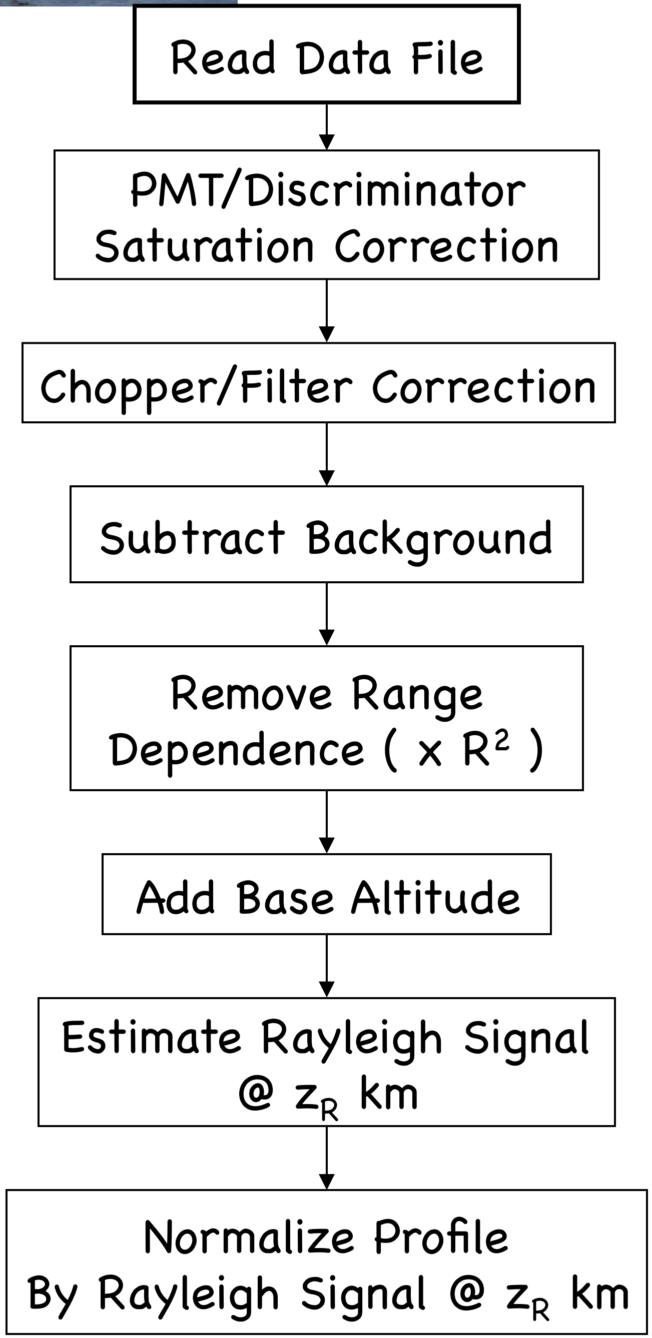
Lecture 29. Lidar Data Inversion (2)

- Pre-process and Profile-process
- Main Process Procedure to Derive T and V_R Using Ratio Doppler Technique
- Derivations of n_c from narrowband resonance Doppler lidar
- Derivation of β
- Derivation of n_c from broadband resonance lidar
- Error analysis for photon noise
- Summary



Preprocess Procedure and Profile-Process Procedure for Na/Fe/K Doppler Lidar

- Read data: for each set, and calculate T, W, and n for each set
- PMT/Discriminator saturation correction
- Chopper/Filter correction

Integration

- Background estimate and subtraction
- Range-dependence removal (xR^2, not z^2)
- Base altitude adjustment
- Take Rayleigh signal @ z_R (Rayleigh fit or Rayleigh mean)
- Rayleigh normalization $\boxed{N_N(\lambda,z) = \dfrac{N_S(\lambda,z) - N_B}{N_S(\lambda,z_R) - N_B}\dfrac{z^2}{z_R^2}}$

Main Process

- Subtract Rayleigh signals from Na/Fe/K region after counting in the factor of T_C

Basic Clue (1): Lidar Equation & Solution

- From lidar equation and its solution to derive preprocess procedure of lidar data inversion

$$N_S(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_{eff}(\lambda,z)n_c(z)R_B(\lambda) + 4\pi\sigma_R(\pi,\lambda)n_R(z)\right]\Delta z\left(\frac{A}{4\pi z^2}\right) \times \left(T_a^2(\lambda)T_c^2(\lambda,z)\right)\left(\eta(\lambda)G(z)\right) + N_B$$

$$+$$

$$N_S(\lambda,z_R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_R(\pi,\lambda)n_R(z_R)\right]\Delta z\left(\frac{A}{z_R^{\;2}}\right)T_a^{\;2}(\lambda,z_R)\left(\eta(\lambda)G(z_R)\right) + N_B$$

$$\Downarrow$$

$$N_{Norm}(\lambda,z) = \frac{N_{Na}(\lambda,z)}{N_R(\lambda,z_R)T_c^2(\lambda,z)}\frac{z^2}{z_R^{\;2}} = \frac{\sigma_{eff}(\lambda,z)n_c(z)}{\sigma_R(\pi,\lambda)n_R(z_R)}\frac{1}{4\pi}$$

