Lecture 31. Lidar Data Inversion (2)

- Pre-process and Profile-process
- Main Process Procedure to Derive T and V_R Using Ratio Doppler Technique
- Derivations of n_c from narrowband resonance Doppler lidar
- Derivation of β
- Derivation of n_c from broadband resonance lidar
- Error analysis for photon noise
- Summary

Preprocess Procedure and Profile-Process Procedure for Na/Fe/K Doppler Lidar

Read Data File
↓
PMT/Discriminator Saturation Correction
↓
Chopper/Filter Correction
↓
Subtract Background
↓
Remove Range Dependence (x R^2)
↓
Add Base Altitude
↓
Estimate Rayleigh Signal @ z_R km
↓
Normalize Profile By Rayleigh Signal @ z_R km

- ❑ Read data: for each set, and calculate T, W, and n for each set
- ❑ PMT/Discriminator saturation correction
- ❑ Chopper/Filter correction

--------------------------------- Integration ----

- ❑ Background estimate and subtraction
- ❑ Range-dependence removal (xR^2, not z^2)
- ❑ Base altitude adjustment
- ❑ Take Rayleigh signal @ z_R (Rayleigh fit or Rayleigh mean)
- ❑ Rayleigh normalization $\boxed{N_N(\lambda,z) = \frac{N_S(\lambda,z) - N_B}{N_S(\lambda,z_R) - N_B}\frac{z^2}{z_R^2}}$

--------------------------------- Main Process ---

- ❑ Subtract Rayleigh signals from Na/Fe/K region after counting in the factor of T_C

Main Ideas to Derive Na T and W

- In the ratio technique, Na number density is cancelled out. So we have two ratios R_T and R_W that are independent of Na density but both dependent on T and W.

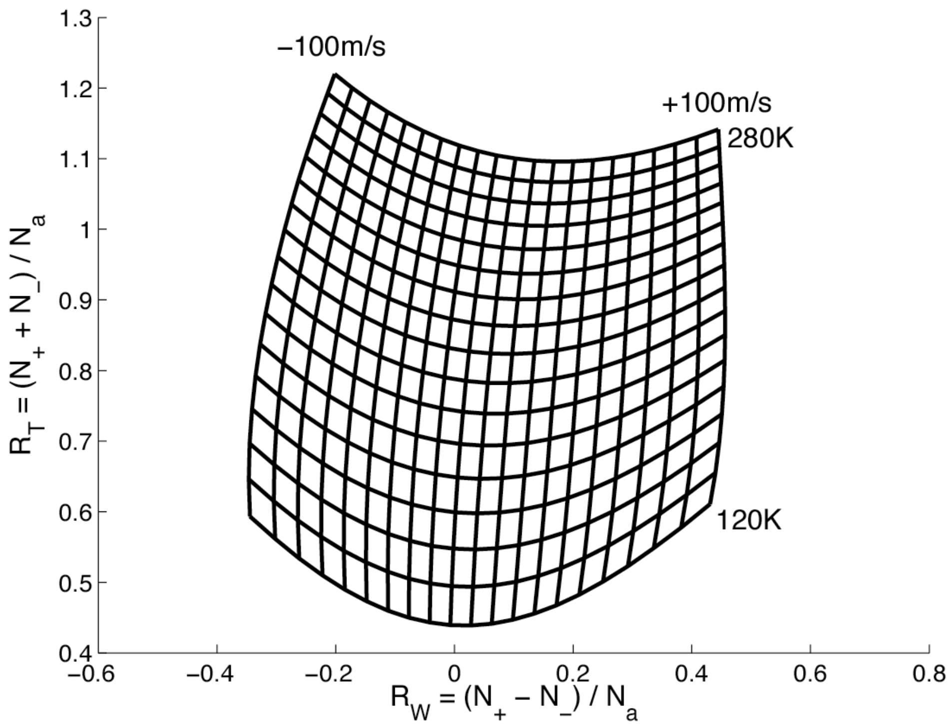
- The idea is to derive temperature and radial wind from these two ratios first, and then derive Na number density using computed temperature and wind at each altitude bin.

- To derive T and W from R_T and R_W, the basic idea is to use look-up table or iteration methods to derive them: (1) compute R_T and R_W from physics point-of-view to generate the table or calibration curves, (2) compute R_T and R_W from actual photon counts, (3) check the table or calibration curves to find the corresponding T and W. (4) If R_T and R_W are out of range, then set to nominal T and W.

- However, because the Na extinction coefficient is involved, the upper bins are related to lower bins, and extinction coefficient is related to Na density and effective cross-section. The solution is to start from the bottom of the Na layer.

