Fundamentals of Spectroscopy for (Optical) Remote Sensing
Homework #5

1. An optically excited sodium atom Na(3P) with a spontaneous radiative lifetime τ(3P) = 16 ns is placed in a cell filled with 10 mbar nitrogen (N_2) gas at a temperature of T = 400 K.

   (1) Calculate the effective lifetime τ_{eff}(3P) if the quenching cross-section for Na(3P)-N_2 collisions is $\sigma_q = 4 \text{ X } 10^{-15}$ cm^2.

   (2) Assume the collision cross-section for Na(3P) with air molecules is the same as Na(3P)-N_2 collisions $\sigma_q = 4 \text{ X } 10^{-15}$ cm^2. In the mesopause region (~ 90 Km), the temperature is T = 200 K, and the atmosphere pressure P = 2 X 10^{-3} mbar. The molecular weight of air molecules is about 28. Calculate the probability $\frac{dP_{ki}^{collision}}{dt}$ of collision-induced transition and the effective lifetime under this condition. Can we safely ignore the quenching effect in the mesopause region for the Na fluorescence based on the computation results?

   (Please refer to our textbook Section 2.7 and Section 11.3 for the details of lifetime measurements and related knowledge for this homework problem.)

2. A sodium atom is placed in a cavity with walls at the temperature T, producing a thermal radiation field with spectral energy density $\rho(\nu)$. At what temperature T are the spontaneous and stimulated transition probabilities equal

   (a) for the transition 3P → 3S (λ = 589 nm) with τ(3P) = 16 ns;

   (b) for the hyperfine transition 3S (F = 3 → F = 2) with τ(3F) = 1 s and ν = 1772 MHz?

3. The beam of a monochromatic laser passes through an absorbing atomic vapor with path length L = 5 cm. If the laser frequency is tuned to the center of an absorbing transition $|i\rangle \rightarrow |k\rangle$ with absorption cross-section $\sigma_0 = 10^{-14} cm^2$, the attenuation of the transmitted laser intensity is 10%. Calculate the atomic density N_i in the absorbing level $|i\rangle$.

4. An excited atomic level E_K is connected with three lower levels E_i by radiative transitions with spontaneous probabilities $A_{k1}=3x10^7$ s^{-1}, $A_{k2}=1x10^7$ s^{-1}, and $A_{k3}=5x10^7$ s^{-1}.

   (1) Calculate the spontaneous lifetime τ_k and the relative populations N_i/N_k under cw excitation of E_k, when $\tau_1 = 5x10^{-7}$ s, $\tau_2 = 6x10^{-9}$ s, and $\tau_3 = 1x10^{-8}$ s.

   (2) Determine the Einstein coefficient B_{0k} for the excitation $|0\rangle \rightarrow |k\rangle$ from a ground state $|0\rangle$ with $\tau_0 = \infty$ at the wavelength λ = 400 nm.

   (3) How large is the absorption cross section σ_{0k} when the absorption line profile has a bandwidth of $\Delta\omega = 9x10^7$ Hz.

5. Let us consider a real problem in the research of Fe resonance lidar: the radiative lifetime for three excited states of ^{56}Fe.

(1) For the excited state $z^5F_5^o$, there are the following allowed transitions with spontaneous transition rates as below:

$$z^5F_5^o \to a^5D_4: \ A_{k1} = 0.162 \times 10^8 s^{-1} \qquad (\lambda = 371.993nm)$$

$$z^5F_5^o \to a^5F_5: \ A_{k2} = 5.50 \times 10^{-4} \times 10^8 s^{-1} \qquad (\lambda = 501.207nm)$$

$$z^5F_5^o \to a^5F_4: \ A_{k3} = 1.14 \times 10^{-4} \times 10^8 s^{-1} \qquad (\lambda = 512.736nm)$$

$$z^5F_5^o \to a^3F_4: \ A_{k4} = 1.8 \times 10^{-6} \times 10^8 s^{-1} \qquad (\lambda = 671.031nm)$$

Please calculate the radiative lifetime of the excited state $z^5F_5^o$.

(2) For the excited state $z^5F_4^o$, there are the following allowed transitions with spontaneous transition rates as below:

$$z^5F_4^o \to a^5D_3: \ A_{k1} = 0.142 \times 10^8 s^{-1} \qquad (\lambda = 373.713nm)$$

$$z^5F_4^o \to a^5D_4: \ A_{k2} = 0.0138 \times 10^8 s^{-1} \qquad (\lambda = 367.991nm)$$

$$z^5F_4^o \to a^5F_4: \ A_{k3} = 4.66 \times 10^{-4} \times 10^8 s^{-1} \qquad (\lambda = 505.163nm)$$

$$z^5F_4^o \to a^5F_5: \ A_{k4} = 1.39 \times 10^{-4} \times 10^8 s^{-1} \qquad (\lambda = 493.969nm)$$

$$z^5F_4^o \to a^3F_4: \ A_{k5} = 2.4 \times 10^{-6} \times 10^8 s^{-1} \qquad (\lambda = 658.122nm)$$

Please calculate the radiative lifetime of the excited state $z^5F_4^o$.

