Lecture 04. Fundamentals of Lidar Remote Sensing (2) – "Lidar Equation"

- Introduction
- Physical Picture of Lidar Equation
- Fundamental Lidar Equation
- Different Forms of Lidar Equation
- An Example of Applying Lidar Equation
- Summary and Questions

Introduction

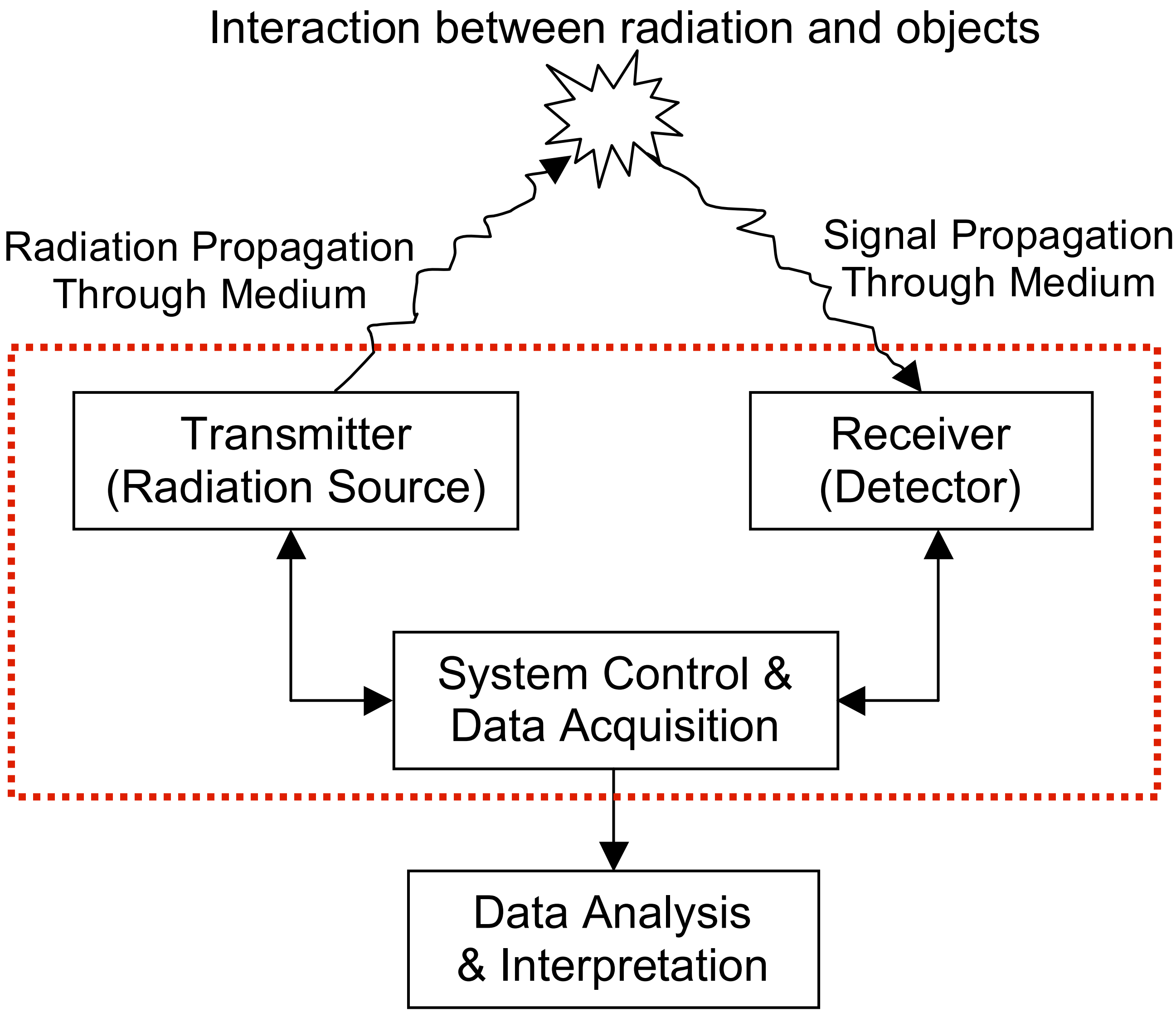
- Lidar equation is the fundamental equation in the lidar field to relate the received photon counts (or light power) to the transmitted laser photon numbers (or laser power), the light transmission in atmosphere or medium, the physical interaction between light and objects, the photon receiving probability, and the lidar system efficiency and geometry, etc.