Main Process Procedure

- Compute Doppler calibration curves from physics



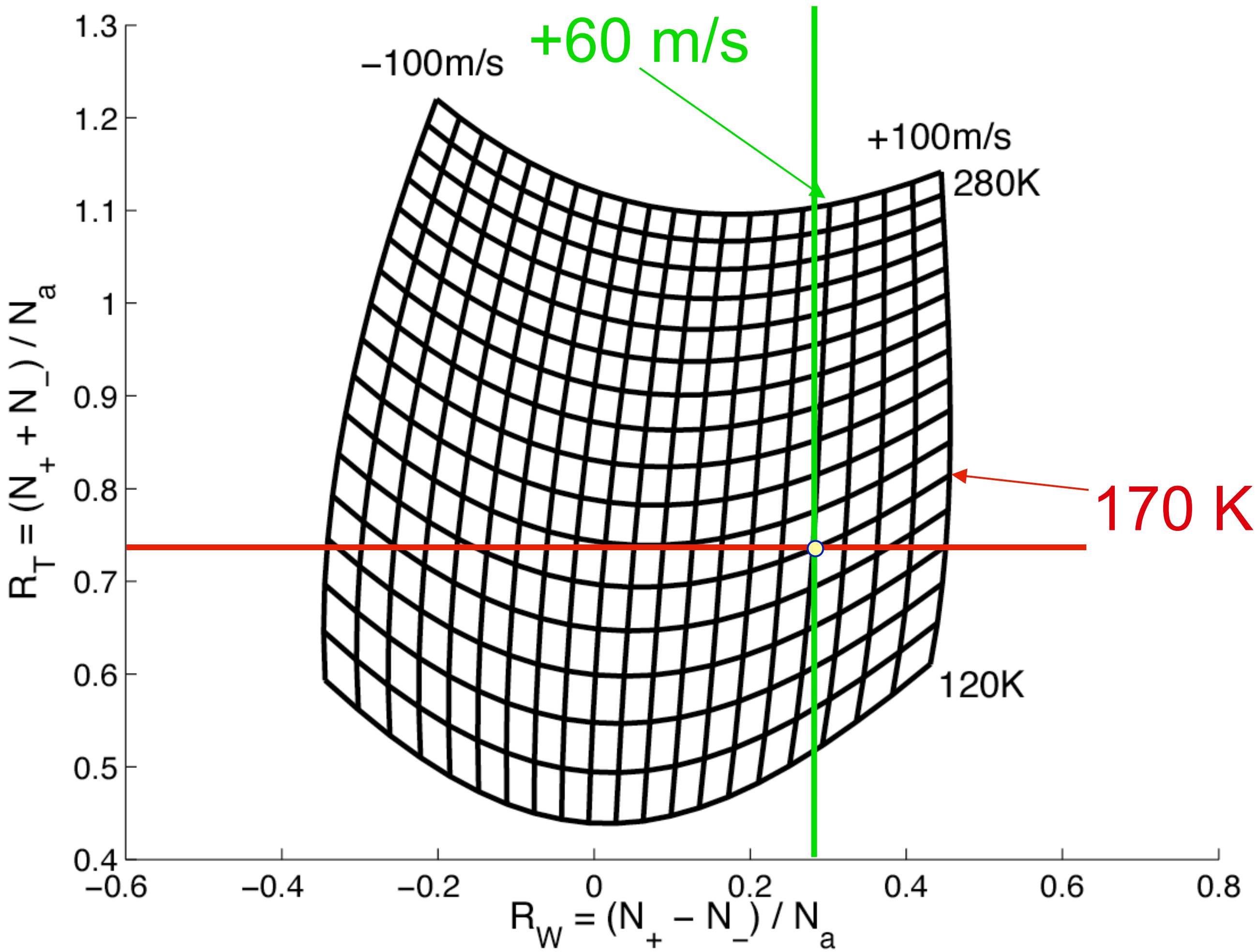
$$R_W = \frac{\sigma_{eff}(f_+, z) - \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

$$R_T = \frac{\sigma_{eff}(f_+, z) + \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

$$\sigma_{\text{eff}}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_{\text{e}}} \frac{e^2 f}{4\epsilon_0 m_{\text{e}} c} \sum_{n=1}^{6} A_n \exp\left(- \frac{\left[\nu_n - \nu\left(1 - \frac{v_{\text{R}}}{c}\right)\right]^2}{2\sigma_{\text{e}}^2} \right)$$

Main Process Procedure

- Compute actual ratios R_T and R_W from photon counts
- Look up these two ratios on the calibration curves to infer the corresponding Temperature and Wind from isoline/isogram.



Constituent Density

- Normalized Photon Count to the density estimation

$$n_c(z) = \left[\frac{N_S(\lambda,z) - N_B}{N_S(\lambda,z_R) - N_B} \cdot \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)} \right] \cdot \frac{4\pi\sigma_R(\pi,\lambda) n_R(z_R)}{\sigma_{eff}(\lambda,z) R_B(\lambda)}$$

Normalized Photon Count
From the preprocess

Temperature and wind dependent

→ we need to estimate the temperature and wind first in order to estimate the density

- The effective cross-section

$$\sigma_{\mathrm{eff}}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{e}}} \frac{e^2 f}{4\epsilon_0 m_{\mathrm{e}} c} \sum_{n=1}^{6} A_n \exp\left(- \frac{\left[\nu_n - \nu\left(1 - \frac{v_{\mathrm{R}}}{c}\right)\right]^2}{2\sigma_{\mathrm{e}}^2} \right)$$

Main Process

Load Atmosphere n_R, T, P Profiles from MSIS00

Start from Na layer bottom
$T_C (z=z_b) = 1$
Calculate Nnorm ($z=z_b$) from photon counts and MSIS number density for each freq

$$N_{Norm}(\lambda,z) = \frac{N_S(\lambda,z) - N_B}{N_S(\lambda,z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}$$

Calculate R_T and R_W from N_{Norm}

Are ratios reasonable?

