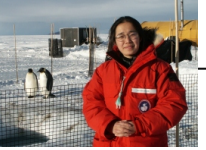
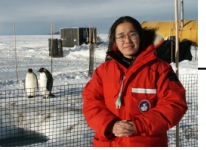


Lecture 15. Temperature Lidar (4) Doppler Techniques

- Doppler effects in absorption and backscatter coefficient vs. cross-section
- Doppler Technique to Measure Temperature and Wind
  - Doppler Shift and Broadening in Resonance Absorption
  - Doppler Shift and Broadening in Resonance Fluorescence
  - Doppler Shift and Broadening in Rayleigh Scattering
- Resonance Fluorescence Doppler vs Rayleigh Doppler
  - Doppler Effects in Fe Boltzmann Lidar
  - Wavelength Scan vs. Scanning Technique
- Summary



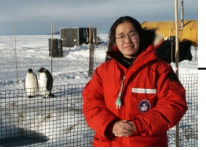
Doppler Broadening in Absorption and Backscatter Coefficients

- It is accurate to say that absorption coefficient and backscatter coefficient experience Doppler broadening. However, only statistically averaged absorption and effective cross sections experience Doppler broadening.
- Absorption and effective cross sections for single atom/molecule experience Doppler shift but not Doppler broadening.
- Absorption and total backscatter coefficients are given by

$$\alpha_{ik} = \sigma_{ik}(N_i - \frac{g_i}{g_k}N_k) \approx \sigma_{ik}N_i$$

$$\beta_T = \sigma_{eff}R_B N_i$$

- Absorption coefficient α_{ik}, total backscatter coefficient β_T
- Absorption cross section σ_{ik}, effective backscatter cross section σ_{eff}
- Population (number density) N_i and N_k



Doppler Shift in Absorption and Backscatter Cross Sections

- In principle, σ_{ik} is the cross section for single atom/molecule, so $\sigma_{ik}(\omega)$ has a Lorentzian shape L(ω, V)

$$\sigma_{ik}(\omega,v) = \sigma_o L(\omega,v)$$

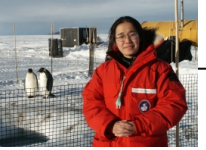
where L(ω, v) is a Lorentzian shape and function of velocity v.

- Single atom absorption/effective cross section experiences Doppler shift, but it does NOT experience Doppler broadening. Single atom cross-section has a Lorentzian shape with narrow natural linewidth, resulted from the finite radiative lifetime of the excited states of atom/molecule.

- N_i can be written as the population distribution along velocity $N_i(v)$, which is a Gaussian shape under Maxwellian distribution. Here N_0 is the total population on energy level E_i, and V_R is the center radial velocity:

$$N_i(v,V_R) = N_o G(v,V_R)$$

- Doppler broadening comes from the fact that different atoms have different velocities in the atmosphere, so causing different Doppler shifts. Averaging over all atoms, it leads to Doppler broadened Gaussian shape.



Doppler Broadening on Absorption and Backscatter Coefficients

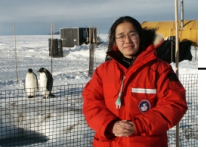
- α_{ik} is the convolution of a Lorentzian absorption cross section with a Gaussian population distribution, which becomes a Voigt profile:

$$\alpha_{ik}(\omega, V_R) = \int_{-\infty}^{+\infty} \sigma_{ik}(\omega, v) N_i(v, V_R) dv = \sigma_o N_o \int_{-\infty}^{+\infty} L(\omega, v) G(v, V_R) dv$$

- But it is common to shift all distribution factors to the absorption cross section, and then N_i will only count the total population.

$$\sigma_{ik,ave} = \sigma_o \int_{-\infty}^{+\infty} L(\omega, v) G(v, V_R) dv = \sigma_{abs}$$

- In this case, it is now a cross section for the atom assembly (but normalized to single molecule), not for single molecule anywhere.



Doppler Broadening on Absorption and Backscatter Coefficients

- Considering the finite laser lineshape (instead of single frequency), the effective absorption coefficient or total scatter coefficient is a convolution of the atomic absorption coefficient with the laser line shape :

$$\beta_{eff}(\omega_L,\omega_o,\lambda) = \int_{-\infty}^{+\infty} \alpha_{ik}(\omega,\omega_o) g_L(\omega,\omega_L) R_{B\lambda}\, d\omega$$

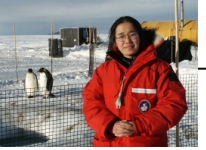
$$\beta_{eff}(\omega_L,\omega_0,\lambda) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \sigma_{ik}(\omega,v) N_i(v,V_R) g_L(\omega,\omega_L)\, dv d\omega$$

$$= N_o \sigma_o \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} L(\omega,v) G(v,V_R) g_L(\omega,\omega_L)\, dv\, d\omega$$

- It is common to shift all distribution factors to the effective backscatter cross section to form an effective cross section for the atom assembly, and then N_i will only count the total population.

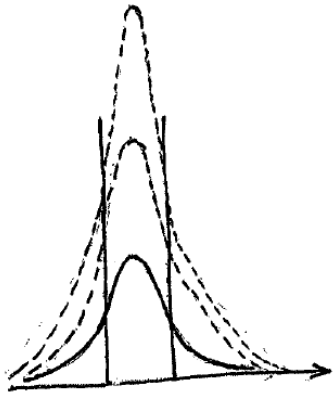
$$\sigma_{eff} = \sigma_o \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} L(\omega,v) G(v,V_R) g_L(\omega,\omega_L)\, dv\, d\omega$$

$$= \int_{-\infty}^{+\infty} \sigma_{abs} g_L(\omega,\omega_L)\, d\omega$$

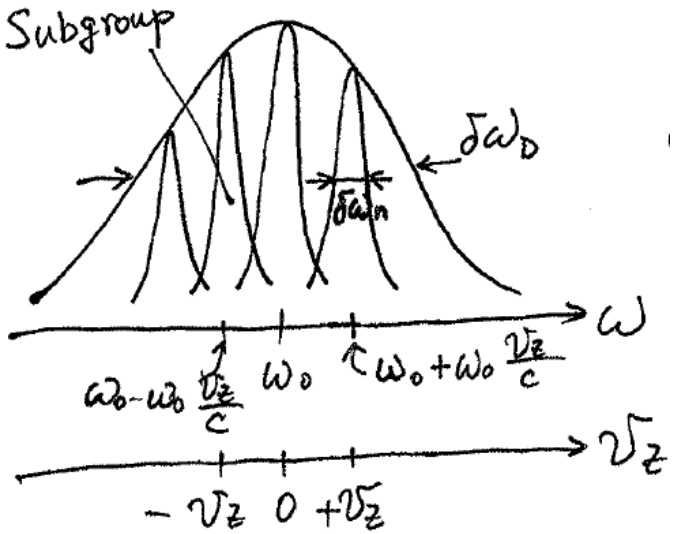


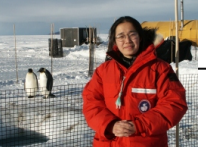
About Doppler-free Spectroscopy

- In Doppler-free or sub-Doppler spectroscopy, we have to use single atom/molecule absorption cross section that is Lorentzian with different velocity distributions of population to derive the sub-Doppler feature.
- How to defeat Doppler broadening to achieve Doppler-free spectroscopy? – Choose a subgroup of atom velocity!
- Homogeneous broadening vs. inhomogeneous broadening