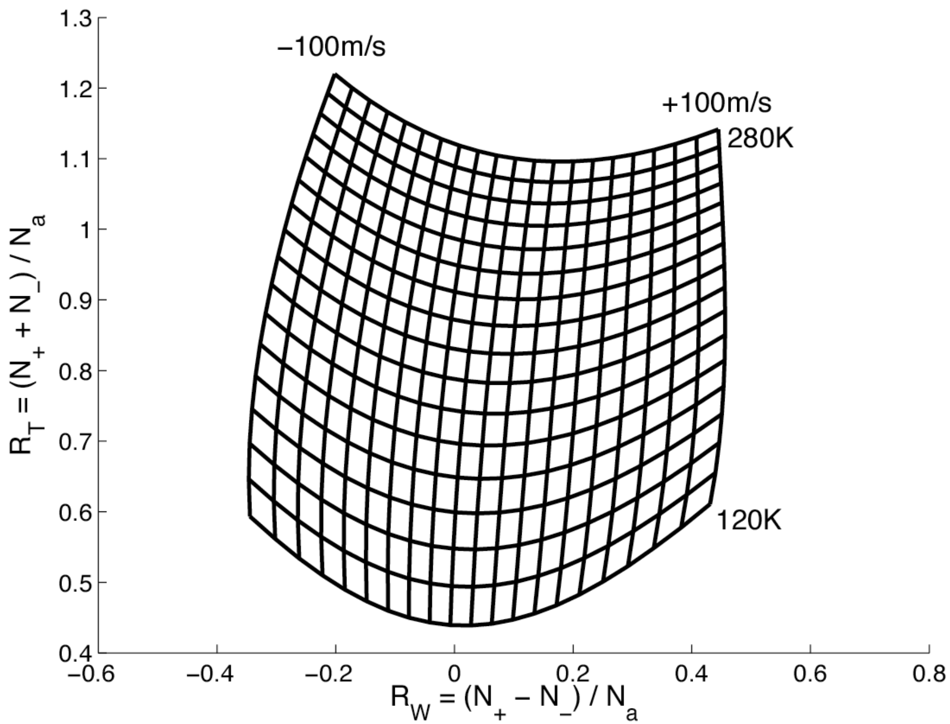
$$= \frac{N_S(\lambda,z) - N_B}{N_S(\lambda,z_R) - N_B}\frac{z^2}{z_R^{\;2}}\frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}$$

Main Ideas to Derive Na T and W

- In the ratio technique, Na number density is cancelled out. So we have two ratios R_T and R_W that are independent of Na density but both dependent on T and W.
- The idea is to derive temperature and radial wind from these two ratios first, and then derive Na number density using computed temperature and wind at each altitude bin.
- To derive T and W from R_T and R_W, the basic idea is to use look-up table or iteration methods to derive them: (1) compute R_T and R_W from physics point-of-view to generate the table or calibration curves, (2) compute R_T and R_W from actual photon counts, (3) check the table or calibration curves to find the corresponding T and W. (4) If R_T and R_W are out of range, then set to nominal T and W.
- However, because the Na extinction coefficient is involved, the upper bins are related to lower bins, and extinction coefficient is related to Na density and effective cross-section. The solution is to start from the bottom of the Na layer.

