

§5.3. Multielectron Atoms

For atoms containing more than one electron, the interactions that we need to consider, in addition to the considerations in hydrogen atom or hydrogen-like ions, are the electrostatic interactions between electrons and the magnetic interactions between electrons.

* The electrostatic interactions come from the Coulomb repulsion force between electrons (in contrast to the Coulomb attraction force between the nucleus and the electrons).

* The magnetic interactions come from the fact that each electron has orbital and spin angular momenta, so has corresponding magnetic moments. These angular momenta (magnetic moments) couple with each other, forming the magnetic interactions. For example, if an atom has two electrons with orbital and spin angular momenta quantum numbers

l_1, S_1, l_2, S_2 . Then there are six kinds of magnetic interactions:

$G_1(S_1 S_2), G_2(l_1 l_2), G_3(l_1 S_1), G_4(l_2 S_2), G_5(l_1 S_2), G_6(l_2 S_1)$

which means spin-spin, orbit-orbit coupling between two electrons (G_1, G_2)

spin-orbit coupling for each individual electron (G_3, G_4)

one electron orbit - another electron spin coupling (G_5, G_6).

Usually, G_5 and G_6 are weak, so can be ignored.

* When atoms contain more than one electron, even for the simplest ones, Helium (He), Schrödinger equation cannot be solved directly, either analytically or numerically. For this reason, the multielectron atomic structure must be based on some approximation model. Here it is necessary to estimate the magnitudes for different interactions so that we can treat the major interactions first, and then treat smaller forces as perturbations to the major interactions.

The electrostatic interactions between the nucleus and the electron and between electrons are on the order of magnitude of

$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{|\vec{r}|} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{|\vec{r}_1 - \vec{r}_2|}$$

$$\approx \frac{1}{4\pi\epsilon_0} \frac{e^2}{a_1} \approx \frac{1.44 \text{ nm} \cdot \text{eV}}{0.053 \text{ nm}} \approx 28 \text{ eV}.$$

Where $a_1 \equiv r_1 = \frac{4\pi\epsilon_0 \hbar^2}{\mu e^2}$ is the first Bohr radius.

The magnetic interactions between electrons or between the spin and orbit of the same electron are about 10^{-3} eV.

Therefore, the electrostatic interaction between electrons is much larger than magnetic interactions, and is in the same order of the electrostatic interaction between nucleus and electron. Thus, we must consider the electrons' electrostatic interaction with the nucleus and electron interaction together.

A suitable schematic treatment is one in which the concept of the individual state of an electron in an atom is accepted, and the state of an atom as a whole is determined by the set of the states of the electrons, taking into account their interaction. In the limit of this approximation, one succeeds in obtaining general information on the system of energy levels possible for a given atom, and on the relative position and grouping of the levels. Also in this limit of approximation, selection rules for radiative transitions are established, which enable one to predict the structure of the spectrum for each element.

1. Central Field Approximation.

Central Field Approximation is the starting point for treating multielectron atoms. The fundamental assumption is that each electron moves independently in a certain effective centrally symmetric field created by the nucleus and all the other electrons. Even without knowing the detailed form of this central field, many results can be obtained on the basis of the general theory of the motion of a particle in the centrally symmetric field.

A more detailed treatment requires a consideration of the noncentral part of the electrostatic interaction between electrons, and also of magnetic interactions, in particular spin-orbit interaction. These interactions are usually considered within perturbation theory as small corrections to the centrally symmetric field. As is known, a perturbation does not alter the number of possible states of a system.

For an atom with nucleus charge Ze and N electrons, the overall atom Hamiltonian operator \hat{H} (only consider the electrostatic interaction) is given by

$$\hat{H} = + \sum_{i=1}^N \frac{\hat{p}_i^2}{2\mu} - \sum_{i=1}^N \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_i} + \sum_{i>j=1}^N \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_{ij}}$$

where $\sum_{i=1}^N$ is to take sum for every electron, \hat{p}_i is the momentum for each electron, $\mu = \frac{m_e M}{m_e + M}$ is the reduced mass of electron (M is the nuclear mass), r_i is the distance of the i th electron from the nucleus, r_{ij} is the

mutual distance of the i th and j th electrons.