Example: 3 atoms' contribution same $\delta\omega$, but 3 times of peak intensity.





Doppler Technique to Measure Temperature and Wind

- Doppler effect is commonly experienced by moving particles, such as atoms, molecules, and aerosols. It is the apparent frequency change of radiation that is perceived by the particles moving relative to the source of the radiation. This is called Doppler shift or Doppler frequency shift.

$\nu = \nu_0 (1 - \frac{v}{c})$

- Doppler frequency shift is proportional to the radial velocity along the line of sight (LOS) of the radiation -

$= \nu_0 - \frac{v}{c/\nu_0}$

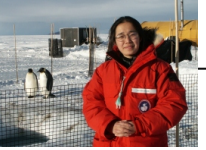
$$\omega = \omega_0 - \vec{k} \cdot \vec{v}$$

$$\Delta\omega = \omega - \omega_0 = -\vec{k} \cdot \vec{v} = -\omega_0 (v/c)\cos\theta$$

$$\Delta\nu = -\nu_0 (v/c)\cos\theta = -(v/\lambda_0)\cos\theta$$

$= \nu_0 - \frac{v}{\lambda_0}$

where ω_0 is the radiation frequency at rest, ω is the shifted frequency, k is the wave vector of the radiation ($k=2\pi/\lambda$), and v is the particle velocity.



Doppler Technique

- Due to particles' thermal motions in the atmosphere, the distribution of perceived frequencies for all particles mirrors their velocity distribution. According to the Maxwellian velocity distribution (Gaussian),

$$P(v_R \rightarrow v_R + dv_R) \propto \exp\left(-Mv_R^2/2k_BT\right)dv_R$$

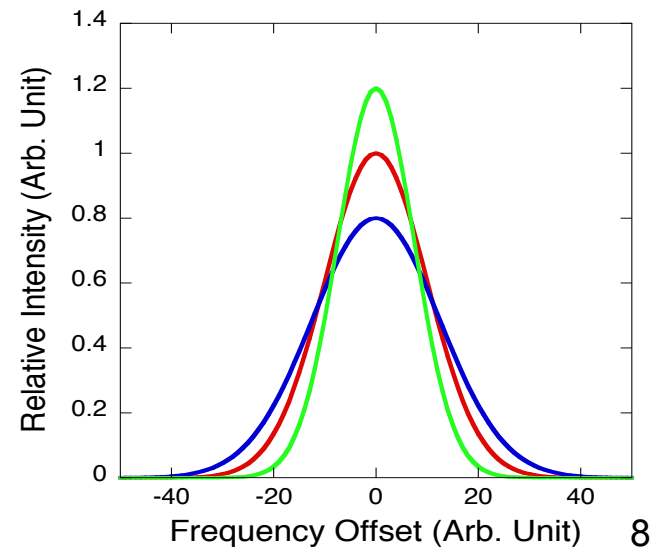
$$\omega = \omega_0 + \vec{k}\cdot\vec{v} = \omega_0\left(1+\frac{v_R}{c}\right) \Rightarrow v_R = \frac{\omega-\omega_0}{\omega_0/c} = \frac{\nu-\nu_0}{\nu_0/c}$$

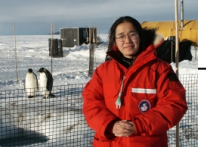
- Substituting v_R into the probability distribution, we obtain the power spectral density distribution (i.e., intensity versus the perceived frequency by moving particles) as a Gaussian lineshape,

$$I \propto \exp\left(-\frac{M(\nu-\nu_0)^2}{2k_BT(\nu_0/c)^2}\right)(c/\nu_0)d\nu$$

- This is called Doppler broadening of a line. The peak is at $\omega = \omega_0$ and the rms width is

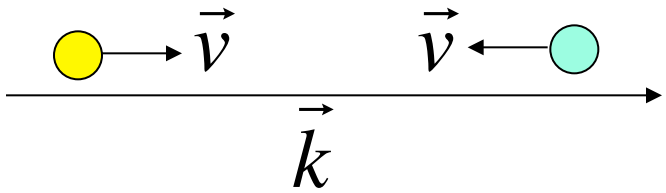
$$\sigma_{rms} = \frac{\nu_0}{c}\sqrt{\frac{k_BT}{M}} = \frac{1}{\lambda_0}\sqrt{\frac{k_BT}{M}}$$





Doppler Shift in Absorption

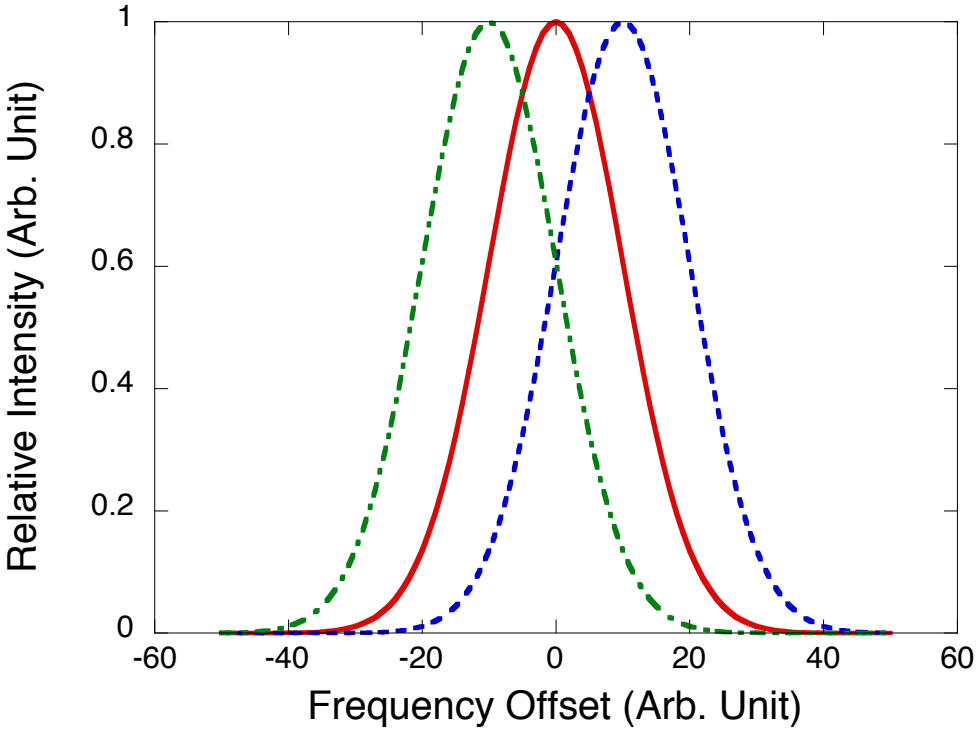
$$\Delta\omega = \omega - \omega_0 = -\vec{k}\cdot\vec{v} = -\omega_0 \frac{v\cos\theta}{c} \qquad (12.13)$$