Yes

Find T and W from the Table

No

Set to nominal values or MSIS
T = 200 K, W = 0 m/s

Calculate Na density $n_c(z)$

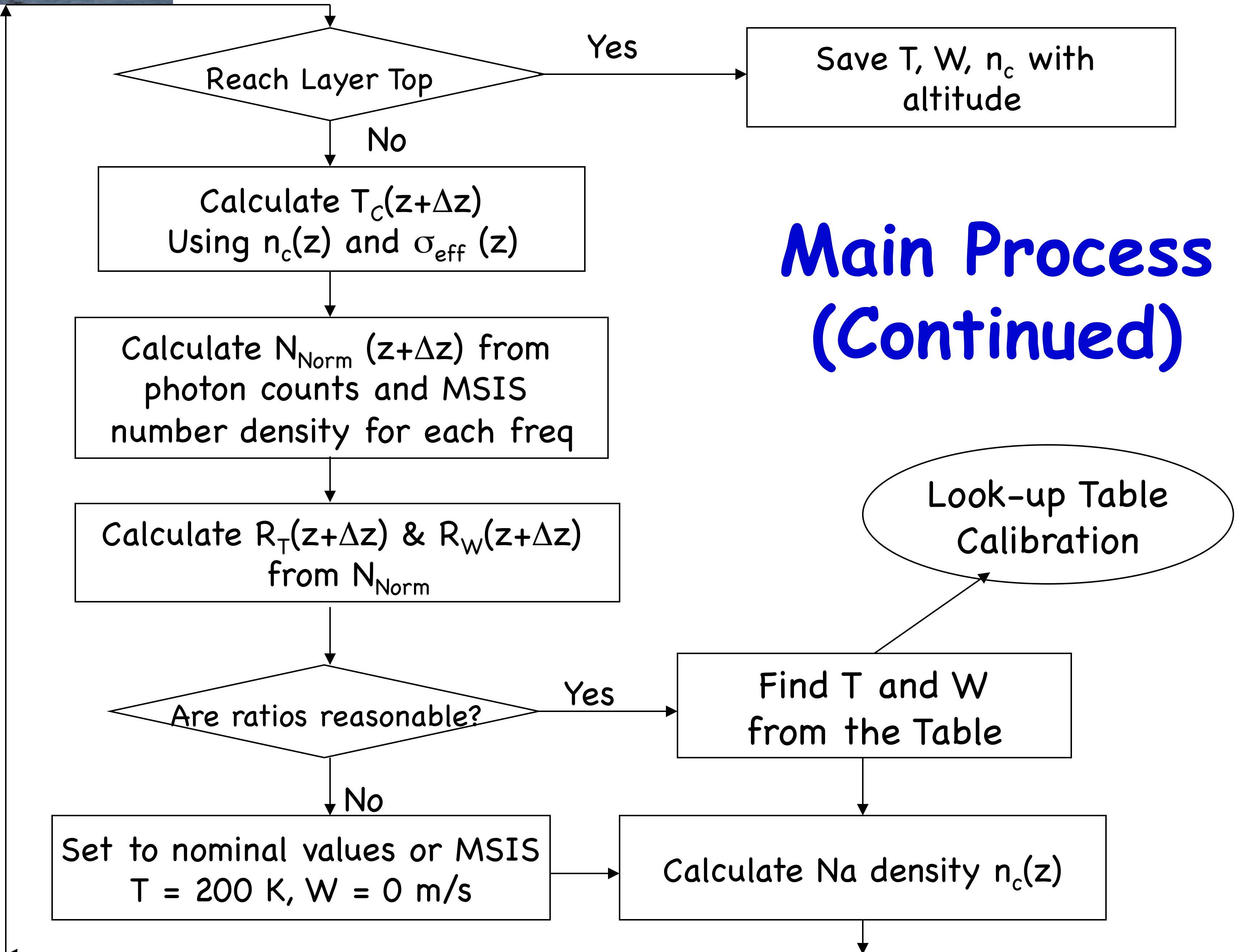
Create look-up table or calibration curves From physics

$$R_T = \frac{\sigma_{eff}(f_+,z) + \sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}$$

$$R_W = \frac{\sigma_{eff}(f_+,z) - \sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}$$

Look-up Table Calibration

Main Process (Continued)



Derivation of T_C (Transmission Caused by Constituent Extinction)

- T_C (caused by constituent absorption) can be derived from

$$T_c(\lambda,z) = \exp\left(-\int_{z_{bottom}}^{z} \sigma_{eff}(\lambda,z) n_c(z)\,\mathrm{d}z\right) = \exp\left(-\sum_{z_{bottom}}^{z} \sigma_{eff}(\lambda,z) n_c(z)\Delta z\right)$$