The first term is the sum of all electrons kinetic energy; the second term is the electrostatic interaction between the nucleus and each electron. The third term is the electrostatic interaction between electrons. Notice that among electrons, the electrostatic interaction is Coulomb repulsion force, so the potential energy is positive, i.e., $+\sum_{i>j=1}^N \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_{ij}}$. This is opposite to the Coulomb attraction interaction between the nucleus and the electrons: $-\sum_{i=1}^N \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_i}$.

The electrostatic interaction between electrons is usually noncentral field, and every electron motion is related with other electrons. Such complicated interactions and motions have no direct solution. We must simplify it using two assumptions. (1) Each electron moves independently in the mean field produced by nucleus and other electrons; (2) the mean field is a centrally symmetric electric field. Let the i th electron experience the central field $U_i(r_i)$,

then the atomic Hamiltonian operator is simplified to

$$\hat{H} = \sum_{i=1}^N \frac{\hat{p}_i^2}{2\mu} - \sum_{i=1}^N \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_i} + \sum_i U_i(r_i)$$

For each electron, the time-independent Schrödinger equation (i.e., the energy eigen-value equation) is

$$\left[\frac{\hat{p}_i^2}{2\mu} - \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_i} + U_i(r_i) \right] u_i = E_i u_i$$

where u_i is the wave function for each electron.

The total wave function of an atom is

$$\Phi = u_1(\vec{r}_1) u_2(\vec{r}_2) \dots u_N(\vec{r}_N),$$

and the total energy of the atom is (under central field approximation)

$$E = \sum_{i=1}^N E_i.$$

The key step is to solve the equation for each individual electron. Since $U_i(r_i)$ is only dependent on r_i , but independent of θ_i, ϕ_i , we can apply the separation of variables to solve the equation. Let

$$u_i(\vec{r}_i) = R_i(r_i) Y_i(\theta_i, \phi_i),$$

then we obtain two equations:

$$\left\{ \begin{array}{l} \left[-\frac{\hbar^2}{2Mr_i^2} \frac{d}{dr_i} \left(r_i^2 \frac{d}{dr_i} \right) + \frac{\alpha_i \hbar^2}{2Mr_i^2} - \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_i} + U_i(r_i) \right] R_i(r_i) \\ = E_i R_i(r_i) \\ \Delta_i^2 Y_i(\theta_i, \phi_i) = \alpha_i \hbar^2 Y_i(\theta_i, \phi_i) \end{array} \right.$$

Compared to hydrogen equation, we only add $U_i(r_i)$ in the first equation. Similar to hydrogen equation solutions,

the solution is $u_i = R_{n_i l_i}(r_i) Y_{l_i m_i}(\theta_i, \phi_i)$

where n_i, l_i, m_i are the principal quantum number, orbital angular momentum quantum number, and orbital magnetic quantum number of the i th electron. $Y_{l_i m_i}$ is universal for all

atoms, and $\alpha_i = l_i(l_i + 1)$, $l_i = 0, 1, 2, \dots, n_i - 1$

$$|m_i| \leq l_i, \quad m_i = l_i, l_i - 1, \dots, -l_i$$

* Similar to hydrogen, the quantum number l_i determines the orbital angular momentum of the electron, in the sense of

$$\hat{L}_i^2 u_i = l_i(l_i + 1) \hbar^2 u_i,$$

i.e., the eigenvalue of the square of orbital angular momentum is $l_i(l_i + 1) \hbar^2$.

* The magnetic quantum number m_i , as hydrogen, determines the z -component of angular momentum along the z -axis of coordinates, i.e., $L_z = m_i \hbar$. m_i represents the spatial orientation of the orbital angular momentum.