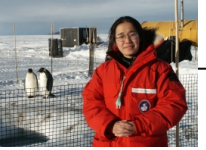
Emitter and receiver move towards each other:

-Blue shift in perceived radiation frequency

-Red shift in absorption peak frequency



- The velocity measurements of lidar, radar, and sodar all base on the Doppler shift principle !



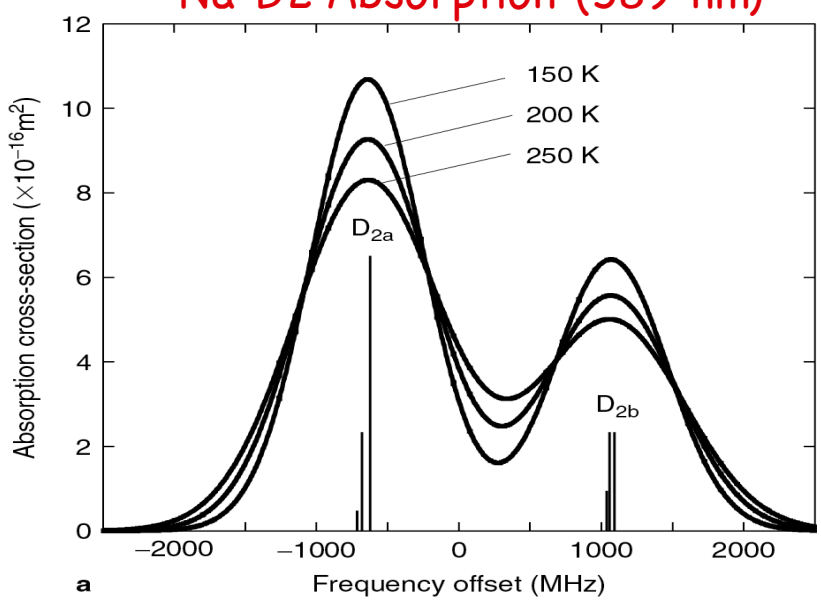
Doppler Broadening in Resonance Absorption Lines

$$\sigma_{rms} = \frac{\nu_0}{c}\sqrt{\frac{k_B T}{M}} = \frac{1}{\lambda_0}\sqrt{\frac{k_B T}{M}}$$

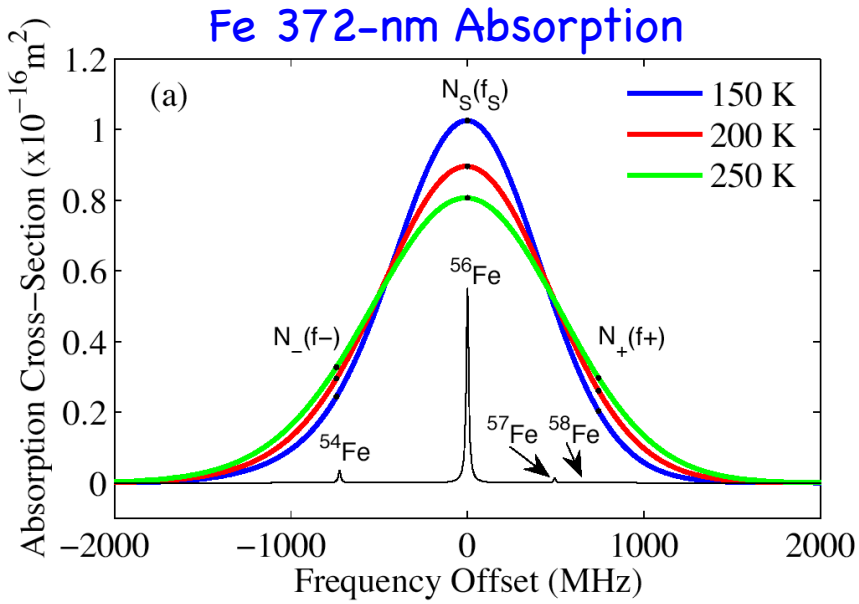
T ↗ ⇨ σ_{rms} ↗

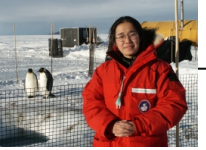
M ↗ ⇨ σ_{rms} ↘

Na D2 Absorption (589 nm)



Fe 372-nm Absorption





Doppler Shift and Broadening in Resonance Fluorescence

- When an atom emits a resonance fluorescence photon, the photon has Doppler shift relative to the center freq. of the atomic absorption line as

$$\omega = \omega_0 + \vec{k}\cdot\vec{v} = \omega_0\left(1+\frac{v_R}{c}\right) \Rightarrow v_R = \frac{\omega-\omega_0}{\omega_0/c} = \frac{\nu-\nu_0}{\nu_0/c}$$

- According to the Maxwellian velocity distribution, the relative probability that an atom/molecule in a gas at temperature T has its velocity component along the line of sight between v_R and v_R+dv_R is

$$P(v_R \rightarrow v_R + dv_R) \propto \exp\left(-Mv_R^2/2k_BT\right)dv_R$$

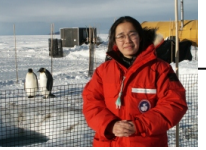
- Substitute the v_R equation into the Maxwellian distribution,

$$I \propto \exp\left(-\frac{M(\nu-\nu_0)^2}{2k_BT(\nu_0/c)^2}\right)(c/\nu_0)d\nu$$

