1. Prove that the light emitted from a certain $J_a \rightarrow J_b$ transition is unpolarized and isotropic, provided the populations of all the upper $M_a$ levels are the same.

2. Calculate the oscillator strength for the following transitions:
   (1) Calculate the oscillator strengths for the absorption transitions from $3^2S_{1/2} \rightarrow 3^2P_{3/2}$ (D$_2$ line) and from $3^2S_{1/2} \rightarrow 3^2P_{1/2}$ (D$_1$ line) of a Na atom. Why is the D$_1$ oscillator strength smaller than D$_2$ line? Related Na parameters are the radiative lifetime ($\tau = 16.23$ ns) of the excited state $3^2P_{3/2}$ and the D$_2$ wavelength $\lambda = 589.1583$ nm; the radiative lifetime ($\tau = 16.29$ ns) of the excited state $3^2P_{1/2}$ and the D$_1$ wavelength $\lambda = 589.7558$ nm.
   (2) Calculate the oscillator strength for the absorption transition from a $^5D_4 \rightarrow ^5F_5$ of an Fe atom. Compare this Fe oscillator strength to the Na oscillator strengths calculated above, what is the main reason for the Fe oscillator strength to be much smaller? Related Fe parameters are the radiative lifetime ($\tau = 61.0$ ns) of the excited state $^5F_5$ and the Fe transition wavelength $\lambda = 372.0993$ nm.

3. An excited atomic level $|E_K\rangle$ is connected with three lower levels $E_n|n\rangle$ by radiative transitions with spontaneous probabilities $A_{k1}=3\times10^7$ s$^{-1}$, $A_{k2}=1\times10^7$ s$^{-1}$, and $A_{k3}=5\times10^7$ s$^{-1}$, and connected with the ground state $|0\rangle$ by $A_{k0}=4\times10^7$ s$^{-1}$.
   (1) Calculate the spontaneous lifetime $\tau_k$ and the relative populations $N_n/N_k$ under cw excitation of $|E_k\rangle$, when $\tau_1 = 500$ ns, $\tau_2 = 6$ ns, and $\tau_3 = 10$ ns (radiative lifetime of three lower levels).
   (2) Determine the Einstein coefficient $B_{0k}$ for the excitation $|0\rangle \rightarrow |k\rangle$ from the ground state $|0\rangle$ with $\tau_0 = \infty$ and with the statistical weights $g_0 = 1$ and $g_1 = 3$. At which spectral energy density $\rho_\nu$ is the induced absorption rate equal to the spontaneous decay rate of level $|E_k\rangle$? What is the intensity of a laser with a bandwidth of 10 MHz at this radiation density?
   (3) How large is the absorption cross section $\sigma_{0k}$ if the absorption linewidth is solely determined by the lifetime of the upper level?

Note: (2) and (3) are optional. You may work on them to get extra points.

4. Let us consider the energy level structure of a sodium (Na) atom only to the fine structure. Place this Na atom in a weak magnetic field, thus there will be Zeeman splitting to the ground state (3S) and the first excited state (3P). Refer to HWK #4, problem No. 2. Draw the energy level diagram of the Na ground state and first excited state including fine structure and Zeeman splitting (using $m_J$ to mark the magnetic energy levels). Please pay attention to the order of the magnetic energy levels.
   (1) Given the selection rules for electric dipole transitions:

   $\Delta n$ is arbitrary, $\Delta S = 0$, $\Delta L = 0$, $\pm 1$, $\Delta J = 0$, $\pm 1$, and $\Delta m_J = 0$, $\pm 1$,

Please determine how many transition lines are allowable from $^2P_{1/2}$ to the ground state $^2S_{1/2}$ and from $^2P_{3/2}$ to the ground state $^2S_{1/2}$, and draw the transition lines on the energy level diagram.
   (2) Given the photon polarization rules in the electric dipole transitions: $\Delta m = 0$ gives linear polarization ($\pi$), and $\Delta m = \pm 1$ gives circular polarization ($\sigma$), please use $\pi$ and $\sigma$ mark the polarization properties for each allowable transition lines. How many linearly polarized lines and how many...
circularly polarized lines do you get for \( ^2P_{1/2} \) to \( ^2S_{1/2} \) and \( ^2P_{3/2} \) to \( ^2S_{1/2} \), respectively?

5. Please do a brief research on the Fabry-Perot Interferometer (FPI) measurements of MLT wind using airglow OI 5577 Å, 6300 Å, and 6356 Å lines. Try to address the following issues:
   (1) Draw energy diagram of OI and explain how the diagram is formed and what kind of transition each line belongs to.
   (2) Try to find approximate transition rate for each line.
   (3) Explain the principle of FPI to measure wind from these airglow signals.