Lecture 31. Lidar Data Inversion (2)

- Pre-process and Profile-process
- Main Process Procedure to Derive T and $V_R$ Using Ratio Doppler Technique
- Derivations of $n_c$ from narrowband resonance Doppler lidar
- Derivation of $\beta$
- Derivation of $n_c$ from broadband resonance lidar
- Error analysis for photon noise
- Summary
Preprocess Procedure and Profile-Process Procedure for Na/Fe/K Doppler Lidar

- Read data: for each set, and calculate $T$, $W$, and $n$ for each set
- PMT/Discriminator saturation correction
- Chopper/Filter correction
- Background estimate and subtraction
- Range-dependence removal ($xR^2$, not $z^2$)
- Base altitude adjustment
- Take Rayleigh signal @ $z_R$ (Rayleigh fit or Rayleigh mean)
- Rayleigh normalization

$$N_N(\lambda, z) = \frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} \frac{z^2}{z_R^2}$$

- Subtract Rayleigh signals from Na/Fe/K region after counting in the factor of $T_C$
Main Ideas to Derive Na T and W

- In the ratio technique, Na number density is cancelled out. So we have two ratios $R_T$ and $R_W$ that are independent of Na density but both dependent on $T$ and $W$.

- The idea is to derive temperature and radial wind from these two ratios first, and then derive Na number density using computed temperature and wind at each altitude bin.

- To derive $T$ and $W$ from $R_T$ and $R_W$, the basic idea is to use look-up table or iteration methods to derive them: (1) compute $R_T$ and $R_W$ from physics point-of-view to generate the table or calibration curves, (2) compute $R_T$ and $R_W$ from actual photon counts, (3) check the table or calibration curves to find the corresponding $T$ and $W$. (4) If $R_T$ and $R_W$ are out of range, then set to nominal $T$ and $W$.

- However, because the Na extinction coefficient is involved, the upper bins are related to lower bins, and extinction coefficient is related to Na density and effective cross-section. The solution is to start from the bottom of the Na layer.
Main Process Procedure

- Compute Doppler calibration curves from physics

\[ R_T = \frac{\sigma_{\text{eff}}(f_+, z) + \sigma_{\text{eff}}(f_-, z)}{\sigma_{\text{eff}}(f_a, z)} \]

\[ R_W = \frac{\sigma_{\text{eff}}(f_+, z) - \sigma_{\text{eff}}(f_-, z)}{\sigma_{\text{eff}}(f_a, z)} \]

\[ \sigma_{\text{eff}}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_e} \frac{e^2 f}{4\epsilon_0 m_e c} \sum_{n=1}^{6} A_n \exp \left( - \frac{\nu_n - \nu(1 - \frac{v_R}{c})}{2\sigma_e^2} \right) \]
Main Process Procedure

- Compute actual ratios $R_T$ and $R_W$ from photon counts
- Look up these two ratios on the calibration curves to infer the corresponding Temperature and Wind from isoline/isogram.
Constituent Density

- Normalized Photon Count to the density estimation

\[
n_c(z) = \left[ \frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} \cdot \frac{z^2}{z_R^2} \cdot \frac{1}{I_c(\lambda, z)} \right] \cdot \frac{4\pi \sigma_R(\pi, \lambda) n_R(z_R)}{\sigma_{\text{eff}}(\lambda, z) R_B(\lambda)} \cdot \frac{n_R(z)}{n_R(z_R)} \cdot \frac{z^2}{z_R^2} \cdot \frac{1}{I_c(\lambda, z)}
\]

Normalized Photon Count
From the preprocess

Temperature and wind
dependent

\[ \rightarrow \text{we need to estimate the temperature and wind first in order to estimate the density} \]

- The effective cross-section

\[
\sigma_{\text{eff}}(\nu) = \frac{1}{\sqrt{2\pi} \sigma_e} \frac{e^2f}{4\epsilon_0 m_e c} \sum_{n=1}^{6} A_n \exp \left( - \frac{\left[ \nu_n - \nu \left(1 - \frac{v_R}{c}\right) \right]^2}{2\sigma_e^2} \right)
\]
Main Process

Load Atmosphere $n_R, T, P$
Profiles from MSIS00

Start from Na layer bottom
$T_C (z=z_b) = 1$
Calculate $N_{\text{norm}} (z=z_b)$ from
 photon counts and MSIS
number density for each freq

$N_{\text{norm}}(\lambda, z) = \frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda, z)} - \frac{n_R(z)}{n_R(z_R)}$

Calculate $R_T$ and $R_W$ from $N_{\text{norm}}$

Look-up Table
Calibration

Are ratios reasonable?
Yes

Find $T$ and $W$
from the Table

Calculate Na density $n_c(z)$

No

Set to nominal values or MSIS
$T = 200$ K, $W = 0$ m/s
Main Process (Continued)

Calculate $N_{\text{Norm}}(z+\Delta z)$ from photon counts and MSIS number density for each freq

Calculate $R_T(z+\Delta z)$ & $R_W(z+\Delta z)$ from $N_{\text{Norm}}$

Are ratios reasonable?