- Therefore, the rms width of the Doppler broadening is

$$\sigma_{rms} = \nu_0/c\sqrt{k_BT/M} = \frac{1}{\lambda_0}\sqrt{k_BT/M}$$

1 time



Doppler Shift in Rayleigh Scattering

- Refer to textbook 5.2.2.4 Lidar wind vs radar wind measurements

Momentum Conservation $m\vec{v}_1 + \hbar\vec{k}_1 = m\vec{v}_2 + \hbar\vec{k}_2$

Energy Conservation $\frac{1}{2}mv_1^2 + \hbar\omega_1 = \frac{1}{2}mv_2^2 + \hbar\omega_2$

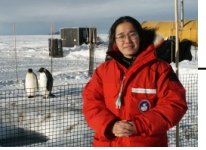
$$\boxed{\omega_1 = \omega_2 + \vec{k}_1 \cdot \vec{v}_1 - \vec{k}_2 \cdot \vec{v}_2 + \frac{\hbar k_1^2}{2m} - \frac{\hbar k_2^2}{2m}}$$

- For Rayleigh or radar backscatter signals, we have

$$\vec{k}_2 \approx -\vec{k}_1 \qquad \vec{v}_2 \approx \vec{v}_1$$

- The frequency shift for Rayleigh or radar backscattering is

$$\boxed{\Delta\omega_{Rayleigh,backscatter} = \omega_2 - \omega_1 = -2\vec{k}_1 \cdot \vec{v}_1}$$



Doppler Broadening in Rayleigh Scatter

- To derive the Doppler broadening, let's write the Doppler shift as

$$\omega = \omega_0\left(1 - \frac{2v_R}{c}\right) \Rightarrow v_R = \frac{\omega_0 - \omega}{2\omega_0/c} = \frac{\nu_0 - \nu}{2\nu_0/c}$$

- According to the Maxwellian velocity distribution, the relative probability that an atom/molecule in a gas at temperature T has its velocity component along the line of sight between v_R and v_R+dv_R is

$$P(v_R \to v_R + dv_R) \propto \exp\left(-Mv_R^2/2k_BT\right)dv_R$$

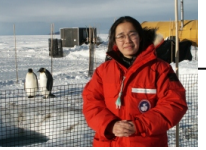
- Substitute the v_R equation into the Maxwellian distribution,

$$I \propto \exp\left(-\frac{M(\nu_0 - \nu)^2}{2k_BT(2\nu_0/c)^2}\right)(c/2\nu_0)d\nu$$

- Therefore, the rms width of the Doppler broadening is

$$\sigma_{rms} = 2\nu_0/c\sqrt{k_BT/M} = \frac{2}{\lambda_0}\sqrt{k_BT/M}$$

2 times !



Doppler Effect in Rayleigh Scattering

- In the atmosphere when aerosols present, the lidar returns contains a narrow spike near the laser frequency caused by aerosol scattering riding on a Doppler broadened molecular scattering profile.

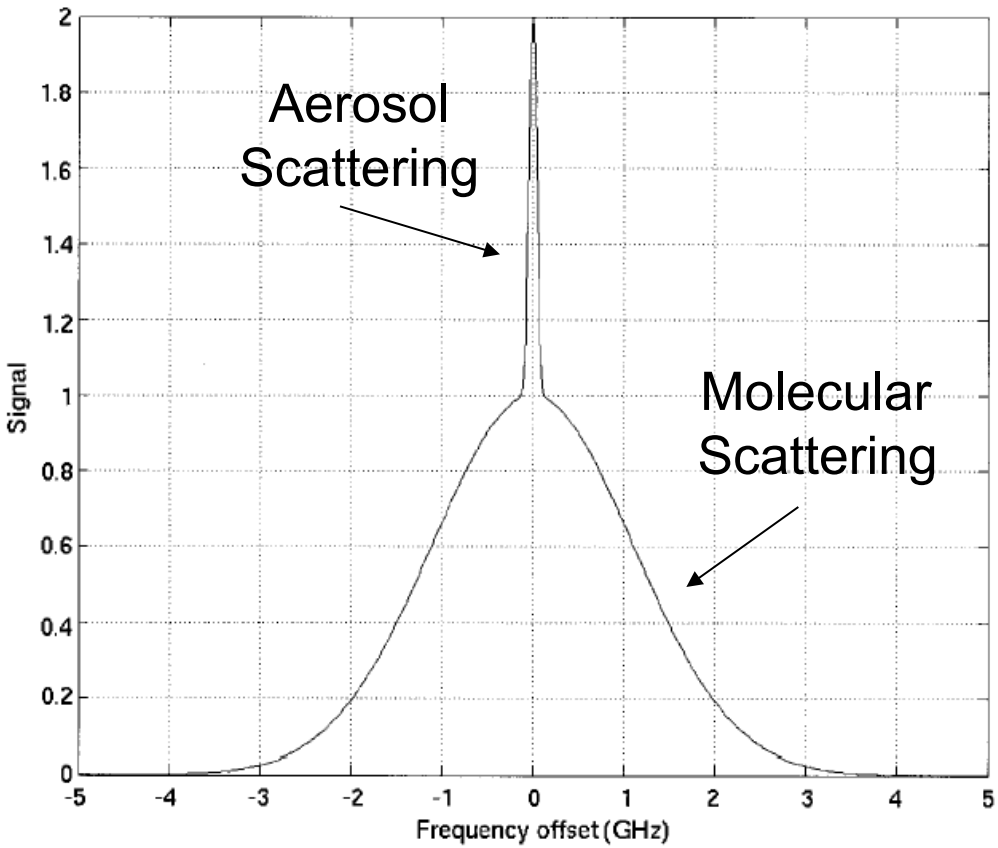
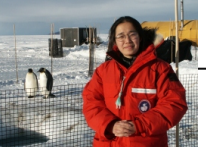


Fig. 5.1. Spectral profile of backscattering from a mixture of molecules and aerosols for a temperature of 300 K. The spectral width of the narrow aerosol return is normally determined by the line width of the transmitting laser.

At T = 300 K, the Doppler broadened FWHM for Rayleigh scattering is 2.58GHz, not 1.29GHz.
Why?

Because Rayleigh backscatter signals have 2 times of Doppler shift!