* Even with the central field potential, it is difficult to solve the $R_i(r_i)$ equation that also determines the energy eigenvalues E_i . In QM, approximation methods are used to solve the equation (mainly deriving the central field $U_i(r_i)$ and the individual electron wave function $u_i(\vec{r}_i) = R_i(r_i) Y_i(\theta_i, \phi_i)$).

A popular method is the so-called self-consistent-field approximation, or Hartree-Fock method. You may refer to the book "Quantum Theory of Atomic Structure" by John C. Slater (1960) for details. We will skip the details but introduce its results, mainly on the energy eigenvalues E_i .

* The main results for central field approximation include

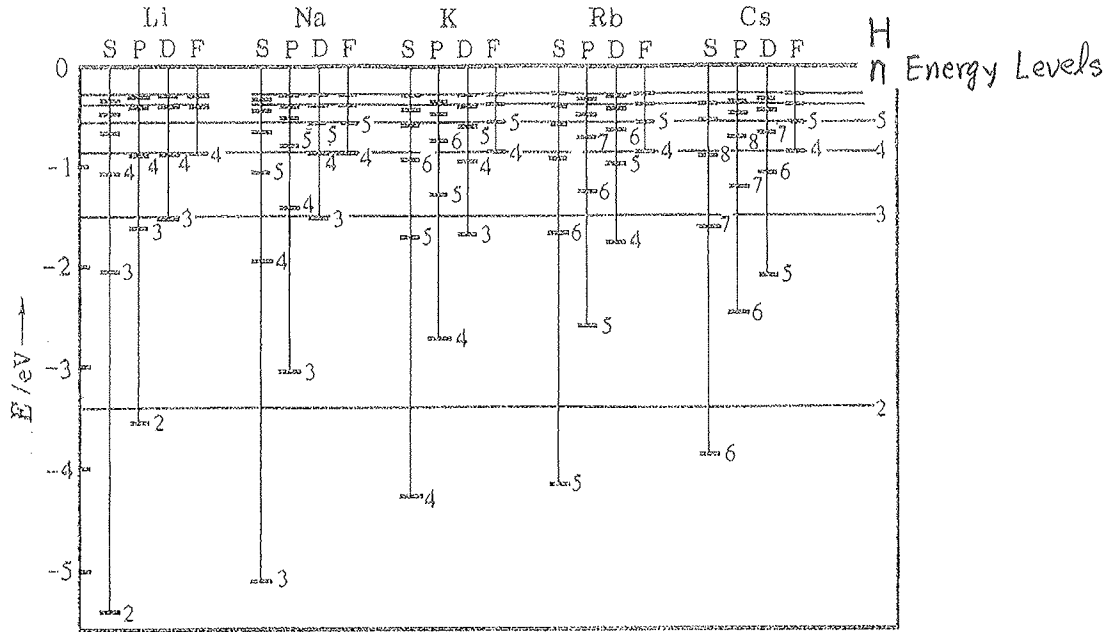
- ① Similar to hydrogen, the energy eigenvalues are independent of m . The independence of m for the energy has a simple physical meaning: In a central field, all directions in space are equivalent, and therefore the energy cannot depend on the

Spatial orientation of the angular momentum.

- ② Unlike the case of hydrogen, the energy from central field approximation depends on both n_i and l_i . FYI, the independence of l for the energy in hydrogen-like ions, is a specific property of the Coulomb field, and does not occur in the general case of a centrally symmetric field. In other words, Coulomb field potential ($\sim \frac{1}{r}$) is a special central field, and it leads to the energy eigenvalue E_n only depends on the principal quantum number n , but independent of l . However, central field does not have to be Coulomb form, i.e.; it contains other forms, such as $U(r) \sim r$, or $U(r) \sim r^2$, or $U(r) \sim \frac{1}{r^2}$, etc, as long as $U(r)$ is spherically symmetric. Such non-Coulomb central field will cause the energy eigenvalues to depend on l . This comes from the eigenvalue equation of $R_i(r_i) \rightarrow R_{n_i l_i}(r_i)$ and the radial wave function.
- ③ For the same n_i , energy levels with larger l_i are higher.
For the same l_i , energy levels with larger n_i are higher.
- ④ Each individual electron is characterized by three quantum numbers: n_i , l_i , and m_i .