- The lidar equation is based on the physical picture of lidar remote sensing, and derived under two assumptions: independent scattering and single scattering.

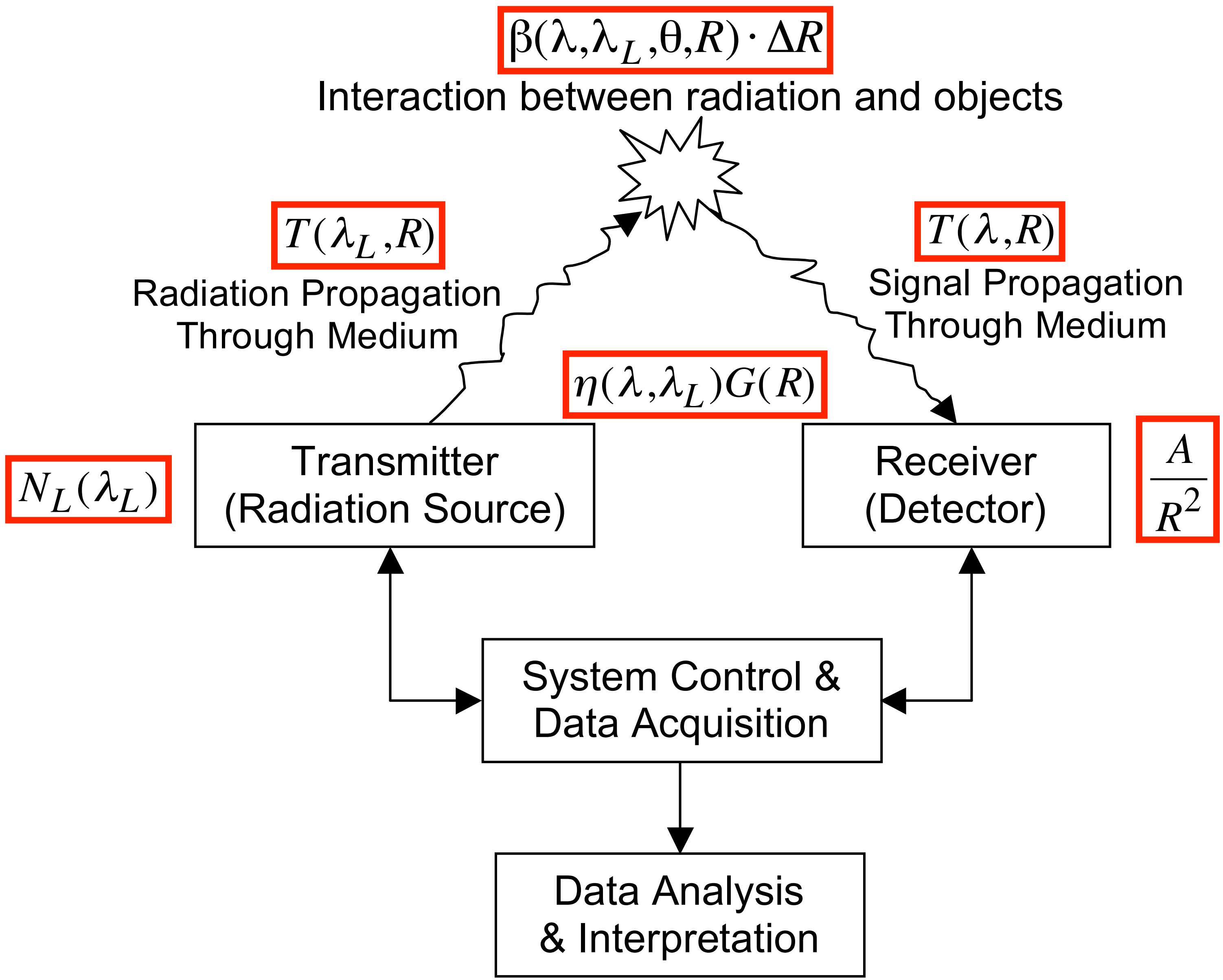
- Different lidars may use different forms of the lidar equation, but all come from the same picture.

