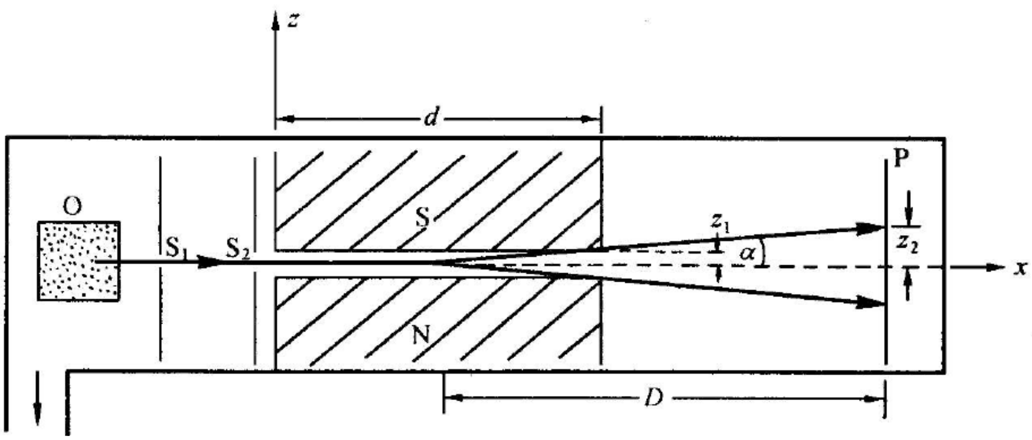
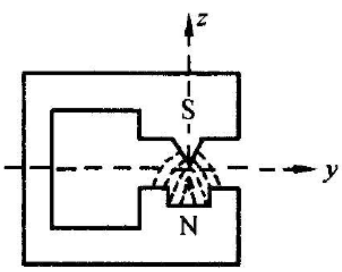
**Fundamentals of Spectroscopy for (Optical) Remote Sensing**
**Homework #3 (Atomic Spectroscopy)**

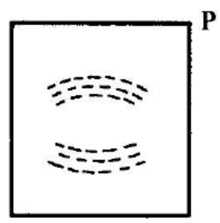
1. Assume an atom has the total spin angular momentum number S=1 and the total orbital angular momentum number L = 3 of electrons. If there exists L-S coupling in this atom, please write down (1) all possible total angular momentum number J for the electrons, (2) the atomic energy state terms, and (3) the possible magnetic quantum number $m_J$ for each J.
2. For atomic states $^2S_{1/2}$, $^2P_{1/2}$, $^2P_{3/2}$, $^2D_{3/2}$, and $^2D_{5/2}$, please (1) calculate the Lande g-factor $g_J$ and then (2) derive all possible z-components $m_J g_J$ for each state. You may list them in a table.
3. For an atom in the energy state $^2D_{3/2}$, please drive the atomic magnetic moment $\vec{\mu}$ and the possible values of its projection in z direction, i.e., $\mu_z$. You may express the results in the unit of Bohr magneton $\mu_B$.
4. In the Stern-Gerlach experiment (see figure below), a narrow beam of silver atoms in the ground state ($^2S_{1/2}$) pass through an inhomogeneous magnetic field (perpendicular to the atomic beam x-direction) and strike on a screen. The magneton longitudinal range is d = 10 cm, the range from the magneton's center to the screen is D = 25 cm. The silver atoms have speed of 400 m/s, and the pattern splitting on the screen is 2.0 mm. How much is the gradient of the magnetic field along z-direction (i.e., $\frac{\partial B_z}{\partial z}$)?



(a)



(b)



(c)

5. Fine structure and hyperfine structure of hydrogen atoms.
   (1) Calculate the energy levels of n = 1, 2, and 3 for hydrogen atom when only considering the electrostatic interaction (Coulomb force) between the nucleus and the electron. The potential energy at infinite is zero.

(2) Calculate the fine structure for each energy level of n = 1, 2, and 3. The relativity mass correction ($\Delta E_m$), the Darwin term ($\Delta E_d$), and the spin-orbit coupling ($\Delta E_{ls}$) should be calculated, and then the sum $\Delta E$ of these three terms should be derived. Please show the procedures how you do the calculation, and then put the results into a table.

(3) Draw a diagram to show the coarse and fine structures of the energy levels (only the final results for the fine structure). Please mark in the energy level diagram the state symbols and the shift relative to the coarse energy levels (n = 1, 2, 3). Use Joule and MHz as the energy unit.

(4) Derive the hyperfine structure for the hydrogen ground state, and calculate the hyperfine splitting between the hyperfine states. Convert the energy split to frequency and wavelength.

6. (1) Please calculate the isotope (mass) shift of hydrogen atoms for $^1$H, $^2$H, and $^3$H for n = 1 and 2, relative to the energy levels of $R_\infty$. Use the unit of $R_\infty hc \dfrac{m_e}{M_p}$, and put the results into a table.

(2) Draw a diagram to show the obtained isotope shifts. The main point is to show the shifts of each isotope relative to the energy levels with $R_\infty$ and which isotope has higher or lower energy levels.