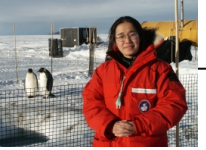
Courtesy of Dr. Ed Eloranta
University of Wisconsin



Resonance Fluorescence Doppler versus Rayleigh Doppler

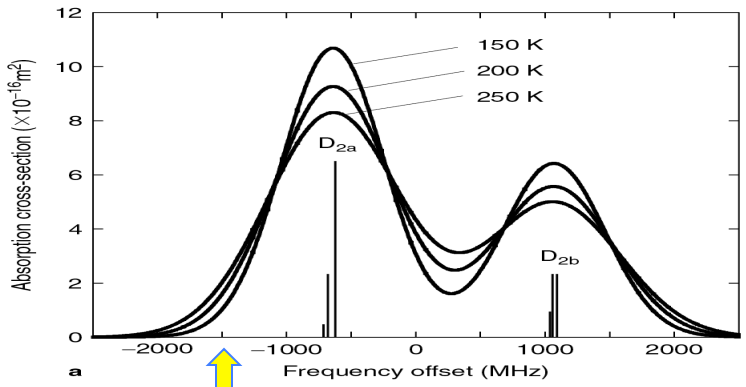
- Atomic absorption lines provide a natural frequency analyzer or frequency discrimination. This is because the absorption cross section undergoes Doppler shift and Doppler broadening. Thus, when a narrowband laser scans through the absorption lines, different absorption and fluorescence strength will be resulted at different laser frequencies. By using a broadband receiver to collect the returned resonance fluorescence, we can easily obtain the line shape of the absorption cross section so that we can infer wind and temperature. There is no need to measure the fluorescence spectrum. - Resonance fluorescence Doppler technique

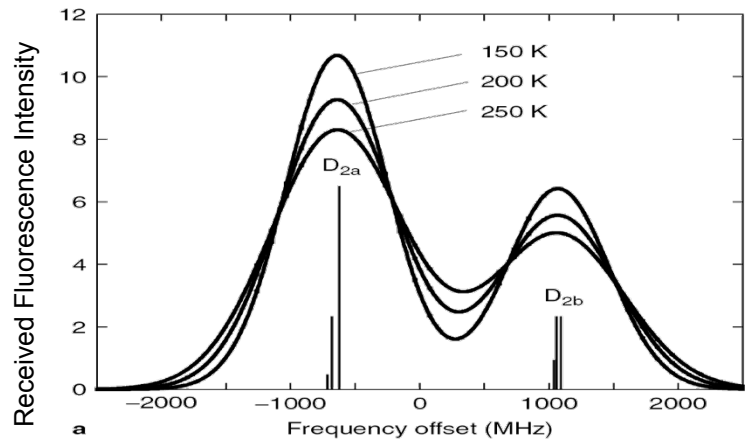
- Rayleigh scattering also undergoes Doppler shift and broadening, however, it is not frequency discriminated. In other words, when scanning a laser frequency, the backscattered Rayleigh signal gives nearly the same Doppler broadened line width, independent of laser frequency. Thus, the atmosphere molecule scattering does not provide frequency discrimination. A frequency analyzer must be implemented into the lidar receiver to discriminate the return light frequency, i.e., analyze Rayleigh scattering spectrum to infer wind and temperature. - Rayleigh Doppler technique

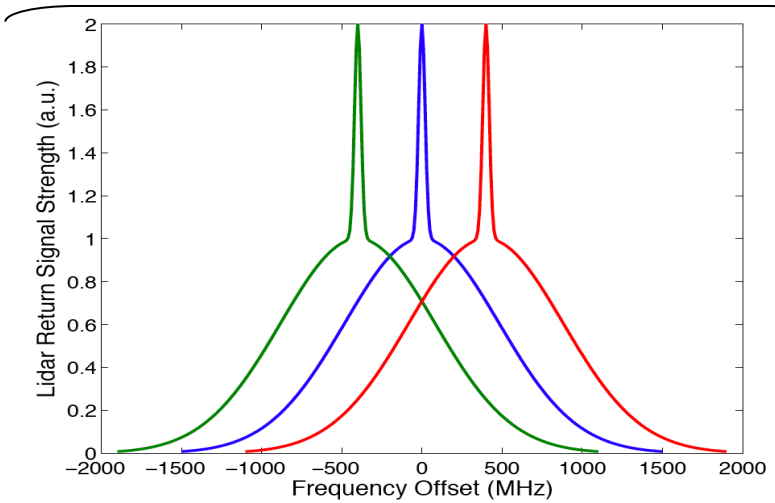


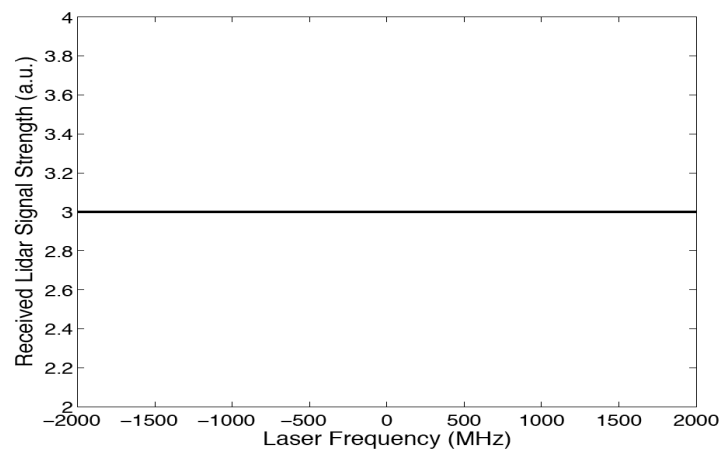
Resonance Fluorescence Doppler versus Rayleigh Doppler

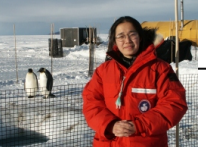
- How will the lidar return signal strength (vs. laser frequency) change when the lidar receiver is broadband and we scan the narrowband laser frequency - in resonance fluorescence case and in Rayleigh scattering case?