A good reference for Lecture 4 is the Laser Remote Sensing textbook Chapter 5, Section 5.2.1 (pages 191-206).

Picture of Lidar Remote Sensing



Physical Picture of Lidar Equation



Factors in Lidar Equation

The received photon counts N_S are related to the factors

$$N_S(\lambda, R) \propto$$

$N_L(\lambda_L)$	Transmitted laser photon number
$T(\lambda_L, R)$	Laser photon transmission through medium
$\beta(\lambda, \lambda_L, \theta, R) \cdot \Delta R$	Probability of a transmitted photon to be scattered
$T(\lambda, R)$	Signal photon transmission through medium
$\dfrac{A}{R^2}$	Probability of a scattered photon to be collected
$\eta(\lambda, \lambda_L) G(R)$	Lidar system efficiency and geometry factor

Considerations for Lidar Equation

- In general, the interaction between the light photons and the particles is a scattering process.
- The expected photon counts are proportional to the product of the

(1) transmitted laser photon number,

(2) probability that a transmitted photon is scattered,

(3) probability that a scattered photon is collected,

(4) light transmission through medium, and

(5) overall system efficiency.

- Background photon counts and detector noise also contribute to the expected photon counts.

Lidar Equation Development

- With the picture of lidar remote sensing in mind, one can follow the photon path to develop a lidar equation to quantify how the received photon number or light power is related to the transmitted photons, light transmission, scattering probability, receiver probability, system efficiency and geometry factors as well as background light and detector noise --

$$\boxed{N_S(\lambda,R) = N_L(\lambda_L)\cdot \eta(\lambda_L)\cdot T(\lambda_L,R)\cdot \left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot T(\lambda,R)\cdot \frac{A}{R^2}\cdot \eta(\lambda)\cdot G(R) + N_B} \tag{4.1}$$

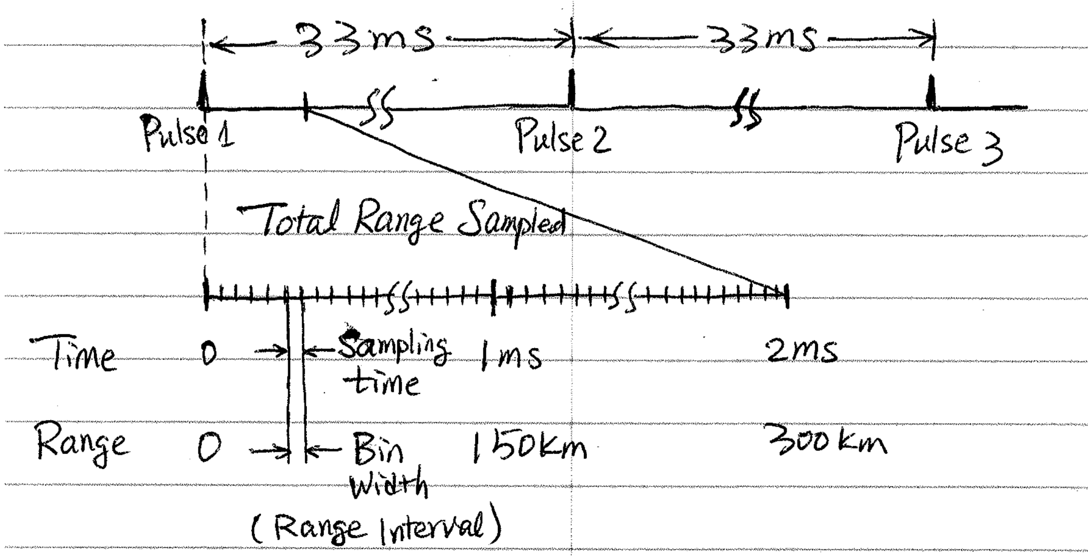
- In this development, photon or light power is regarded as scalar quantities, so the calculation sequence does not matter. However, if vector or matrix is involved, e.g., for polarization study, then it is necessary to consider the matrix computation sequence for the lidar equation.