(Please see an example in the next page)

Example:



Energy Levels of Alkali Atoms

As stated above, the central field approximation tells us that

- ① The energy levels depend on both n and l quantum numbers.
- ② For the same n , higher l corresponds to higher energy levels, e.g., for $n=3$ in the Li atom,

$$3s < 3p < 3d$$

- ③ For the same l , larger n corresponds to higher energy levels. e.g., for $l=0$ (S states),

$$2s < 3s < 4s < 5s$$

Plotted on the far rightmost column is the energy levels of hydrogen, as a comparison. For H, the $n=1$ level is not shown, but we recall the ground state of H ($n=1, l=0$), has energy of -13.6 eV , much lower than alkali atoms. This means H atom is much more stable because you need much more energy to reject the electron (ionization energy).

2. Electron Spin Quantum Number \Rightarrow Full Description of Electron States

Besides n_i , l_i , and m_i , we must consider the electron spin as we did in the hydrogen. Each electron has a spin angular momentum represented by spin quantum number $S_i = \frac{1}{2}$. Corresponding to S , we have the spin magnetic quantum number m_{s_i} , representing the component of spin angular momentum along the z -axis.

Thus, we have five quantum numbers to describe each electron state: n, l, S, m_l, m_s .

(Note: we remove i in the subscript, as we know we are talking about each electron now; we use m_l to replace the m_i to represent the z -component of l)

Since electron's spin is always $S = \frac{1}{2}$, the real distinguishable quantum numbers are the four quantum numbers

$$n, l, m_l, m_s,$$

We use these four quantum numbers to describe the state of an electron as $|n l m_l m_s\rangle$.

From these quantum numbers, let us derive the total stationary state number corresponding to each n :

① For each n , $l = 0, 1, 2, \dots, n-1$ (i.e., n of l)

② For each l , $m_l = l, l-1, \dots, -l$ (i.e., $2l+1$ of m_l)

③ For each m_l , $m_s = \pm \frac{1}{2}$, (i.e., 2 of m_s)

\therefore The total number of possible stationary states is

$$N_n = \sum_{l=0}^{n-1} 2(2l+1) = 2n^2.$$

3. Pauli Exclusion Principle and Atomic Shell Structure

Pauli Exclusion Principle is a fundamental principle in atomic structure theory. The general statement of this principle is that for any system consisting of Fermions (particles with spin of $\frac{1}{2}\hbar$ or odd times of $\frac{1}{2}\hbar$, e.g., electrons, protons, and neutrons), there cannot be more than one Fermion in the same state of $|n, l, m_l, m_s\rangle$. Therefore, each state $|n, l, m_l, m_s\rangle$ can have a maximum of one electron or none.

For each n , the maximum number of possible stationary state is $2n^2$. So maximum $2n^2$ electrons can have the same principal quantum number n .

Example: For $n=1$, maximum ($2n^2 = 2 \times 1^2 = 2$) 2 electrons can have $n=1$. This is because:

for $n=1$, we must have $l=0$ (because $l=0, 1, \dots, n-1$)

So $m_l=0$. Since $m_s = \pm \frac{1}{2}$, the possible states are

$|n, l, m_l, m_s\rangle = |1, 0, 0, \frac{1}{2}\rangle$ and $|1, 0, 0, -\frac{1}{2}\rangle$.

Therefore, only 2 electrons can be in these two states.