Main Process Procedure

- Compute Doppler calibration curves from physics



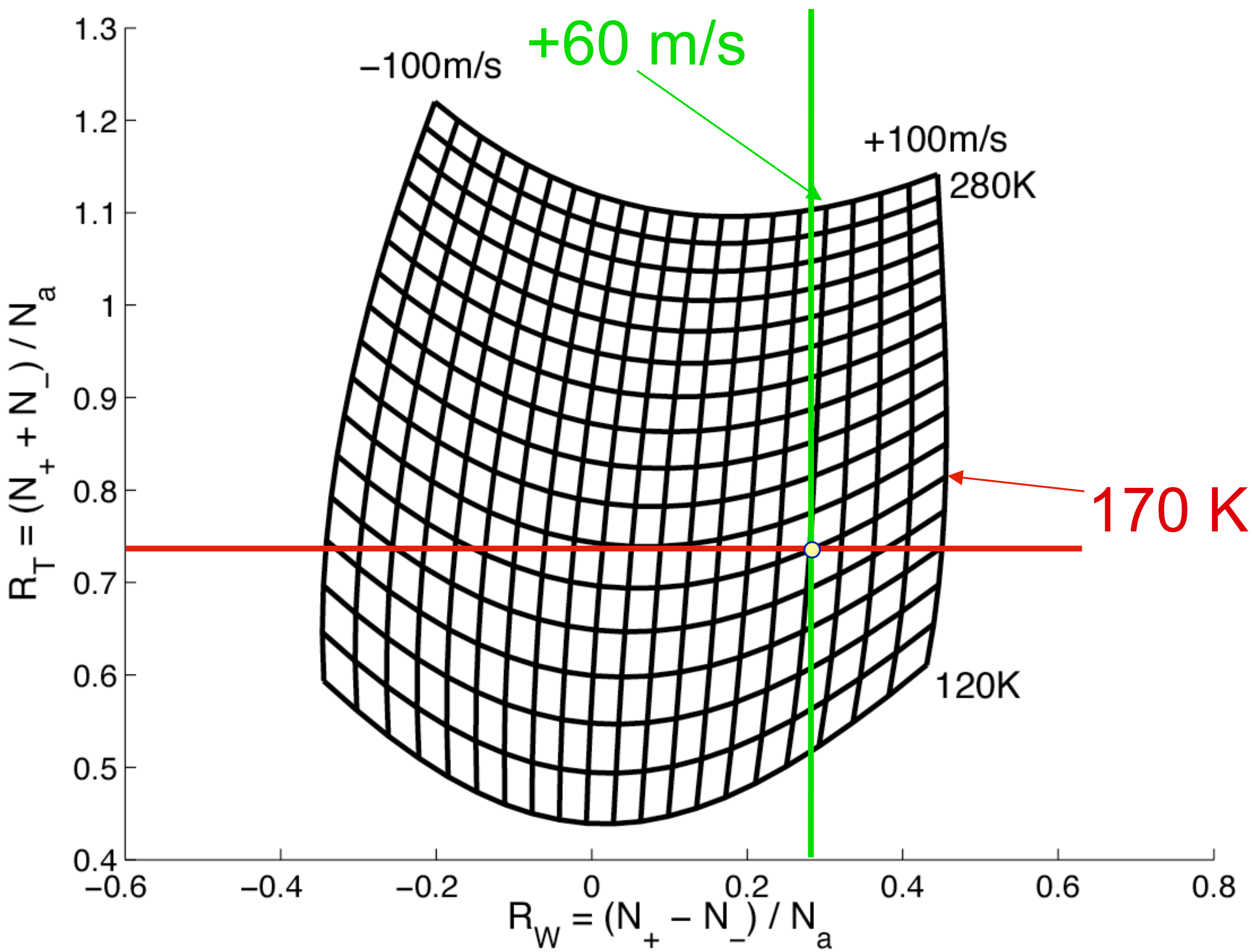
$$R_W = \frac{\sigma_{eff}(f_+, z) - \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

$$R_T = \frac{\sigma_{eff}(f_+, z) + \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

$$\sigma_{\rm eff}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_{\rm e}} \frac{e^2 f}{4\epsilon_0 m_{\rm e} c} \sum_{n=1}^{6} A_n \exp\left(-\frac{\left[\nu_n - \nu\left(1 - \frac{v_{\rm R}}{c}\right)\right]^2}{2\sigma_{\rm e}^2}\right)$$

Main Process Procedure

- Compute actual ratios R_T and R_W from photon counts
- Look up these two ratios on the calibration curves to infer the corresponding Temperature and Wind from isoline/isogram.



Constituent Density

- Normalized Photon Count to the density estimation

$$n_c(z) = \left[\frac{N_S(\lambda,z) - N_B}{N_R(\lambda,z_R) - N_B} \cdot \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)} \right] \cdot \frac{4\pi\sigma_R(\pi,\lambda) n_R(z_R)}{\sigma_{eff}(\lambda,z) R_B(\lambda)}$$

Normalized Photon Count
From the preprocess

Temperature and wind dependent

→ we need to estimate the temperature and wind first in order to estimate the density

- The effective cross-section

$$\sigma_{\mathrm{eff}}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{e}}} \frac{e^2 f}{4\epsilon_0 m_{\mathrm{e}} c} \sum_{n=1}^{6} A_n \exp\left(- \frac{\left[\nu_n - \nu\left(1 - \frac{v_{\mathrm{R}}}{c}\right)\right]^2}{2\sigma_{\mathrm{e}}^2} \right)$$

Main Process

Load Atmosphere n_R, T, P Profiles from MSIS00

Start from Na layer bottom
T_C ($z=z_b$) = 1
Calculate Nnorm ($z=z_b$) from photon counts and MSIS number density for each freq

$$N_{Norm}(\lambda,z) = \frac{N_S(\lambda,z)-N_B}{N_S(\lambda,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}$$

Calculate R_T and R_W from N_{Norm}

Are ratios reasonable?