ΔR in Lidar Equation

- Refer to our textbook Section 5.2.1.1 (page 192)
- ΔR is the thickness/width of the range bin, range interval or scattering layer thickness. It can also be called bin width or range resolution, determined by range gating choices.
- ΔR is related to the sampling time $\tau_{sampling}$ by $\Delta R = c\tau_{sampling}/2$.
- In principle, ΔR can be arbitrary, up to the choice of users. That is, users can choose any sampling time $\tau_{sampling}$ as long as the receiver digitizer can handle ($\tau_{sampling} \geq \tau_{digitizer}$). It does not have to be limited by the laser pulse duration time τ_{pulse} as $\tau_{sampling}$ can be longer than, equal to, or shorter than τ_{pulse}.
- For atmospheric distributed (or diffuse) scatters, the minimum meaningful ΔR is limited by the pulse duration time $c\tau_{pulse}/2$. For discrete targets, the minimum meaningful ΔR is limited by the digitizer resolution $\tau_{digitizer}$ (in the range of ns or ps) $c\tau_{digitizer}/2$.

Illustration of Time Scales in Lidar

A pulsed laser with 30 Hz repetition rate is used as an example to illustrate the different time scale terms in lidar equation.



RR = 30 Hz

Pulse Interval = 1/30 = 33 ms

Assume

$T_{digitizer} = 2\,ns \rightarrow$ minimum $\Delta R = 30\,cm$

$T_{pulse} = 5\,ns \rightarrow \Delta R = 75\,cm$

$T_{sampling} = 160\,ns \rightarrow \Delta R = 24\,m$

Minimum $T_{sampling} \geq T_{digitizer}$, while

$T_{sampling} > T_{pulse}$, $T_{sampling} = T_{pulse}$, $T_{sampling} < T_{pulse}$

Fundamental Lidar Equation

$$N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B \quad (4.2)$$

- **N_S -- expected photon counts detected at λ and R**
- **1st term -- the transmitted laser photon number;**
- **2nd term -- the probability of a transmitted photon to be scattered by the objects into a unit solid angle;**
- **3rd term -- the probability of a scatter photon to be collected by the receiving telescope;**
- **4th term -- the light transmission through medium for the transmitted laser and return signal photons;**
- **5th term -- the overall system efficiency;**
- **6th term N_B -- background and detector noise counts.**

Basic Assumptions in Lidar Equation

- The lidar equation is developed under two assumptions: the scattering processes are independent, and only single scattering occurs.

- Independent scattering means that particles are separated adequately and undergo random motion so that the contribution to the total scattered energy by many particles have no phase relation or no phase coherence. Thus, the total intensity is simply a sum of the intensity scattered from each particle.

- Single scattering implies that a photon is scattered only once. Multiple scatter is excluded in the considerations of this basic lidar equation.

Cases Violating Assumptions of Independent & Single Scattering

- Non-independent scattering: Coherent Doppler lidars consider target speckle noise and scintillation due to atmospheric turbulence. That is, elastic scattering from aerosols can have phase coherence, so causing interference. Consequently, the total intensity received is not a simple sum of individual particles' contributions. Such effects limit the effective collection area of telescope (see textbook Section 7.3.8).

- Multiple scattering of a photon occurs in clouds or dense atmosphere/medium. Not only does it change the scattering probability but also causes time delay.

1st Term: Transmitted Photon Number

$$N_S(\lambda,R) = \boxed{N_L(\lambda_L)} \cdot \left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right] \cdot \frac{A}{R^2} \cdot \left[T(\lambda_L,R)T(\lambda,R)\right] \cdot \left[\eta(\lambda,\lambda_L)G(R)\right] + N_B$$

$$\boxed{N_L(\lambda_L) = \left(\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right)} \qquad (4.3)$$

$$\frac{\text{Laser Power x time bin length}}{\text{Planck constant x Laser frequency}}$$

$$= \frac{\text{Transmitted laser energy within time bin}}{\text{Single laser photon energy}}$$

= Transmitted laser photon number within time bin length

Let's estimate how many photons are sent out with a 20 mJ laser pulse at 589 nm

2nd Term: Probability to be Scattered

$$N_S(\lambda,R) = N_L(\lambda_L)\cdot\boxed{\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]}\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

Angular scattering probability – the probability that a transmitted photon is scattered by scatters into a unit solid angle.