If an atom has more than 2 electrons, then the other electrons must go $n > 1$, e.g., $n=2$ or 3 . Let us calculate for the first a few n , the maximum allowable electron numbers:

$$n = 1, 2, 3, 4, 5, 6, 7$$

$$2n^2 = 2, 8, 18, 32, 50, 72, 98$$

In atomic structure theory, we name the principal quantum number n corresponding to each main shell.

$$n = 1, 2, 3, 4, 5, 6, \dots$$

$$\text{Main shell: } K, L, M, N, O, P, \dots$$

But please be aware that the shell is just a description to the principal quantum number n that mainly determines the majority part of energy, and roughly describes the distance of the electrons relative to the nucleus. Electrons do not really move on some shells within the atom, because we know for Quantum Mechanics, electrons have probability to occur in nearly every position inside the atom.

For each n , each $l = 0, 1, 2, \dots, n-1$ forms a sub-shell. The maximum allowable electron number for a l sub-shell is $2(2l+1)$, because of the $(2l+1)$ of m_l , and 2 of m_s ($m_s = \pm \frac{1}{2}$).

$$\therefore l=0, \text{ s shell, } 2(2l+1) = 2 \quad (\text{s})$$

$$l=1, \text{ p shell, } 2(2l+1) = 6 \quad (\text{p})$$

$$l=2, \text{ d shell, } 2(2l+1) = 10 \quad (\text{d})$$

$$l=3, \text{ f shell, } 2(2l+1) = 14 \quad (\text{f})$$

$$l=4, \text{ g shell, } 2(2l+1) = 18 \quad (\text{g})$$

$$l=5, \text{ h shell, } 2(2l+1) = 22 \quad (\text{h})$$

Note: please do not confuse these letters for denoting the orbital angular momentum with the main shell letters (K, L, M, N, \dots).

4. Electron Configuration and Periodic Table of Elements

The results of energy eigenvalue E_i derived from the central field approximation are the energy eigenvalues of each individual electron. Recall our assumption of central field approximation: Each electron moves independently in the mean field. What we really care about is the total energy of the whole atom. Under the central field approximation, the atom total energy is the sum of each electron energy:

$$E = \sum_{i=1}^N E_i.$$

Since each electron energy E_i depends on n and l quantum numbers, the atom total energy will be determined by the arrangement (or combination or distribution) of multi-electrons' n and l values. We use $n_1 l_1, n_2 l_2, \dots, n_N l_N$ to represent each electron (N of them) states. The combination of $n_1 l_1, n_2 l_2, \dots, n_N l_N$ is called the electron configuration. In atomic structure theory, n is written as its number, while l is represented by a letter, e.g., $1s$ means ($n=1, l=0$); $2p$ means ($n=2, l=1$), etc.

When we have more than one electrons, e.g., one is in $1s$, and another is $2p$, then the electron configuration is written as $1s 2p$. If both electrons are in $1s$, we can write $1s 1s$ as $1s^2$.

Since E_i depends on $n_i l_i$, the atom total energy E is mainly determined by the assignment of Electron Configuration

$n_1 l_1 n_2 l_2 \dots n_N l_N.$

1S	1P	1D	3S	3P	3D
$\overline{1s, 3s}$	$\overline{1s, 3p}$	$\overline{1s, 3d}$	$\overline{1s, 3s}$	$\overline{1s, 3p}$	$\overline{1s, 3d}$
$\overline{1s, 2s}$	$\overline{1s, 2p}$		$\overline{1s, 2s}$	$\overline{1s, 2p}$	
$\overline{(1s)^2}$					

$$(1s)^2 \equiv 1s^2$$

He Electron Configuration versus He energy levels.

The reason that we say the atom energy levels are mainly determined by the electron configuration (not totally), is that this is under central field approximation. When other interactions (e.g., noncentral electrostatic or magnetic interactions) are considered, there will be corrections to the $E = \sum_{i=1}^N E_i$ energy levels. These corrections may result in shift and splitting.

* You may notice in the figure of Helium shown above, there is always one electron in $1s$ state. This is because it needs tremendous energy to excite both electrons out of ground states ($1s$). This is usually not achievable in experiments.

For the spectroscopy we are interested, always $1s$ electron is in Helium with the 2nd electron in various states, which determines different energy levels of the entire atom.