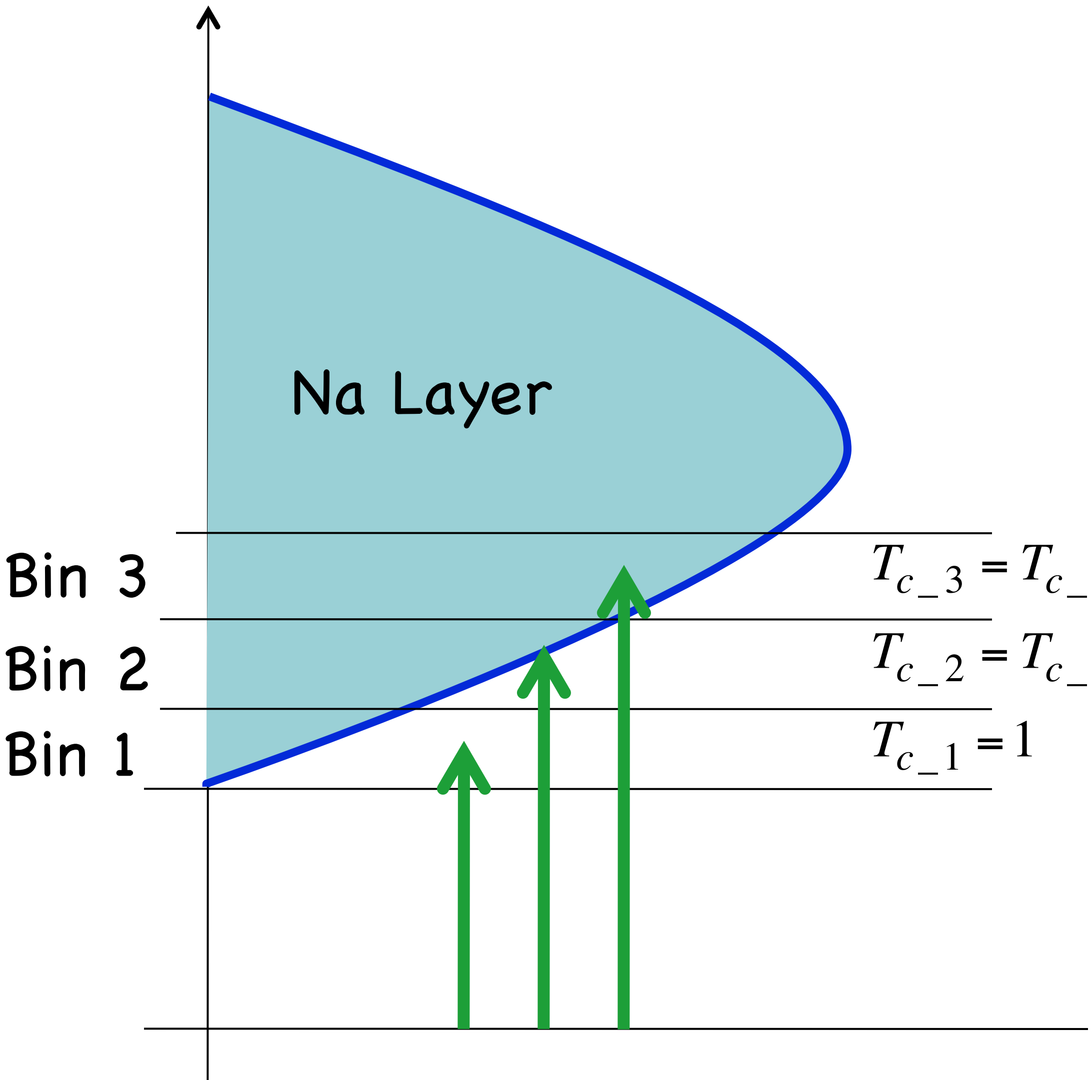
+

$$n_c(z) = \left[\frac{N_S(\lambda,z)-N_B}{N_S(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^2}\frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}\right]\cdot\frac{4\pi\sigma_R(\pi,\lambda)n_R(z_R)}{\sigma_{eff}(\lambda,z)R_B(\lambda)}$$

$$\sigma_e = \sqrt{\sigma_D^2 + \sigma_L^2}$$

$$T_c(\lambda,z) = \exp\left(-\sum_{z_{bottom}}^{z}\left[\frac{N_S(\lambda,z)-N_B}{N_S(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^2}\frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}\right]\cdot\frac{4\pi\sigma_R(\pi,\lambda)n_R(z_R)}{R_B(\lambda)}\Delta z\right)$$

A Step in the Main Process: Estimating Transmission (T_c)



$$T_{c\_3} = T_{c\_2} \times \exp(-\sigma_{eff\_2} \cdot n_{c\_2} \cdot \Delta z)$$

$$T_{c\_2} = T_{c\_1} \times \exp(-\sigma_{eff\_1} \cdot n_{c\_1} \cdot \Delta z)$$

$$T_{c\_1} = 1$$

Main Process Step 1: Starting Point

1. Set transmission (T_c) at the bottom of Na layer to be 1
2. Calculate the normalized photon count for each frequency

$$\boxed{N_{Norm}(\lambda,z) = \frac{N_S(\lambda,z) - N_B}{N_S(\lambda,z_R) - N_B} \frac{z^2}{z_R{}^2} \frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}}$$

3. Take ratios R_T and R_W from normalized photon counts

$$\boxed{R_T = \frac{N_{Norm}(f_+,z) + N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}} \qquad \boxed{R_W = \frac{N_{Norm}(f_+,z) - N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}}$$

4. Estimate the temperature and wind using the calibration curves computed from physics

Main Process Step 2: Bin-by-Bin Procedure

5. Calculate the effective cross section using temperature and wind derived
6. Using the effective cross-section and T_C = 1 (at the bottom), calculate the Na density.

$$\boxed{n_c(z) = \left[\frac{N_S(\lambda,z)-N_B}{N_S(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^2}\frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}\right]\cdot\frac{4\pi\sigma_R(\pi,\lambda)n_R(z_R)}{\sigma_{eff}(\lambda)R_B(\lambda)}}$$

7. From effective cross-section and Na density, calculate the transmission T_C for the next bin.

$$\boxed{T_c(\lambda,z) = \exp\left(-\int_{z_{bottom}}^{z}\sigma_{eff}(\lambda,z)n_c(z)\,\mathrm{d}z\right) = \exp\left(-\sum_{z_{bottom}}^{z}\sigma_{eff}(\lambda,z)n_c(z)\Delta z\right)}$$

Na Density Derivation

- The Na density can be inferred from the peak freq signal

$$n_{Na}(z) = \frac{N_{norm}(f_a, z)}{\sigma_a} 4\pi n_R(z_R)\sigma_R = \frac{N_{norm}(f_a, z)}{\sigma_a} 4\pi \times 2.938 \times 10^{-32} \frac{P(z_R)}{T(z_R)} \cdot \frac{1}{\lambda^{4.0117}}$$

- The Na density can also be inferred from a weighted average of all three frequency signals.
- The weighted effective cross-section is

$$\sigma_{eff\_wgt} = \sigma_a + \alpha\sigma_+ + \beta\sigma_-$$

where α and β are chosen so that

$$\frac{\partial \sigma_{eff\_wgt}}{\partial T} = 0; \qquad \frac{\partial \sigma_{eff\_wgt}}{\partial v_R} = 0$$