Angular scattering probability =
volume scatter coefficient β
x scattering layer thickness ΔR (4.4)

Volume Scatter Coefficient β

Volume scatter coefficient β is equal to

$$\boxed{\beta(\lambda,\lambda_L,R) = \sum_i \left[\frac{d\sigma_i(\lambda_L,\theta)}{d\Omega} n_i(R) p_i(\lambda) \right]} \quad (\mathrm{m^{-1}sr^{-1}}) \qquad (4.5)$$

$\frac{d\sigma_i(\lambda_L)}{d\Omega}$ is the differential scatter cross-section of single particle in species i at scattering angle θ ($\mathrm{m^2sr^{-1}}$)

$n_i(R)$ is the number density of scatter species i ($\mathrm{m^{-3}}$)

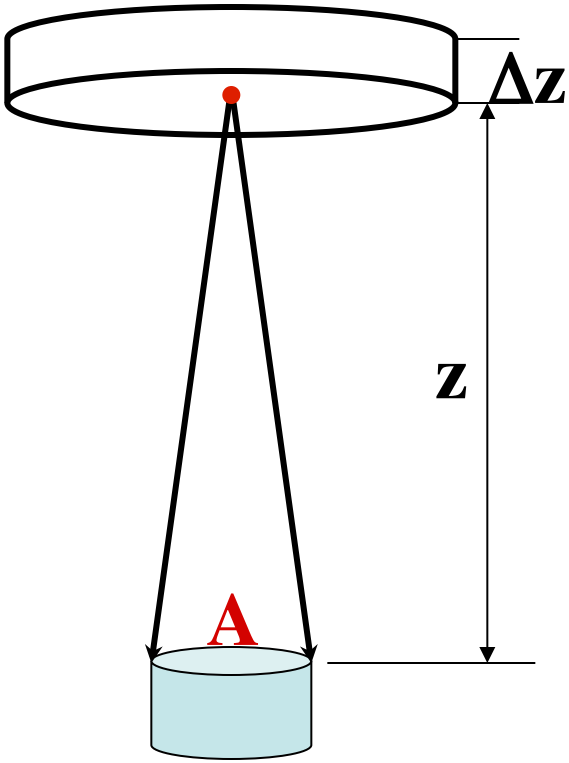
$p_i(\lambda)$ is the probability of the scattered photons falling into the wavelength λ.

Volume scatter coefficient β is the probability per unit distance travel that a photon is scattered into wavelength λ in unit solid angle at angle θ.

3rd Term: Probability to be Collected

$$N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\boxed{\frac{A}{R^2}}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

The probability that a scatter photon is collected by the receiving telescope,
i.e., the solid angle subtended by the receiver aperture to the scatterer.



4th Term: Light Transmission

$$N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\boxed{\left[T(\lambda_L,R)T(\lambda,R)\right]}\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

The atmospheric transmission of laser light at outgoing wavelength λ_L and return signal at wavelength λ

Transmission for laser light

Transmission for return signal

$$\boxed{\begin{aligned} T(\lambda_L,R) &= \exp\left[-\int_0^R \alpha(\lambda_L,r)dr\right] \\ T(\lambda,R) &= \exp\left[-\int_0^R \alpha(\lambda,r)dr\right] \end{aligned}} \quad (4.6)$$

Where $\alpha(\lambda_L, R)$ and $\alpha(\lambda, R)$ are extinction coefficients (m^{-1})

Extinction Coefficient α

$$\boxed{\alpha(\lambda,R) = \sum_i \left[\sigma_{i,ext}(\lambda) n_i(R)\right]} \qquad (4.7)$$

$\sigma_{i,ext}(\lambda)$ is the extinction cross-section of species i

$n_i(R)$ is the number density of species i

Extinction = Absorption + Scattering (Integrated)

$$\sigma_{i,ext}(\lambda) = \sigma_{i,abs}(\lambda) + \sigma_{i,sca}(\lambda) \qquad (4.8)$$

Total Extinction = Aerosol Extinction + Molecule Extinction

$$\alpha(\lambda,R) = \alpha_{aer,abs}(\lambda,R) + \alpha_{aer,sca}(\lambda,R) + \alpha_{mol,abs}(\lambda,R) + \alpha_{mol,sca}(\lambda,R)$$

(4.9)

5th Term: Overall Efficiency

$$N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\boxed{\left[\eta(\lambda,\lambda_L)G(R)\right]}+N_B$$

