Fundamentals of Spectroscopy for (Optical) Remote Sensing
Homework #6

1. From the semi-classical description and time-dependent perturbation theory, derive the expression of transition probability $P_{if}(t)$ and transition probability per unit time $W_{if}(t)$ for a radiative transition from an initial state $|\Phi_i\rangle$ to a final state $|\Phi_f\rangle$ under the interaction of a radiation field with an atom (for time t → large enough). The total Hamiltonian operation is

$$\hat{H} = \hat{H}_o + \hat{H}',$$

where $\hat{H}_o|\varphi_n\rangle = E_n|\varphi_n\rangle$, $|\Phi_n(t)\rangle = |\varphi_n\rangle e^{-iE_n t/\hbar}$, and $\hat{H}' = \hat{F}(e^{2i\omega t} + e^{-2i\omega t})$. $\hat{H}'$ is regarded as a perturbation to $\hat{H}_o$ and added to the atom at $t = 0$. Please show the detailed derivation procedure.

2. Derive the relationship between Einstein A and B coefficients from the semi-classical theory and from the full quantum treatment.

3. Prove the commutation relationship: $[\hat{x}, \hat{p}_x^2] = 2i\hbar\hat{p}_x$ and $[\hat{\vec{r}}, \hat{H}_a] = \frac{i\hbar}{\mu}\hat{\vec{p}}$, where $\hat{H}_a = \frac{\hat{\vec{p}}^2}{2\mu} + \hat{U}$. Note $\hat{U}$ is a function of $\hat{\vec{r}}$.

4. Prove that the light emitted from a certain J_a → J_b transition is unpolarized and isotropic, provided the populations of all the upper M_a levels are the same.

5. Calculate the oscillator strength for the following transitions:
(1) Calculate the oscillator strengths for the absorption transitions from $3^2S_{1/2}$ → $3^2P_{3/2}$ (D_2 line) and from $3^2S_{1/2}$ → $3^2P_{1/2}$ (D_1 line) of a Na atom. Why is the D_1 oscillator strength smaller than D_2 line?
Related Na parameters are the radiative lifetime (τ = 16.23 ns) of the excited state $3^2P_{3/2}$ and the D_2 wavelength λ = 589.1583 nm; the radiative lifetime (τ = 16.29 ns) of the excited state $3^2P_{1/2}$ and the D_1 wavelength λ = 589.7558 nm.
(2) Calculate the oscillator strength for the absorption transition from a^5D_4 → z^5F_5 of an Fe atom. Compare this Fe oscillator strength to the Na oscillator strengths calculated above, what is the main reason for the Fe oscillator strength to be much smaller? Related Fe parameters are the radiative lifetime (τ = 61.0 ns) of the excited state z^5F_5 and the Fe transition wavelength λ = 372.0993 nm.