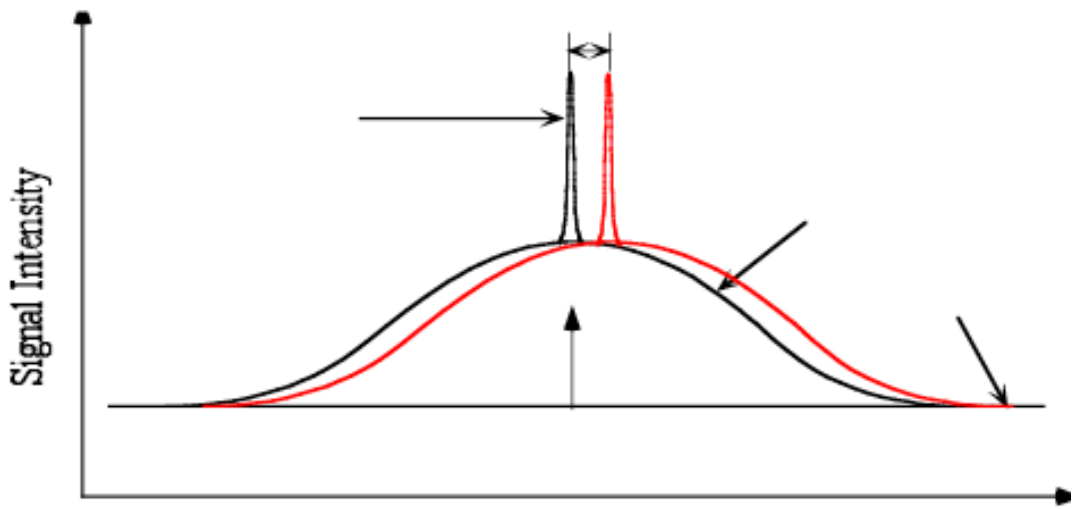


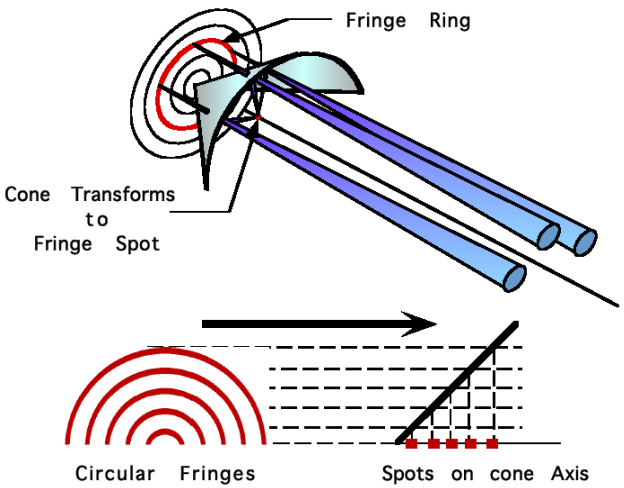




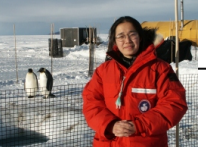
Fringe Imaging

- Fringe imaging is to use high resolution Fabry-Perot etalon to image the lidar returns, i.e., turn spectral distribution to spatial distribution.
- Fringe width is used to derive temperature



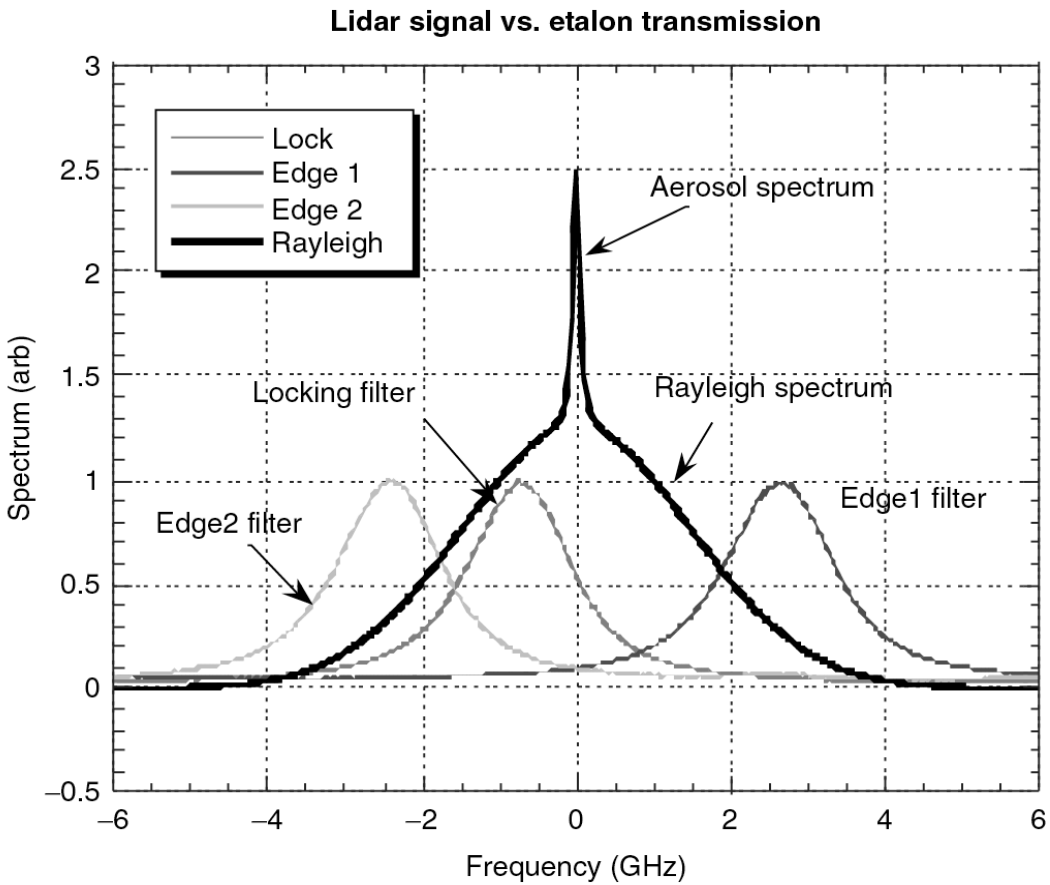


- Diameters of the circles give the Doppler shift when compared with a known wavelength fringe, while the fringe width is an indication of Rayleigh temperature.
- Current issues: suffer low signal levels above 50 km because of decreasing atmospheric density and "waste" of photons in fringes.



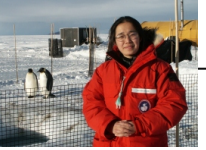
Edge Filter

- Edge filter is to use either high resolution Fabry-Perot etalons or atomic/molecular vapor cell filters to reject part of the return spectra while passing the other part of the spectra to two different channels. The temperature information is then derived from the ratio of signals from these two channels.



$$\frac{N_1}{N_2} = \frac{\xi_1 f_{m1}(T,P,V_R)}{\xi_2 f_{m2}(T,P,V_R)}$$

If V_R is known, then T can be derived.
If T is known, then V_R can be derived.