$\boxed{\eta(\lambda,\lambda_L) = \eta_T(\lambda_L)\cdot\eta_R(\lambda)}$ is the lidar hardware optical efficiency

(4.10) e.g., mirrors, lens, filters, detectors, etc

$\boxed{G(R)}$ is the geometrical form factor, mainly concerning the overlap of the area of laser irradiation with the field of view of the receiver optics

6th Term: Background Noise

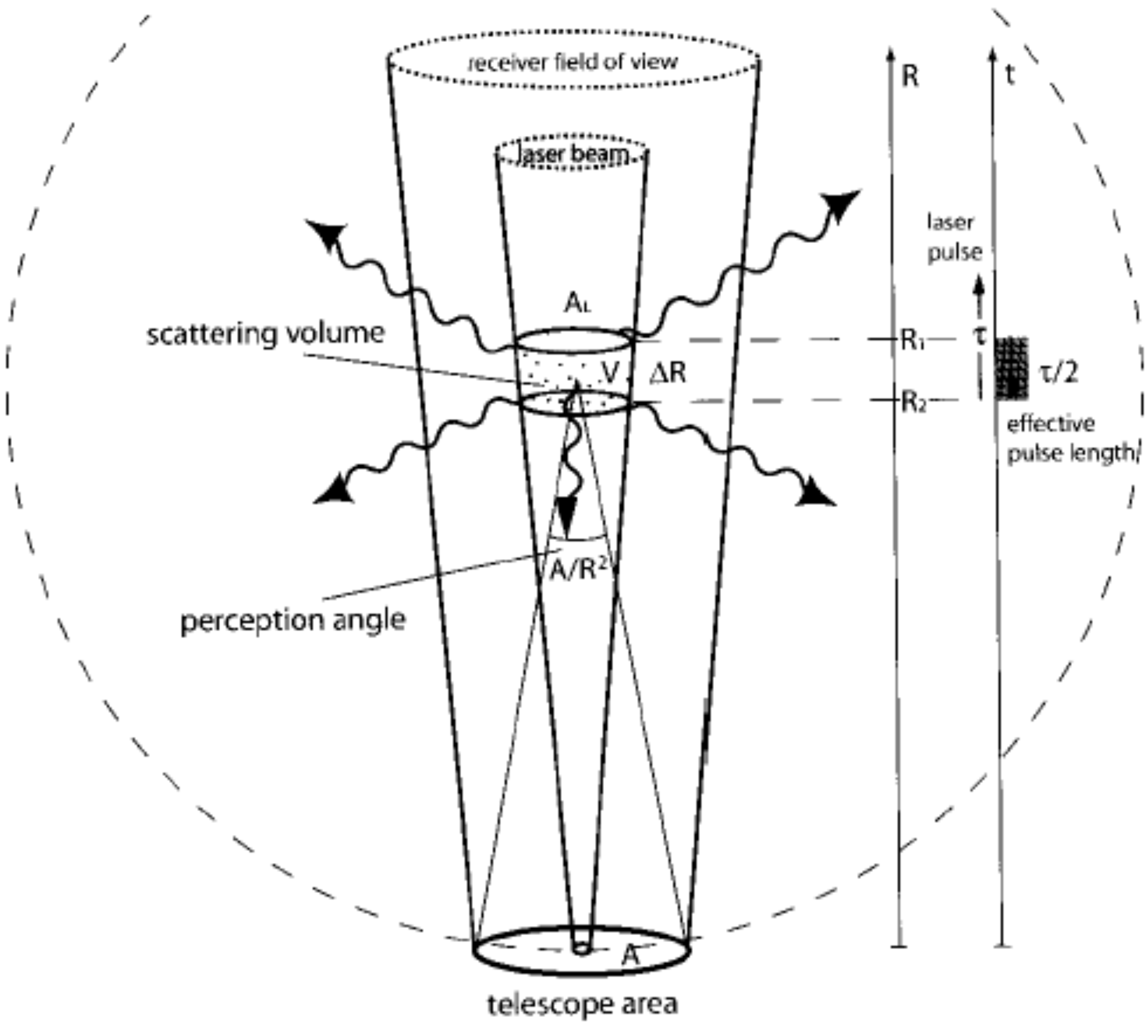
$$N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+\boxed{N_B}$$

$\boxed{N_B}$ is the expected photon counts due to background noise, detector and circuit shot noise, etc.

