Fundamentals of Spectroscopy for (Optical) Remote Sensing
Homework #7

1. For a spontaneous transition from the upper level $|k\rangle$ to the lower level $|i\rangle$,

   (1) If the radiative lifetime of $|k\rangle$ is 61.0 ns and the $|i\rangle$ is the ground state, derive the natural linewidth of this transition $\Delta\nu_n$.

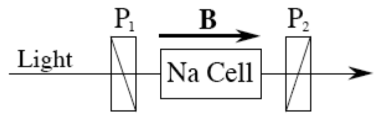
   (2) If the radiative lifetime of $|k\rangle$ is 10.0 ns and the $|i\rangle$ is not the ground state but has a radiative lifetime of 50.0 ns, derive the natural linewidth of this transition $\Delta\nu_n$.

2. Derive the Doppler linewidth for the following spectral lines:
   (1) Vacuum ultraviolet (UV): for the Lyman α line (2p → 1s transition in the hydrogen atom) in a discharge with temperature T = 1500 K, molecular weight M = 1, wavelength λ = 121.6 nm, derive the Doppler linewidth $\Delta\nu_D$ in frequency domain.
   (2) Visible spectral region: for the Na D line (3p → 3s transition of the Na atom) in the mesopause region at temperature T = 200 K, λ = 589.1583 nm, derive the Doppler linewidth $\Delta\nu_D$.
   (3) Infrared region: for a vibrational transition between two rovibronic levels of the CO_2 molecule in a CO_2 cell at temperature T = 500 K, λ = 10 μm, derive the Doppler linewidth $\Delta\nu_D$.

3. A beam of NO_2 molecules with velocity $\bar{\upsilon}$ = 1000 m/s passes through a focused laser beam with a waist w = 0.1 mm. Please derive the transit time broadening $\Delta\nu_{tt}$.

4. An excited atom with spontaneous lifetime τ suffers quenching collisions. (1) Show that the line profile stays Lorentzian and doubles its linewidth if the mean time between two collision is $\bar{t}_c = \tau$. (2) Calculate the pressure of N_2 molecules at T = 400 K for which $\bar{t}_c = \tau$ for collisions Na^*+N_2 with the quenching cross section $\sigma_a = 4 \times 10^{-15}$ cm^2. (Note: Na* means the Na in an excited state.)

5. A Na Faraday filter is an atomic resonance filter based on the resonant Faraday effect. A Na vapor cell is placed within a permanent magnetic field between two crossed polarizers (see the figure below). The magnetic field is oriented parallel to the optical axis. It will pass the light with frequency on the resonance of Na transition line (e.g., D_2), while block the light with frequency outside the resonance line.



Please explain the principle of Faraday filter (also called FADOF): from the aspects of dispersion relations and Zeeman splitting of energy levels to explain how the Faraday filter rotates light polarization so selectively passes light with resonant frequency while blocks the others. Draw some diagrams and show some related equations to help the explanation. (*Note: the Na Faraday filter is the device that made the daytime measurements of Na lidar possible since it cuts down the solar background so well*.)