Doppler Effects in Fe Boltzmann Lidar

- Pure Fe signal and pure Rayleigh signal in Fe region are

$$N_{Fe}(\lambda,z)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_{eff}(\lambda,T,\sigma_L)R_{B\lambda}n_{Fe}(z)\right]\Delta z\left(\frac{A}{4\pi z^2}\right)\left(T_a^2(\lambda)T_c^2(\lambda,z)\right)\left(\eta(\lambda)G(z)\right)$$

- Pure Rayleigh signal in molecular scattering region is

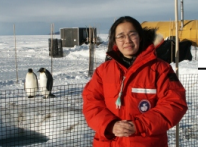
$$N_R(\lambda,z_R)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_R(\pi,\lambda)n_R(z_R)\right]\Delta z\left(\frac{A}{z_R^{\ 2}}\right)T_a^{\ 2}(\lambda,z_R)\left(\eta(\lambda)G(z_R)\right)$$

- From physics point of view, the normalized Fe count is

$$N_{Norm}(\lambda,z)=\frac{N_{Fe}(\lambda,z)}{N_R(\lambda,z_R)T_c^2(\lambda,z)}=\frac{\sigma_{eff}(\lambda,T,\sigma_L)R_{B\lambda}n_{Fe}(\lambda,z)}{4\pi\sigma_R(\lambda,)n_R(z_R)}$$

- Note that for the two channels (372 and 374 nm) of Fe Boltzmann lidar, Boltzmann factor is in the Fe number density, while both effective cross-sections (coefficients) experience Doppler shift and Doppler broadening.

- The particles (atoms or molecules) on each energy level have a velocity distribution given by Maxwell velocity distribution, while the total population of each energy level obey the Boltzmann distribution law.



Doppler Effects in Fe Boltzmann Lidar

- For two channels of Fe Boltzmann lidar,

$$N_{Norm}(\lambda_{372},z)=\frac{N_{Fe}(\lambda_{372},z)}{N_R(\lambda_{372},z_R)T_c^2(\lambda_{372},z)}=\frac{\sigma_{eff}(\lambda_{372},T,\sigma_L)R_{B\lambda_{372}}n_{Fe}(\lambda_{372},z)}{4\pi\sigma_R(\lambda_{372},)n_R(z_R)}$$

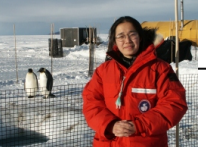
$$N_{Norm}(\lambda_{374},z)=\frac{N_{Fe}(\lambda_{374},z)}{N_R(\lambda_{374},z_R)T_c^2(\lambda_{374},z)}=\frac{\sigma_{eff}(\lambda_{374},T,\sigma_L)R_{B\lambda_{374}}n_{Fe}(\lambda_{374},z)}{4\pi\sigma_R(\lambda_{374},)n_R(z_R)}$$

$$=\frac{N_{Fe}(\lambda_{374},z)}{N_R(\lambda_{374},z_R)T_c^2(\lambda_{374},z)}=\frac{\sigma_{eff}(\lambda_{374},T,\sigma_L)R_{B\lambda_{374}}n_{Fe}(\lambda_{372},z)}{4\pi\sigma_R(\lambda_{374},)n_R(z_R)}\frac{g_2}{g_1}\exp(-\Delta E/k_BT)$$

- Take the Boltzmann temperature ratio as

$$\boxed{R_T(z)=\frac{N_{norm}(\lambda_{374},z)}{N_{norm}(\lambda_{372},z)}=\frac{g_2}{g_1}\frac{R_{B374}}{R_{B372}}\left(\frac{\lambda_{374}}{\lambda_{372}}\right)^{4.0117}\frac{\sigma_{eff}(\lambda_{374},T,\sigma_{L374})}{\sigma_{eff}(\lambda_{372},T,\sigma_{L372})}\exp\left(-\Delta E/k_BT\right)}$$

- Using the Fe atoms in the MLT as an "Fe vapor cell", if we scan the laser frequency of a single channel of Fe Boltzmann lidar, we will see a convoluted Doppler broadening of the return Fe signals. This wavelength scan can be used to infer temperature if the laser lineshape is well known, or can be used to infer the laser lineshape if the temperature is known.



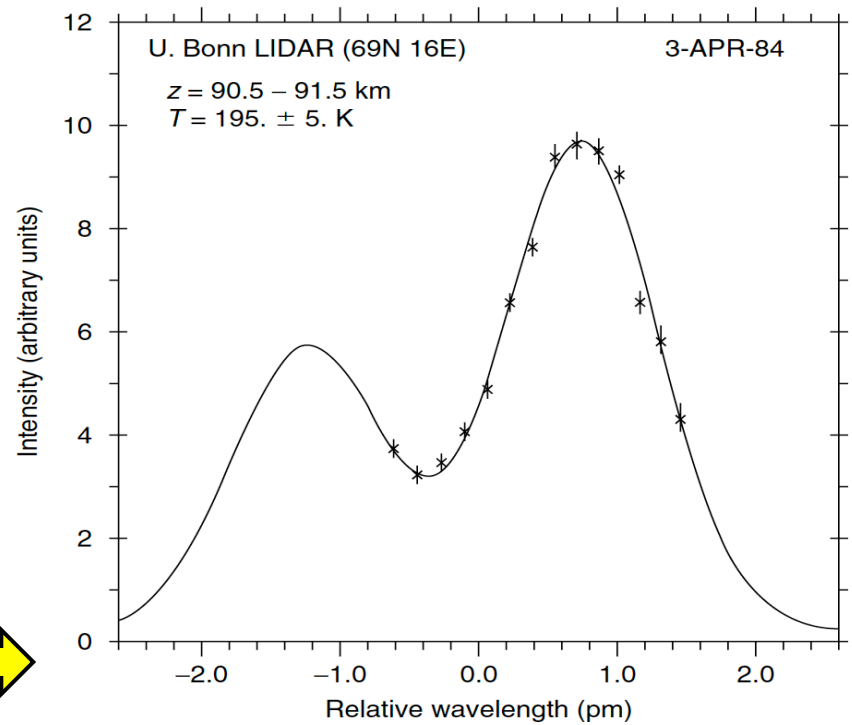
Doppler Scanning Technique

$$N_{Na}(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda)n_{Na}(z)\Delta z\right)\left(\frac{A}{4\pi z^2}\right)\left(\eta(\lambda)T_a^{\,2}(\lambda)T_c^{\,2}(\lambda,z)G(z)\right)$$

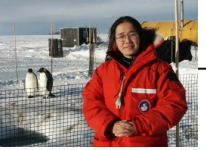
$$N_R(\lambda,z_R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_R(\pi,\lambda)n_R(z_R)\Delta z\right)\left(\frac{A}{z_R^{\,2}}\right)\left(\eta(\lambda)T_a^{\,2}(\lambda,z_R)G(z_R)\right)$$

$$\sigma_{eff}(\lambda,z) = \frac{C(z)}{T_c^{\,2}(\lambda,z)}\frac{N_{Na}(\lambda,z)}{N_R(\lambda,z_R)}$$