Up-looking lidar: main background noise comes from solar scattering, solar straight radiation, star/city light.

Down-looking lidar: besides above noise, it could have three extra backgrounds: (1) Specular reflection from water/ice surface; (2) Laser reflectance from ground; (3) Solar reflectance from ground.

Illustration of LIDAR Equation



-- Courtesy of Ulla Wandinger [Introduction to Lidar]

Different Forms of Lidar Equation

- The main difference between upper and lower atmosphere lidars lies in the treatment of backscatter coefficient and atmosphere transmission (extinction).
- Upper atmosphere lidar cares about the backscatter coefficient more than anything else, because (1) the lower atmosphere transmission is cancelled out during Rayleigh normalization, and (2) the extinction caused by atomic absorption can be precisely calculated, thus, extinction is not an issue to upper atmosphere lidar.
- Lower atmosphere lidar relies on both backscatter coefficient and atmospheric extinction, as these are what they care about or something that cannot be cancelled out.

General Form of Lidar Equation

$$\boxed{N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B}$$

(4.2) — Lidar Equation in Photon Count and $\beta(\theta)$

$$\boxed{P_S(\lambda,R) = P_L(\lambda_L)\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+P_B}$$

(4.11) — Lidar Equation in Light Power and β (θ)

$$\boxed{N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta_T(\lambda,\lambda_L,R)\Delta R\right]\cdot\frac{A}{4\pi R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B}$$

(4.12) — Lidar Equation in Photon Count and β_T

General Form of LIDAR Equation

Angular volume scattering coefficient

$$\beta(\lambda,\lambda_L,\theta,R)=\sum_i\left[\frac{d\sigma_i(\lambda_L)}{d\Omega}n_i(R)p_i(\lambda)\right] \tag{4.5}$$

Total scattering coefficient

$$\begin{aligned}\beta_T(\lambda,\lambda_L,R)&=\int_0^{4\pi}\beta(\lambda,\lambda_L,\theta,R)d\Omega\\&=\int_0^{4\pi}\sum_i\left[\frac{d\sigma_i(\lambda_L)}{d\Omega}n_i(R)p_i(\lambda)\right]d\Omega\\&=\sum_i\left[\sigma_i(\lambda_L)n_i(R)p_i(\lambda)\right]\end{aligned} \tag{4.13}$$

Fluorescence Form of Lidar Equation

$$\boxed{N_S(\lambda,R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda,R)n_c(z)R_B(\lambda)\Delta R\right)\left(\frac{A}{4\pi R^2}\right)\left(T_a^2(\lambda,R)T_c^2(\lambda,R)\right)\left(\eta(\lambda)G(R)\right)+N_B} \quad (4.14)$$

- Here, $T_c(R)$ is the transmission resulted from the constituent extinction

$$\boxed{T_c(R) = \exp\left(-\int_{R_{bottom}}^{R} \sigma_{eff}(\lambda,r')n_c(r')\,\mathrm{d}r'\right) = \exp\left(-\int_{R_{bottom}}^{R} \alpha_c(\lambda,r')\,\mathrm{d}r'\right)} \quad (4.15)$$

- Here, $\alpha(\lambda,R)$ is the extinction coefficient caused by the constituent absorption.

$$\boxed{\alpha_c(\lambda,R) = \sigma_{eff}(\lambda,R)n_c(R)} \quad (4.16)$$

- Resonance fluorescence and laser-induced-fluorescence are NOT instantaneous processes, but have delays due to the radiative lifetime of the excited states.

General Lidar Equation in β and α

$$\boxed{\begin{aligned}N_S(\lambda,R) = \left[\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right]\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\left(\frac{A}{R^2}\right) \\ \cdot\exp\left[-\int_0^R \alpha(\lambda_L,r')dr'\right]\exp\left[-\int_0^R \alpha(\lambda,r')dr'\right]\left[\eta(\lambda,\lambda_L)G(R)\right] + N_B\end{aligned}}$$

β is the volume scatter coefficient (4.17)

α is the extinction coefficient

Volume scatter coefficient $$\boxed{\beta(\lambda,\lambda_L,R) = \sum_i \left[\frac{d\sigma_i(\lambda_L)}{d\Omega} n_i(R) p_i(\lambda)\right]} \quad (4.5)$$