- The Na density is then calculated by

$$n_{Na}(z) = 4\pi n_R(z_R)\sigma_R \frac{N_{norm}(f_a, z) + \alpha N_{norm}(f_+, z) + \beta N_{norm}(f_-, z)}{\sigma_a + \alpha\sigma_+ + \beta\sigma_-}$$

Process Procedure for Deriving β

Read Data File

PMT Saturation Correction

Chopper Correction

Integrate and/or smooth profiles to improve SNR

Subtract Background

Remove Range Dependence (x R^2)

Add Base Altitude

Take Rayleigh Signal @ z_R km

Normalize Profile By Rayleigh Signal @ z_R km

Load Atmosphere n_R, T, P Profiles from MSIS00

Calculate $R(z)$ and $\beta(z)$

Smooth $R(z)$ and $\beta(z)$ By Hamming Window FWHM = 250 m

Calculate z_c, σ_{rms}, β_{total}

Process Procedure for β of PMC

Load Atmosphere n_R, T, P Profiles from MSIS00

↓

Calculate $R(z)$ and $\beta(z)$

↓

Smooth $R(z)$ and $\beta(z)$ By Hamming Window FWHM = 250 m

↓

Calculate z_c, σ_{rms}, β_{total}

$$R = \frac{[N_S(z) - N_B]\cdot z^2}{[N_S(z_{RN}) - N_B]\cdot z_{RN}^2}\cdot\frac{n_R(z_{RN})}{n_R(z)}$$

$$\beta_{PMC}(z) = \left[\frac{[N_S(z) - N_B]\cdot z^2}{[N_S(z_{RN}) - N_B]\cdot z_{RN}^2} - \frac{n_R(z)}{n_R(z_{RN})}\right]\cdot\beta_R(z_{RN})$$

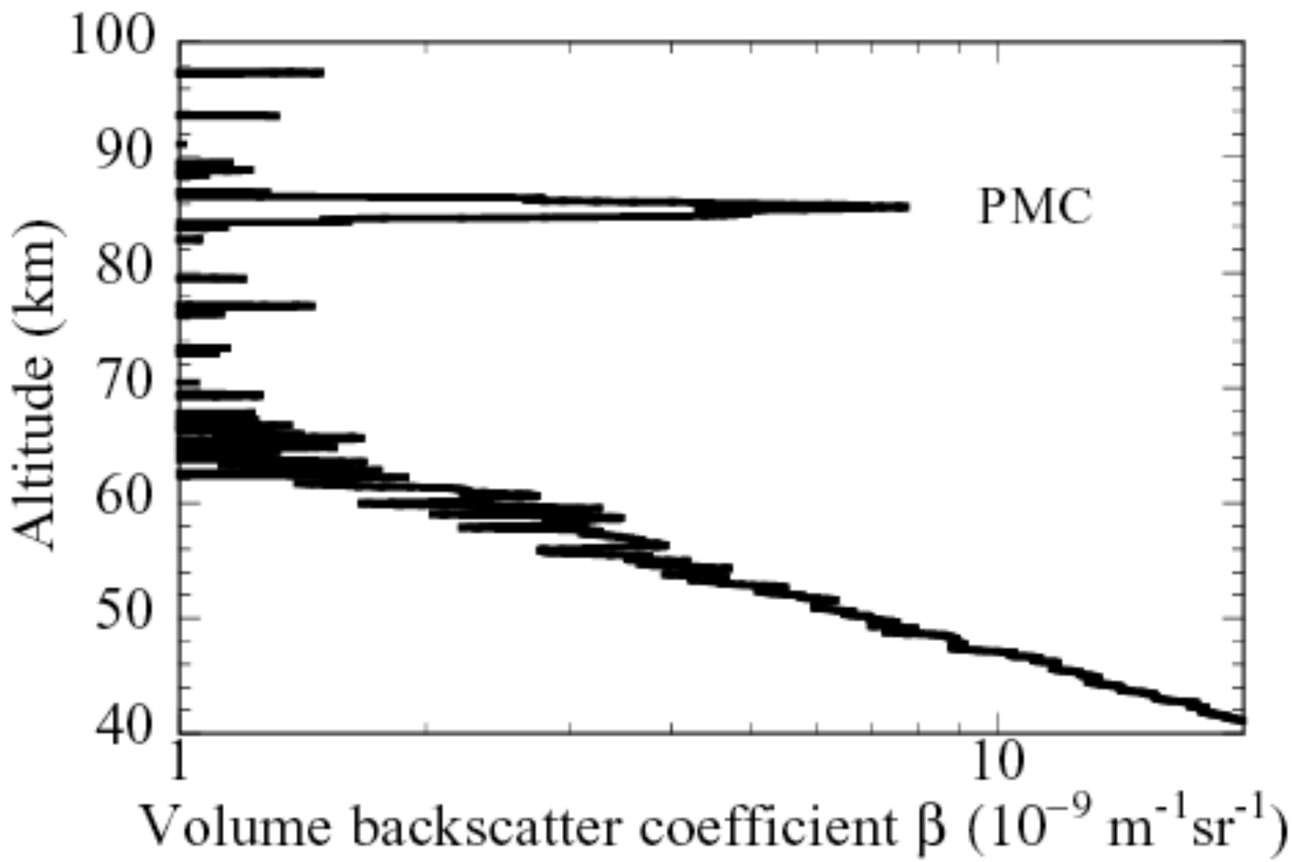
$$\beta_R(z_{RN},\pi) = \frac{\beta}{4\pi}P(\pi) = 2.938\times 10^{-32}\frac{P(z_{RN})}{T(z_{RN})}\cdot\frac{1}{\lambda^{4.0117}}$$