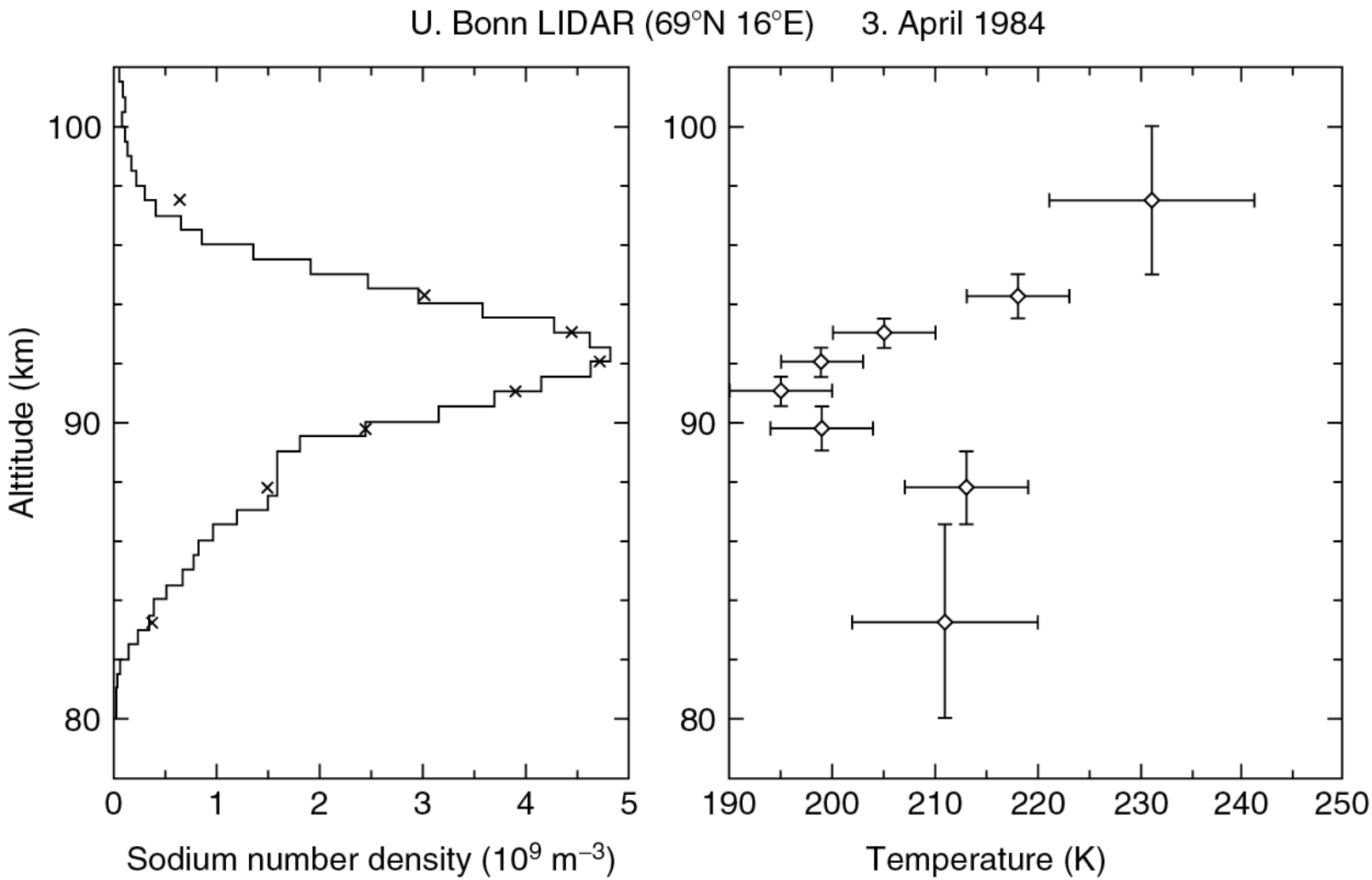
where $C(z) = \dfrac{\sigma_R(\pi,\lambda)n_R(z_R)}{n_{Na}(z)}\dfrac{4\pi z^2}{z_R^{\,2}}$

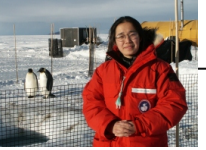


Least-square fitting gives temp
[Fricke and von Zahn, JATP, 1985]



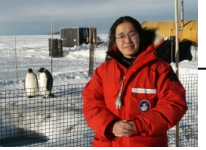
Scanning Na Lidar Results





Various Physical Interactions and Techniques are Applied to Doppler Tech

- When resonance absorption and fluorescence is available (not quenched), the resonance Doppler lidar can be employed to probe Doppler broadening and Doppler shift. It can be atomic resonance fluorescence or molecular resonance fluorescence in the upper atmosphere.
- If molecular absorption is available, then DIAL lidar can be employed to infer the Doppler broadening in molecular absorption for temperature. This is doable in the lower atmosphere.
- Rayleigh scattering experiences Doppler broadening, so it can be measured using various techniques (e.g., edge filter, interferometers, etc.) to infer the temperature.
- High-spectral-resolution lidar (HSRL) can help remove aerosol contamination to reveal Doppler broadening for temperature.
- Vibrational-rotational Raman scattering should experience similar Doppler broadening as Rayleigh scattering, but it has shifted frequencies of return signals, which may help avoid the contamination of aerosol scattering to Rayleigh scattering.



Summary

- It is accurate to say that absorption or backscatter coefficient experiences Doppler broadening, while single atom/molecule experiences Doppler shift. The velocity distribution of particles brings in the Doppler broadening in an assembly.
- Doppler technique utilizes the Doppler effect (frequency shift and linewidth broadening) by moving particles to infer wind and temperature information. It is widely applied in lidar, radar and sodar technique as well as passive optical remote sensing.
- Absorption experiences 1 time Doppler shift and broadening, while Rayleigh and radar scattering experiences 2 times of Doppler shift and broadening.
- Doppler effects also exist in Fe Boltzmann lidar, where the population on each energy level obeys Maxwellian velocity distribution, while the total population of each energy level obeys Boltzmann distribution law.
- Resonance fluorescence Doppler lidar technique applies scanning or ratio technique to infer the temperature and wind from the Doppler spectroscopy, while the Doppler spectroscopy is inferred from intensity ratio at different frequencies.