Set to nominal values or MSIS $T = 200$ K, $W = 0$ m/s

Reach Layer Top

No

Yes

Save $T$, $W$, $n_c$ with altitude

Find $T$ and $W$ from the Table

Calculate Na density $n_c(z)$

Look-up Table Calibration
Derivation of $T_C$ (Transmission Caused by Constituent Extinction)

$T_C(\lambda, z) = \exp\left(-\int_{z_{\text{bottom}}}^{z} \sigma_{\text{eff}}(\lambda, z)n_c(z)\,dz\right) = \exp\left(-\sum_{z_{\text{bottom}}}^{z} \sigma_{\text{eff}}(\lambda, z)n_c(z)\Delta z\right)$

$T_C(\lambda, z) = \exp\left(-\sum_{z_{\text{bottom}}}^{z} \sigma_{\text{eff}}(\lambda, z)n_c(z)\Delta z\right)$

$\sigma_e = \sqrt{\sigma_D^2 + \sigma_L^2}$

$n_c(z) = \left[\frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} \cdot \frac{z^2}{z_R^2} \cdot \frac{1}{T_c^2(\lambda, z)} - \frac{n_R(z)}{n_R(z_R)} \right] \cdot \frac{4\pi \sigma_R(\pi, \lambda)n_R(z_R)}{\sigma_{\text{eff}}(\lambda, z)R_B(\lambda)}$
A Step in the Main Process: Estimating Transmission ($T_C$)

Na Layer

$T_{c\_3} = T_{c\_2} \times \exp(-\sigma_{eff\_2} \cdot n_{c\_2} \cdot \Delta z)$

$T_{c\_2} = T_{c\_1} \times \exp(-\sigma_{eff\_1} \cdot n_{c\_1} \cdot \Delta z)$

$T_{c\_1} = 1$
Main Process Step 1: Starting Point

1. Set transmission ($T_c$) at the bottom of Na layer to be 1

2. Calculate the normalized photon count for each frequency

$$N_{Norm}(\lambda,z) = \frac{N_S(\lambda,z) - N_B}{N_S(\lambda,z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}$$

3. Take ratios $R_T$ and $R_W$ from normalized photon counts

$$R_T = \frac{N_{Norm}(f_+,z) + N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}$$

$$R_W = \frac{N_{Norm}(f_+,z) - N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}$$

4. Estimate the temperature and wind using the calibration curves computed from physics
Main Process Step 2: Bin-by-Bin Procedure

5. Calculate the effective cross section using temperature and wind derived

6. Using the effective cross-section and $T_C = 1$ (at the bottom), calculate the Na density.

$$n_c(z) = \frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} \cdot \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda, z)} - n_R(z) \cdot \frac{4\pi\sigma_R(\pi, \lambda)n_R(z_R)}{\sigma_{eff}(\lambda)R_B(\lambda)}$$

7. From effective cross-section and Na density, calculate the transmission $T_C$ for the next bin.

$$T_c(\lambda, z) = \exp\left(-\int_{z_{bottom}}^{z} \sigma_{eff}(\lambda, z)n_c(z) \, dz\right) = \exp\left(-\sum_{z_{bottom}}^{z} \sigma_{eff}(\lambda, z)n_c(z) \Delta z\right)$$
Na Density Derivation

- The Na density can be inferred from the peak freq signal

\[
n_{Na}(z) = \frac{N_{\text{norm}}(f_a, z)}{\sigma_a} 4\pi n_R(z_R)\sigma_R = \frac{N_{\text{norm}}(f_a, z)}{\sigma_a} 4\pi \times 2.938 \times 10^{-32} \frac{P(z_R)}{T(z_R)} \cdot \frac{1}{\lambda^{4.0117}}
\]

- The Na density can also be inferred from a weighted average of all three frequency signals.