$$z_c = \frac{\sum_i \beta_{PMC}(z_i)\cdot z_i}{\sum_i \beta_{PMC}(z_i)}$$

$$\sigma_{rms} = \sqrt{\frac{\sum_i (z_i - z_c)^2\beta_{PMC}(z_i)}{\sum_i \beta_{PMC}(z_i)}}$$

$$\beta_{total} = \int \beta_{PMC}(z)dz$$

Example Result: South Pole PMC



Process Procedure for n_c using broadband lidar

1. Load Atmosphere n_R, T, P Profiles from MSIS00
2. Start from layer bottom $T_C(z = z_b) = 1$ Calculate $\sigma_{eff}(z = z_b)$
3. Calculate $n_c(z)$
4. Calculate $T_C(z+\Delta z)$ using $\sigma_{eff}(z)$ Calculate $\sigma_{eff}(z+\Delta z)$
5. Calculate $n_c(z+\Delta z)$ Using $T_C(z+\Delta z)$ and $\sigma_{eff}(z+\Delta z)$
6. Reach Layer Top — No: return to step 4; Yes: continue
7. Smooth $n_c(z)$ By Hamming Window
8. Save $n_c(z)$ with altitude
9. Calculate Abundance, etc

Process Procedure for n_c

- Computation of effective cross-section (concerning laser shape, assuming nominal T and W)
- Spatial resolution - binning or smoothing
- temporal resolution – integration or smoothing

-- in order to improve SNR

- Transmission T_C (extinction coefficient)
- Calculate density
- Calculate abundance, peak altitude, etc.

To Improve SNR

- In order to improve signal-to-noise ratio (SNR), we have to sacrifice spatial and/or temporal resolutions.
- Spatial resolution
  - integration (binning)
  - smoothing
- temporal resolution
  - integration
  - smoothing

Analysis of Error Caused by Photon Noise

- Use the temperature error derivation for 3-freq Na lidar as an example to explain the error analysis procedure using a differentiation method. For 3-frequency technique, we have the temperature ratio

$$R_T = \frac{\sigma_{eff}(f_+) + \sigma_{eff}(f_-)}{\sigma_{eff}(f_a)} = \frac{N(f_+) + N(f_-)}{N(f_a)}$$

- Temperature error caused by photon noise is given by (1st order apprx.)

$$\Delta T = \frac{\partial T}{\partial R_T}\Delta R_T = \frac{R_T}{\partial R_T / \partial T}\frac{\Delta R_T}{R_T}$$

- Define the temperature sensitivity as

$$\boxed{S_T = \frac{\partial R_T / \partial T}{R_T}}$$

We have $$(\Delta T)_{rms} = \frac{\partial T}{\partial R_T}\Delta R_T = \underbrace{\frac{R_T}{\partial R_T / \partial T}}_{\text{Error coefficient}}\underbrace{\frac{\Delta R_T}{R_T}}_{R_T \text{ relative error}} = \frac{1}{S_T}\frac{\Delta R_T}{R_T}$$

Error Coefficient and $\Delta R_T/R_T$

- The temperature error coefficient can be derived numerically

$$\frac{R_T}{\partial R_T/\partial T} = \frac{R_T}{[R_T(T+\delta T) - R_T(T)]/\delta T}$$

- We can derive the error of 3-freq R_T caused by photon noise

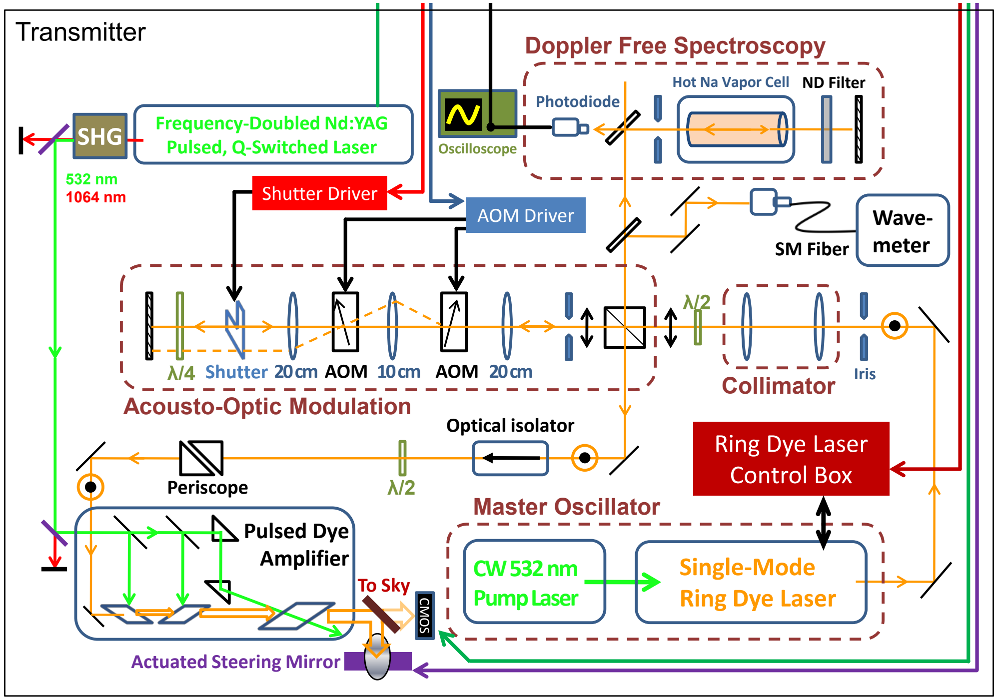
$$\frac{\Delta R_T}{R_T} = \frac{\left(1+\frac{1}{R_T}\right)^{1/2}}{\left(N_{f_a}\right)^{1/2}}\left[1+\frac{B}{N_{f_a}}\frac{\left(1+\frac{2}{R_T^2}\right)}{\left(1+\frac{1}{R_T}\right)}\right]^{1/2}$$