Transmission $$\boxed{T(\lambda_L,R)T(\lambda,R) = \exp\left[-\left(\int_0^R \alpha(\lambda_L,r)dr + \int_0^R \alpha(\lambda,r)dr\right)\right]} \quad (4.18)$$

An Example of Applying Lidar Equation -- Envelope Estimate of Lidar Returns

- Envelope estimate is to calculate integrated photon returns from an entire layer or region using the fundamental lidar equation – non-range-resolved lidar simulation.
- It is a good way to assess a lidar potential and system performance. Envelope estimate provides an idea of what performance can be expected.
- Resonance fluorescence lidar uses the equation

$$\boxed{N_S(\lambda,R)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda,R)n_c(z)R_B(\lambda)\Delta R\right)\left(\frac{A}{4\pi R^2}\right)\left(T_a^2(\lambda,R)T_c^2(\lambda,R)\right)\left(\eta(\lambda)G(R)\right)+N_B} \quad (4.14)$$

Lidar and Atmosphere Parameters

- Arecibo K Doppler lidar transmitter parameters:
  - Laser pulse energy: E_{pulse} = 150 mJ
  - Laser central wavelength: λ_L = 770.1088 nm
  - Transmitter 5 mirrors @ R = 99% ⇨ $R_{tmirror} = (0.99)^5 = 0.95$
- Arecibo K lidar receiver parameters:
  - Primary mirror diameter: D = 80 cm ⇨ A = 0.50 m2
  - Primary mirror reflectivity: $R_{primary}$ = 91%
  - Fiber coupling efficiency: η_{fiber} = 90%
  - Receiver lens transmittance: $T_{Rmirror}$ = 90%
  - Interference filter peak transmission: T_{IF} = 80%
  - APD quantum efficiency: QE = 60%
- K layer and atmosphere information
  - Peak effective cross-section of K D_{1a} line: $\sigma_{eff} = 10 \times 10^{-16}$ m^2
  - K layer column abundance: KAbdn = 6×10^7 cm^{-2} = 6×10^{11} m^{-2}
  - K layer centroid altitude: R = 90 km = 9×10^4 m
  - Lower atmosphere transmission at 770 nm: T_{atmos} = 80%

Envelope Estimate

$$N_S(\lambda,R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda,R)n_c(z)R_B(\lambda)\Delta R\right)\left(\frac{A}{4\pi R^2}\right)\left(T_a^2(\lambda,R)T_c^2(\lambda,R)\right)\left(\eta(\lambda)G(R)\right)+N_B$$

5.81×10^{17} $\quad$ 6×10^{-4} $\quad$ 4.94×10^{-12} $\quad$ 0.64 $\quad$ 0.336

The scattering probability is given by: $P_{scattering} = \sigma_{eff} \times$ Kabdn $= 6 \times 10^{-4}$

Transmitter efficiency $\eta_{transmitter} = (0.99)^5 = 0.95$
Receiver efficiency $\eta_{receiver} = 0.91 \times 0.9 \times 0.9 \times 0.8 \times 0.6 = 0.35$
Overall lidar efficiency $\eta = 0.336$

- The overall lidar return from the entire K layer is

$N_S = 5.81\times10^{17} \times 6\times10^{-4} \times 4.94\times10^{-12} \times 0.64 \times 0.35 = 370$ counts/shot

- These returns originate from 5.8×10^{17} laser photons!!!

Long range $1/R^2$ → Weak signal !

Summary and Questions

- ❑ Lidar equation is the fundamental equation governing the lidar remote sensing field.
- ❑ Lidar equation relates the received photon counts to the transmitted laser photon numbers, light transmission through medium, probability of a transmitted photon to be scattered, properties of scatters, probability of a scattered photon to be collected, and lidar system efficiency and geometry factors.
- ❑ Different lidars may use different forms of the lidar equation, depending on the needs and emphasis.

➢ From the example given, what is the major killer of the signal strength for the upper atmosphere lidars? How will the situation be different in the lower atmosphere lidars?

Chapter 1 of "Laser Remote Sensing" textbook
IntroLidar.pdf
Chapter 5. Sections 5.1 and 5.2.1 textbook