With the electron configuration concept, let us now consider how to fill in the main shells and sub-shells of an atom when it has multi-electrons. Two principles we must follow:

- ① Pauli Exclusion Principle: no more than 1 electron in the same state
- ② Lowest Energy Principle: electrons fill in the levels with energy as low as possible.

According to these two principles,

Electron Configuration

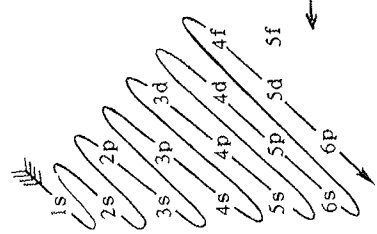
$Z = 1$, H,	one electron,	so ground state is $1s$
$Z = 2$, He,	2 electrons,	Ground state is $1s^2$
$Z = 3$, Li,	3 " ,	Ground state $1s^2 2s$
$Z = 4$, Be,	4 " ,	" $1s^2 2s^2$
$Z = 5$, B,	5 " ,	" $1s^2 2s^2 2p$
$Z = 6$, C,	6 " ,	" $1s^2 2s^2 2p^2$
$Z = 7$, N,	7 " ,	" $1s^2 2s^2 2p^3$
$Z = 8$, O,	8 " ,	" $1s^2 2s^2 2p^4$
$Z = 9$, F,	9 " ,	" $1s^2 2s^2 2p^5$
$Z = 10$, Ne,	10 " ,	" $1s^2 2s^2 2p^6$
$Z = 11$, Na,	11 " ,	" $1s^2 2s^2 2p^6 3s$
$Z = 12$, Mg,	12 " ,	" $1s^2 2s^2 2p^6 3s^2$

Tables in the next page list the electron configurations for the ground states of all elements. Also shown in the next page is the order for filling in electrons in shells/subshells, according to energy levels.

