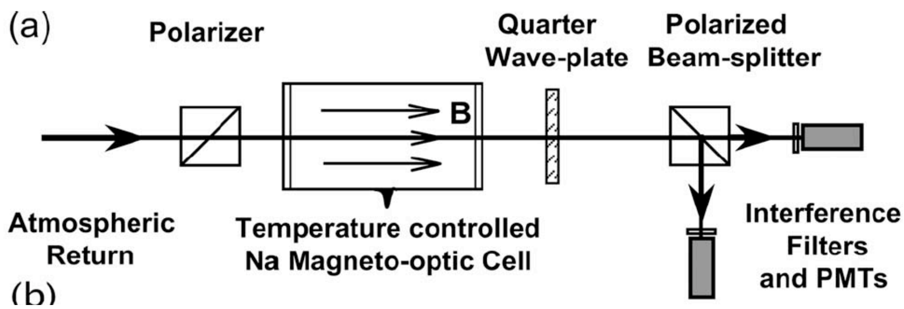

Fig. 2. Na-DEMOF setup

Despite the similarity in the setup, the Faraday filter and the Na-DEMOF use different aspects of the Na resonance—the anomalous dispersion for the former and the absorption for the latter. **Please explain the principle of Faraday filter and the principle of Na-DEMOF.**

(*You may consider from the aspects of dispersion relations and Zeeman splitting of energy levels to explain how the Faraday filter rotates light polarization so selectively passes light with resonant frequency while blocks the others. Draw some diagrams and show some related equations to help the explanation. You may consider from the Na absorption under Zeeman splitting how the two edge filters are formed with the Na vapor cell under magnetic field.*)