- The weighted effective cross-section is

\[
\sigma_{\text{eff - wgt}} = \sigma_a + \alpha\sigma_+ + \beta\sigma_-
\]

where \(\alpha\) and \(\beta\) are chosen so that

\[
\frac{\partial \sigma_{\text{eff - wgt}}}{\partial T} = 0; \quad \frac{\partial \sigma_{\text{eff - wgt}}}{\partial n_R} = 0
\]

- The Na density is then calculated by

\[
n_{Na}(z) = 4\pi n_R(z_R)\sigma_R \frac{N_{\text{norm}}(f_a, z) + \alpha N_{\text{norm}}(f_+, z) + \beta N_{\text{norm}}(f_-, z)}{\sigma_a + \alpha\sigma_+ + \beta\sigma_-}
\]
Process Procedure for Deriving $\beta$

1. **Read Data File**
2. **PMT Saturation Correction**
3. **Chopper Correction**
4. **Subtract Background**
5. **Remove Range Dependence ($x R^2$)**
6. **Add Base Altitude**
7. **Take Rayleigh Signal @ $z_R$ km**
8. **Normalize Profile**

**Integrate and/or smooth profiles to improve SNR**

9. **Load Atmosphere $n_R$, T, P Profiles from MSIS00**
10. **Calculate $R(z)$ and $\beta(z)$**
11. **Smooth $R(z)$ and $\beta(z)$ By Hamming Window FWHM = 250 m**
12. **Calculate $z_c$, $\sigma_{rms}$, $\beta_{total}$**
# Process Procedure for $\beta$ of PMC

1. Load Atmosphere $n_R$, $T$, $P$ profiles from MSIS00

2. Calculate $R(z)$ and $\beta(z)$

3. Smooth $R(z)$ and $\beta(z)$ by Hamming Window, FWHM = 250 m

4. Calculate $z_c$, $\sigma_{rms}$, $\beta_{total}$

## Mathematical Formulas

- $R = \frac{\left[ N_S(z) - N_B \right] \cdot z^2}{\left[ N_S(z_{RN}) - N_B \right] \cdot z_{RN}^2} \cdot \frac{n_R(z_{RN})}{n_R(z)}$

- $\beta_{PMC}(z) = \frac{\left[ N_S(z) - N_B \right] \cdot z^2}{\left[ N_S(z_{RN}) - N_B \right] \cdot z_{RN}^2} \cdot \frac{n_R(z)}{n_R(z_{RN})} \cdot \beta_R(z_{RN})$

- $\beta_R(z_{RN}, \pi) = \frac{\beta}{4\pi} \cdot P(\pi) = 2.938 \times 10^{-32} \cdot \frac{P(z_{RN})}{T(z_{RN})} \cdot \frac{1}{\lambda^{4.0117}}$

- $z_c = \frac{\sum_i \beta_{PMC}(z_i) \cdot z_i}{\sum_i \beta_{PMC}(z_i)}$

- $\sigma_{rms} = \sqrt{\frac{\sum_i \left( z_i - z_c \right)^2 \beta_{PMC}(z_i)}{\sum_i \beta_{PMC}(z_i)}}$

- $\beta_{total} = \int \beta_{PMC}(z) \, dz$
Example Result: South Pole PMC
Load Atmosphere $n_R$, T, P Profiles from MSIS00

Start from layer bottom

$T_C(z = z_b) = 1$

Calculate $\sigma_{\text{eff}}(z = z_b)$

Calculate $n_c(z)$

Calculate $T_C(z + \Delta z)$ using $\sigma_{\text{eff}}(z)$

Calculate $\sigma_{\text{eff}}(z + \Delta z)$

Calculate $n_c(z + \Delta z)$

Using $T_C(z + \Delta z)$ and $\sigma_{\text{eff}}(z + \Delta z)$

Reach Layer Top

Smooth $n_c(z)$ By Hamming Window

Save $n_c(z)$ with altitude

Calculate Abundance, etc

Yes

No
Process Procedure for $n_c$

- Computation of effective cross-section (concerning laser shape, assuming nominal T and W)
- Spatial resolution – binning or smoothing
- Temporal resolution – integration or smoothing
  -- in order to improve SNR
- Transmission $T_c$ (extinction coefficient)
- Calculate density
- Calculate abundance, peak altitude, etc.
To Improve SNR

- In order to improve signal-to-noise ratio (SNR), we have to sacrifice spatial and/or temporal resolutions.