Electron Configurations for Elements

Z	Element	Electron Configuration	Atomic Ground State	Z	Element	Electron Configuration	Atomic Ground State
1	H	1s	$^2S_{1/2}$	35	Br	$[\text{Ar}](3d)^{10}(4s)^2(4p)^5$	$^2P_{3/2}$
2	He	$(1s)^2$	1S_0	36	Kr	$[\text{Ar}](3d)^{10}(4s)^2(4p)^6$	1S_0
3	Li	$[\text{He}]2s$	$^2S_{1/2}$	37	Rb	$[\text{Kr}]5s$	$^2S_{1/2}$
4	Be	$[\text{He}](2s)^2$	1S_0	38	Sr	$[\text{Kr}](5s)^2$	1S_0
5	B	$[\text{He}](2s)^2(2p)$	$^2P_{1/2}$	39	Y	$[\text{Kr}]4d(5s)^2$	$^2D_{3/2}$
6	C	$[\text{He}](2s)^2(2p)^2$	3P_0	40	Zr	$[\text{Kr}](4d)^2(5s)^2$	3F_2
7	N	$[\text{He}](2s)^2(2p)^3$	$^4S_{3/2}$	41	Nb	$[\text{Kr}](4d)^45s$	$^6D_{1/2}$
8	O	$[\text{He}](2s)^2(2p)^4$	3P_2	42	Mo	$[\text{Kr}](4d)^55s$	7S_3
9	F	$[\text{He}](2s)^2(2p)^5$	$^2P_{3/2}$	43	Tc	$[\text{Kr}](4d)^5(5s)^2$	$^6S_{5/2}$
10	Ne	$[\text{He}](2s)^2(2p)^6$	1S_0	44	Rn	$[\text{Kr}](4d)^75s$	5F_3
11	Na	$[\text{Ne}]3s$	$^2S_{1/2}$	45	Rh	$[\text{Kr}](4d)^85s$	$^4F_{9/2}$
12	Mg	$[\text{Ne}](3s)^2$	1S_0	46	Pd	$[\text{Kr}](4d)^{10}$	1S_0
13	Al	$[\text{Ne}](3s)^2(3p)$	$^2P_{1/2}$	47	Ag	$[\text{Kr}](4d)^{10}5s$	$^2S_{1/2}$
14	Si	$[\text{Ne}](3s)^2(3p)^2$	3P_0	48	Cd	$[\text{Kr}](4d)^{10}(5s)^2$	1S_0
15	P	$[\text{Ne}](3s)^2(3p)^3$	$^4S_{3/2}$	49	In	$[\text{Kr}](4d)^{10}(5s)^25p$	$^2P_{1/2}$
16	S	$[\text{Ne}](3s)^2(3p)^4$	3P_2	50	Sn	$[\text{Kr}](4d)^{10}(5s)^2(5p)^2$	3P_0
17	Cl	$[\text{Ne}](3s)^2(3p)^5$	$^2P_{3/2}$	51	Sb	$[\text{Kr}](4d)^{10}(5s)^2(5p)^3$	$^4S_{3/2}$
18	Ar	$[\text{Ne}](3s)^2(3p)^6$	1S_0	52	Te	$[\text{Kr}](4d)^{10}(5s)^2(5p)^4$	3P_2
19	K	$[\text{Ar}]4s$	$^2S_{1/2}$	53	I	$[\text{Kr}](4d)^{10}(5s)^2(5p)^5$	$^2P_{3/2}$
20	Ca	$[\text{Ar}](4s)^2$	1S_0	54	Xe	$[\text{Kr}](4d)^{10}(5s)^2(5p)^6$	1S_0
21	Sc	$[\text{Ar}]3d(4s)^2$	$^2D_{3/2}$	55	Cs	$[\text{Xe}]6s$	$^2S_{1/2}$
22	Ti	$[\text{Ar}](3d)^2(4s)^2$	3F_2	56	Ba	$[\text{Xe}](6s)^2$	1S_0
23	V	$[\text{Ar}](3d)^3(4s)^2$	$^4F_{3/2}$	57	La	$[\text{Xe}]5d(6s)^2$	$^2D_{3/2}$
24	Cr	$[\text{Ar}](3d)^44s$	7S_3	58	Ce	$[\text{Xe}]4f5d(6s)^2$	1G_4
25	Mn	$[\text{Ar}](3d)^5(4s)^2$	$^6S_{5/2}$	59	Pr	$[\text{Xe}](4f)^3(6s)^2$	$^4F_{5/2}$
26	Fe	$[\text{Ar}](3d)^6(4s)^2$	5D_4	60	Nd	$[\text{Xe}](4f)^4(6s)^2$	5I_4
27	Co	$[\text{Ar}](3d)^7(4s)^2$	$^4F_{9/2}$	61	Pm	$[\text{Xe}](4f)^5(6s)^2$	$^6H_{5/2}$
28	Ni	$[\text{Ar}](3d)^8(4s)^2$	3F_4	62	Sm	$[\text{Xe}](4f)^6(6s)^2$	7F_0
29	Cu	$[\text{Ar}](3d)^{10}4s$	$^2S_{1/2}$	63	Eu	$[\text{Xe}](4f)^7(6s)^2$	$^8S_{7/2}$
30	Zn	$[\text{Ar}](3d)^{10}(4s)^2$	$^4S_{3/2}$	64	Gd	$[\text{Xe}](4f)^75d(6s)^2$	9D_2
31	Ga	$[\text{Ar}](3d)^{10}(4s)^24p$	$^2P_{1/2}$	65	Tb	$[\text{Xe}](4f)^95d(6s)^2$	$^6H_{5/2}$
32	Ge	$[\text{Ar}](3d)^{10}(4s)^2(4p)^2$	3P_0	66	Dy	$[\text{Xe}](4f)^{10}(6s)^2$	5I_3
33	As	$[\text{Ar}](3d)^{10}(4s)^2(4p)^3$	$^4S_{3/2}$	67	Ho	$[\text{Xe}](4f)^{11}(6s)^2$	$^4I_{15/2}$
34	Se	$[\text{Ar}](3d)^{10}(4s)^2(4p)^4$	3P_2	68	Er	$[\text{Xe}](4f)^{12}(6s)^2$	3H_6