(3) For the excited state $z^5D_4^o$, there are the following allowed transitions with spontaneous transition rates as below:

$$z^5D_4^o \to a^5D_4: \ A_{k1} = 0.0970 \times 10^8 s^{-1} \qquad (\lambda = 385.991nm)$$

$$z^5D_4^o \to a^5D_3: \ A_{k2} = 0.0108 \times 10^8 s^{-1} \qquad (\lambda = 392.291nm)$$

$$z^5D_4^o \to a^5F_5: \ A_{k3} = 0.0127 \times 10^8 s^{-1} \qquad (\lambda = 526.954nm)$$

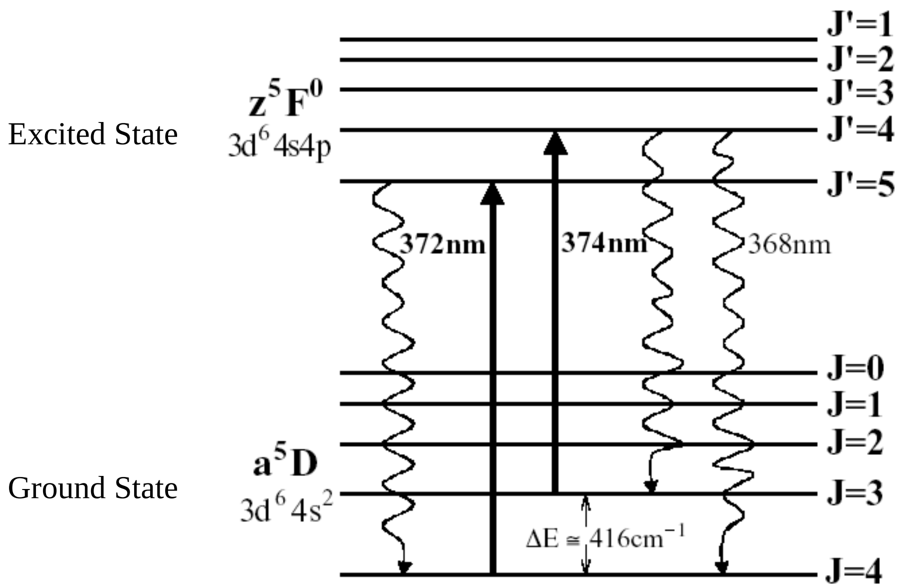
$$z^5D_4^o \to a^5F_4: \ A_{k4} = 0.00259 \times 10^8 s^{-1} \qquad (\lambda = 539.713nm)$$

$z^5D_4^o \rightarrow a^5F_3$: $A_{k5} = 2.7 \times 10^{-4} \times 10^8 s^{-1}$ $(\lambda = 550.146 nm)$

$z^5D_4^o \rightarrow a^3F_4$: $A_{k6} = 2.4 \times 10^{-6} \times 10^8 s^{-1}$ $(\lambda = 718.002 nm)$

Please calculate the radiative lifetime of the excited state $z^5D_4^o$.

(4) In spectroscopy, a term called transition branching ratio (or transition branching fraction) is defined as $R_B = \frac{A_{ki}}{A_k} = \frac{A_{ki}}{1/\tau_k}$ for the transition from $|k\rangle \rightarrow |i\rangle$, where A_{ki} is the spontaneous transition rate for $|k\rangle \rightarrow |i\rangle$ and A_k is the total spontaneous transition rate associated with the upper level $|k\rangle$. Please calculate the branching ratio R_B for the transitions of $z^5F_5^o \rightarrow a^5D_4$ (371.993 nm), $z^5F_4^o \rightarrow a^5D_3$ (373.713 nm), and $z^5D_4^o \rightarrow a^5D_4$ (385.991 nm).



Partially energy level diagram of ^{56}Fe atom.

6. Let us consider the energy level structure of a sodium (Na) atom only to the fine structure. Place this Na atom in a weak magnetic field, thus there will be Zeeman splitting to the ground state (3S) and the first excited state (3P).

(1) Draw the energy level diagram of the Na ground state and first excited state including qualitative considerations of fine structure and Zeeman splitting (using m_J to mark the magnetic energy levels). Please pay attention to the order of the magnetic energy levels.

(2) Given the selection rules for electric dipole transitions:

Δn is arbitrary, $\Delta S = 0$, $\Delta L = 0, \pm 1$, $\Delta J = 0, \pm 1$, and $\Delta m_J = 0, \pm 1$,

Please determine how many transition lines are allowable from $^2P_{1/2}$ to the ground state $^2S_{1/2}$ and from $^2P_{3/2}$ to the ground state $^2S_{1/2}$, and draw the transition lines on the energy level diagram.

(3) Given the photon polarization rules in the dipole transitions: $\Delta m = 0$ gives linear polarization (π), and $\Delta m = \pm 1$ gives circular polarization (σ), please use π and σ mark the polarization properties for each allowable transition lines. How many linearly polarized lines and how many circularly polarized lines do you get for $^2P_{1/2}$ to $^2S_{1/2}$ and $^2P_{3/2}$ to $^2S_{1/2}$, respectively?