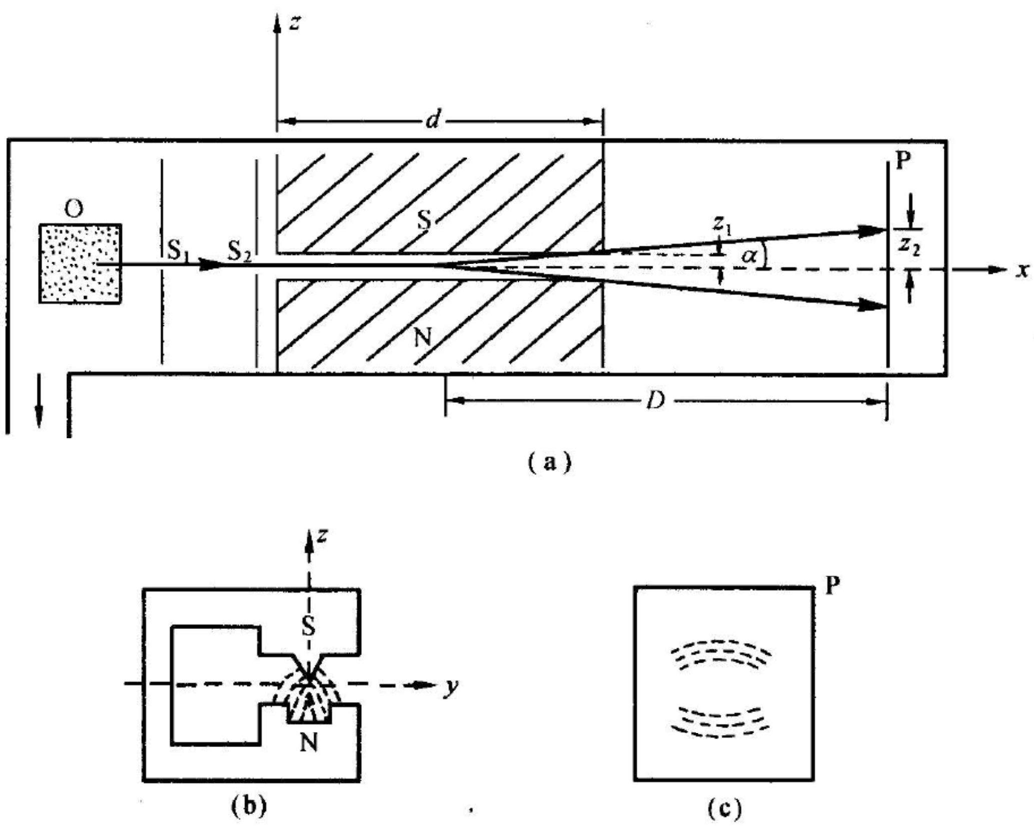
Fundamentals of Spectroscopy for Optical Remote Sensing
Homework #3 (Atomic Structure)

1. Assume an atom has the total spin angular momentum number S=1 and the total orbital angular momentum number L = 3 of electrons. If there exists L-S coupling in this atom, please write down (1) all possible total angular momentum number J for the electrons, (2) the atomic energy state terms, and (3) the possible magnetic quantum number m_J for each J.
2. For atomic states $^2S_{1/2}$, $^2P_{1/2}$, $^2P_{3/2}$, $^2D_{3/2}$, and $^2D_{5/2}$, please (1) calculate the Lande g-factor g_J and then (2) derive all possible z-components $m_J g_J$ for each state. You may list them in a table.
3. For an atom in the energy state $^2D_{3/2}$, please drive the atomic magnetic moment $\vec{\mu}$ and the possible values of its projection in z direction, i.e., μ_z. You may express the results in the unit of Bohr magneton μ_B.
4. In the Stern-Gerlach experiment (see figure below), a narrow beam of silver atoms in the ground state ($^2S_{1/2}$) pass through an inhomogeneous magnetic field (perpendicular to the atomic beam x-direction) and strike on a screen. The magneton longitudinal range is d = 10 cm, the range from the magneton's center to the screen is D = 25 cm. The silver atoms have speed of 400 m/s, and the pattern splitting on the screen is 2.0 mm. How much is the gradient of the magnetic field along z-direction (i.e., $\frac{\partial B_z}{\partial z}$)?



5. Fine structure and hyperfine structure of hydrogen atoms.
   (1) Calculate the energy levels of n = 1, 2, and 3 for hydrogen atom when only considering the electrostatic interaction (Coulomb force) between the nucleus and the electron. The potential energy at infinite is zero.

(2) Calculate the fine structure for each energy level of n = 1, 2, and 3. The relativity mass correction (ΔE_m), the Darwin term (ΔE_d), and the spin-orbit coupling (ΔE_{ls}) should be calculated, and then the sum ΔE of these three terms should be derived. Please show the procedures how you do the calculation, and then put the results into a table.

(3) Draw a diagram to show the coarse and fine structures of the energy levels (only the final results for the fine structure). Please mark in the energy level diagram the state symbols and the shift relative to the coarse energy levels (n = 1, 2, 3). Use Joule and MHz as the energy unit.

(4) Derive the hyperfine structure for the hydrogen ground state, and calculate the hyperfine splitting between the hyperfine states. Convert the energy split to frequency and wavelength.

6. Consider a system of angular momentum J. We confine ourselves in this exercise to a three-dimensional subspace, spanned by the three kets |+1>, |0>, |-1>, common eigenstates of J^2 (eigenvalue $2\hbar^2$) and J_z (eigenvalues $+\hbar, 0, -\hbar$). The Hamiltonian H_0 of the system is

$$H_0 = aJ_z + \frac{b}{\hbar}J_z^2$$

where a and b are two positive constants, which have the dimensions of an angular frequency. What are the energy levels of the system? For what value of the ratio b/a is there degeneracy?

7. For the time-independent perturbation theory, please derive the two equations on lecture notes page II.35 for 1st order correction of the wave function and 2nd order correction of the energy eigenvalue (in the case of non-degenerate states).