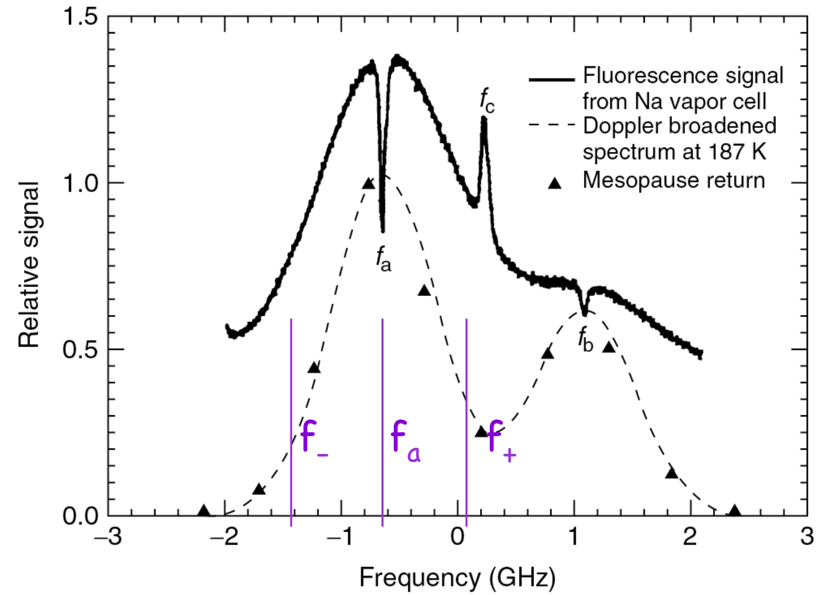
Hint: An example for 2-freq ratio technique, $R_T = N_{fc} / N_{fa}$

$$\frac{\Delta R_T}{R_T} = \frac{\Delta N_{f_c}}{N_{f_c}} - \frac{\Delta N_{f_a}}{N_{f_a}} \qquad \overline{\left(\Delta N_{f_c}\right)^2} = N_{f_c} + B, \quad \overline{\left(\Delta N_{f_a}\right)^2} = N_{f_a} + B$$

$$\left(\frac{\Delta R_T}{R_T}\right)_{rms} = \sqrt{\overline{\left(\frac{\Delta N_{f_c}}{N_{f_c}} - \frac{\Delta N_{f_a}}{N_{f_a}}\right)^2}} = \sqrt{\overline{\left(\frac{\Delta N_{f_c}}{N_{f_c}}\right)^2} + \overline{\left(\frac{\Delta N_{f_a}}{N_{f_a}}\right)^2}}$$

Other Possible Errors or Biases





1) Laser frequency locking error
2) AOM frequency shift error
3) Pulsed laser frequency chirp
4) Laser line shape and linewidth
5) Hanle effect
6) Metal layer saturation
7) Daytime filter function
8) Photodetector and discriminator saturation, etc. …

Influence of Laser Lineshape & Filter Function

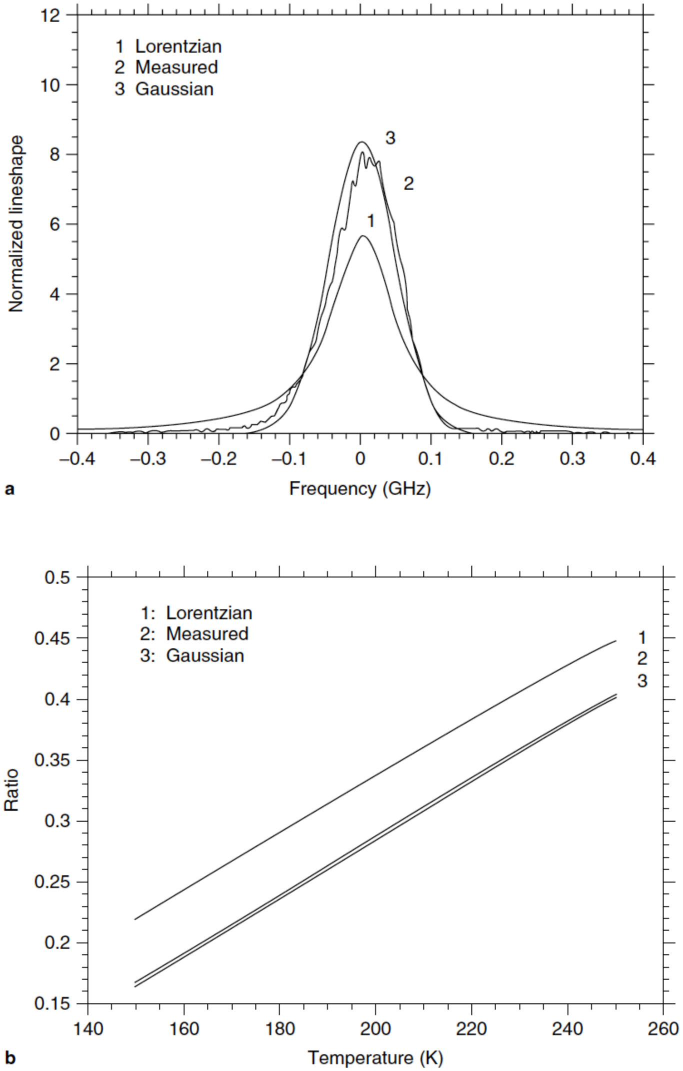


Figure 5.18 (a) Three laser lineshape functions for the Na wind/temperature lidar: a measured lineshape along with two hypothetical line shapes with equal FWHM = 112 MHz. (b) The temperature calibration lines of these lineshapes. (From She, C.Y., Yu, J.R., Latifi, H., and Bills, R.E., *Appl. Opt.*, 31, 2095–2106, 1992. With permission.)