- Spatial resolution
  - integration (binning)
  - smoothing

- Temporal resolution
  - integration
  - smoothing
Analysis of Error Caused by Photon Noise

- Use the temperature error derivation for 3-freq Na lidar as an example to explain the error analysis procedure using a differentiation method. For 3-frequency technique, we have the temperature ratio

\[ R_T = \frac{\sigma_{eff}(f_+)}{\sigma_{eff}(f_a)} = \frac{N(f_+)}{N(f_a)} \]

- Temperature error caused by photon noise is given by (1\textsuperscript{st} order approx.)

\[ \Delta T = \frac{\partial T}{\partial R_T} \Delta R_T = \frac{R_T}{\partial R_T / \partial T} \frac{\Delta R_T}{R_T} \]

- Define the temperature sensitivity as

\[ S_T = \frac{\partial R_T / \partial T}{R_T} \]

We have \((\Delta T)_{rms} = \frac{\partial T}{\partial R_T} \Delta R_T = \frac{R_T}{\partial R_T / \partial T} \frac{\Delta R_T}{R_T} = \frac{1}{S_T} \frac{\Delta R_T}{R_T} \)
Error Coefficient and $\Delta R_T/R_T$

- The temperature error coefficient can be derived numerically

$$\frac{R_T}{\partial R_T/\partial T} = \frac{R_T}{[R_T(T+\delta T) - R_T(T)]/\delta T}$$

- We can derive the error of 3-freq $R_T$ caused by photon noise

$$\frac{\Delta R_T}{R_T} = \left(1 + \frac{1}{R_T}\right)^{1/2} \left[1 + \frac{B}{N_{f_a}} \left(\frac{1 + \frac{2}{R_T^2}}{1 + \frac{1}{R_T}}\right)^{1/2}\right]$$

Hint: An example for 2-freq ratio technique, $R_T = N_{fc} / N_{fa}$

$$\frac{\Delta R_T}{R_T} = \frac{\Delta N_{fc}}{N_{fc}} - \frac{\Delta N_{fa}}{N_{fa}} \quad \left(\frac{\Delta N_{fc}}{N_{fc}}\right)^2 = N_{fc} + B, \quad \left(\frac{\Delta N_{fa}}{N_{fa}}\right)^2 = N_{fa} + B$$

$$\left(\frac{\Delta R_T}{R_T}\right)_{rms} = \sqrt{\left(\frac{\Delta N_{fc}}{N_{fc}} - \frac{\Delta N_{fa}}{N_{fa}}\right)^2} = \sqrt{\left(\frac{\Delta N_{fc}}{N_{fc}}\right)^2 + \left(\frac{\Delta N_{fa}}{N_{fa}}\right)^2}$$
Other Possible Errors or Biases

1) Laser frequency locking error
2) AOM frequency shift error
3) Pulsed laser frequency chirp
4) Laser line shape and linewidth
5) Hanle effect
6) Metal layer saturation
7) Daytime filter function
8) Photodetector and discriminator saturation, etc. ...
Influence of Laser Lineshape & Filter Function

Figure 5.17 Calibration curves for the conversion of measured intensity ratio to temperature in the mesopause region for the regular channel (without Faraday filter) and the Faraday channel (with Faraday filter). (From Chen, H., White, M.A., Krueger, D.A., and She, C.Y., Opt. Lett., 21, 1093–1095, 1996. With permission.)

Figure 5.18 (a) Three laser lineshape functions for the Na wind/temperature lidar: a measured lineshape along with two hypothetical line shapes with equal FWHM = 112 MHz. (b) The temperature calibration lines of these lineshapes. (From She, C.Y., Yu, J.R., Latifi, H., and Bills, R.E., Appl. Opt., 31, 2095–2106, 1992. With permission.)
Metal Layer Saturation vs. Laser Intensity

**Figure 5.19** Na saturation at maximum, minimum, and temperature correction in relation to the energy area density. Basic assumptions are atmospheric temperature 200 K (dashed lines), laser linewidth 130 MHz, and pulse length 10 nsec. Further atmospheric temperatures are 150 K (dotted lines) and 250 K (solid lines). (From von der Gathen, P., *J. Geophys. Res.*, 96 (A3), 3679–3690, 1991. With permission.)
Summary

- The pre-process and profile-process are to convert the raw photon counts to corrected and normalized photon counts in consideration of hardware properties and limitations.

- The main process of $T$ and $V_R$ is to convert the normalized photon counts to $T$ and $V_R$ through iteration or looking-up table methods.

- The main process of $n_c$ or $\beta$ is to convert the normalized photon counts to number density or volume backscatter coefficient, in combination with prior acquired knowledge or model knowledge of certain atmosphere information or atomic/molecular spectroscopy.

- Data inversion procedure consists of the following processes:
  1. pre- and profile-process,
  2. process of $T$ and $V_R$,
  3. process of $n_c$ and $\beta$, etc.