Z	Element	Electron Configuration	Atomic Ground State	Z	Element	Electron Configuration	Atomic Ground State
69	Tm	$[\text{Xe}](4f)^{13}(6s)^2$	$^2F_{7/2}$	90	Th	$[\text{Rn}](6d)^2(7s)^2$	3F_2
70	Yb	$[\text{Xe}](4f)^{14}(6s)^2$	3D_3	91	Pa	$[\text{Rn}](5f)^26d(7s)^2$	$^4K_{11/2}$
71	Lu	$[\text{Xe}](4f)^{14}5d(6s)^2$	$^2D_{3/2}$	92	U	$[\text{Rn}](5f)^36d(7s)^2$	5L_6
72	Hf	$[\text{Xe}](4f)^{14}(5d)^2(6s)^2$	3F_2	93	Np	$[\text{Rn}](5f)^46d(7s)^2$	$^6L_{11/2}$
73	Ta	$[\text{Xe}](4f)^{14}(5d)^3(6s)^2$	$^4F_{3/2}$	94	Pu	$[\text{Rn}](5f)^6(7s)^2$	7F_0
74	W	$[\text{Xe}](4f)^{14}(5d)^4(6s)^2$	6D_0	95	Am	$[\text{Rn}](5f)^7(7s)^2$	$^8S_{7/2}$
75	Re	$[\text{Xe}](4f)^{14}(5d)^5(6s)^2$	$^6S_{5/2}$	96	Cm	$[\text{Rn}](5f)^76d(7s)^2$	9D_2
76	Os	$[\text{Xe}](4f)^{14}(5d)^6(6s)^2$	5D_4	97	Bk	$[\text{Rn}](5f)^9(7s)^2$	$^8H_{17/2}$
77	Ir	$[\text{Xe}](4f)^{14}(5d)^7(6s)^2$	$^4F_{9/2}$	98	Cf	$[\text{Rn}](5f)^{10}(7s)^2$	$^5I_{18}$
78	Pt	$[\text{Xe}](4f)^{14}(5d)^96s$	3D_3	99	Es	$[\text{Rn}](5f)^{11}(7s)^2$	$^4I_{15/2}$
79	Au	$[\text{Xe}](4f)^{14}(5d)^{10}6s$	$^2S_{1/2}$	100	Fm	$[\text{Rn}](5f)^{12}(7s)^2$	3H_6
80	Hg	$[\text{Xe}](4f)^{14}(5d)^{10}(6s)^2$	1S_0	101	Md	$[\text{Rn}](5f)^{12}(7s)^2$	$^2F_{7/2}$
81	Tl	$[\text{Hg}]6p$	$^2P_{1/2}$	102	No	$[\text{Rn}](5f)^{14}(7s)^2$	1S_0
82	Pb	$[\text{Hg}](6p)^2$	3P_0	103	Lr	$[\text{Rn}](5f)^{14}6d(7s)^2$	$^2D_{3/2}$
83	Bi	$[\text{Hg}](6p)^3$	$^4S_{3/2}$	104	Rf	$[\text{Rn}](5f)^{14}(6d)^2(7s)^2?$	$^3F_2?$
84	Po	$[\text{Hg}](6p)^4$	3P_2	105	Db		
85	At	$[\text{Hg}](6p)^5$	$^2P_{3/2}$	106	Sg		
86	Rn	$[\text{Hg}](6p)^6$	1S_0	107	Bh		
87	Fr	$[\text{Rn}]7s$	$^2S_{1/2}$	108	Hs		
88	Ra	$[\text{Rn}](7s)^2$	1S_0	109	Mt		
89	Ac	$[\text{Rn}]6d(7s)^2$	$^3D_{3/2}$				



$L = 0, 1, 2, 3, 4, 5, 6, 7, 8, \dots$
 $S, P, D, F, G, H, I, K, L, \dots$

← Experimental experience of filling orders.