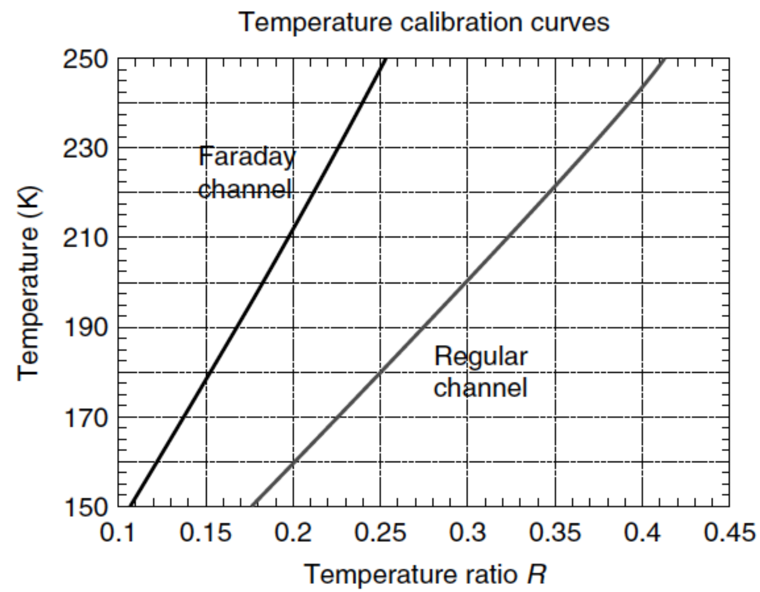


Figure 5.17 Calibration curves for the conversion of measured intensity ratio to temperature in the mesopause region for the regular channel (without Faraday filter) and the Faraday channel (with Faraday filter). (From Chen, H., White, M.A., Krueger, D.A., and She, C.Y., *Opt. Lett.*, 21, 1093–1095, 1996. With permission.)

Metal Layer Saturation vs. Laser Intensity

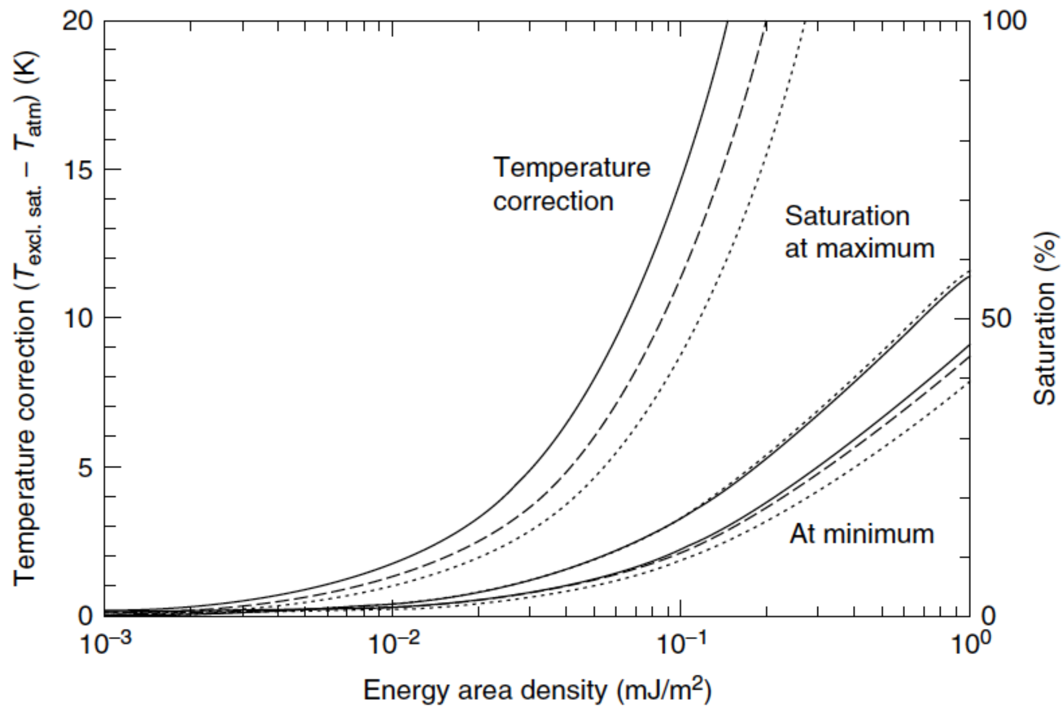


Figure 5.19 Na saturation at maximum, minimum, and temperature correction in relation to the energy area density. Basic assumptions are atmospheric temperature 200 K (dashed lines), laser linewidth 130 MHz, and pulse length 10 nsec. Further atmospheric temperatures are 150 K (dotted lines) and 250 K (solid lines). (From von der Gathen, P., *J. Geophys. Res.*, 96 (A3), 3679–3690, 1991. With permission.)

Summary

- The pre-process and profile-process are to convert the raw photon counts to corrected and normalized photon counts in consideration of hardware properties and limitations.
- The main process of T and V_R is to convert the normalized photon counts to T and V_R through iteration or looking-up table methods.
- The main process of n_c or β is to convert the normalized photon counts to number density or volume backscatter coefficient, in combination with prior acquired knowledge or model knowledge of certain atmosphere information or atomic/molecular spectroscopy.
- Data inversion procedure consists of the following processes:

  (1) pre- and profile-process,

  (2) process of T and V_R,

  (3) process of n_c and β, etc.