Yes

Find T and W from the Table

No

Set to nominal values or MSIS
T = 200 K, W = 0 m/s

Calculate Na density $n_c(z)$

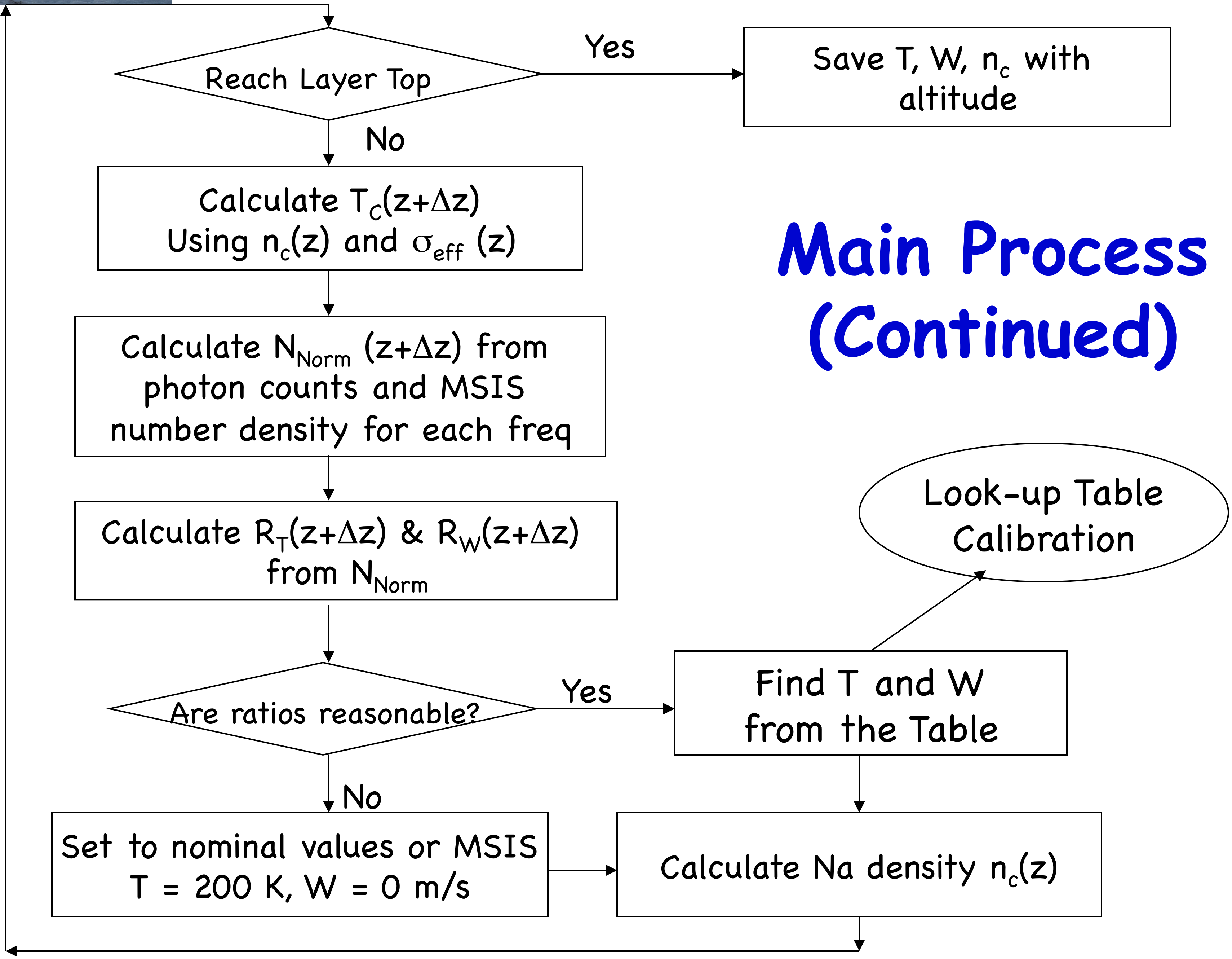
Create look-up table or calibration curves From physics

$$R_T = \frac{\sigma_{eff}(f_+,z)+\sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}$$

$$R_W = \frac{\sigma_{eff}(f_+,z)-\sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}$$

Look-up Table Calibration

Main Process (Continued)



Derivation of T_c (Transmission Caused by Constituent Extinction)

- T_c (caused by constituent absorption) can be derived from

$$\boxed{T_c(\lambda,z) = \exp\left(-\int_{z_{bottom}}^{z} \sigma_{eff}(\lambda,z) n_c(z)\,\mathrm{d}z\right) = \exp\left(-\sum_{z_{bottom}}^{z} \sigma_{eff}(\lambda,z) n_c(z)\Delta z\right)}$$

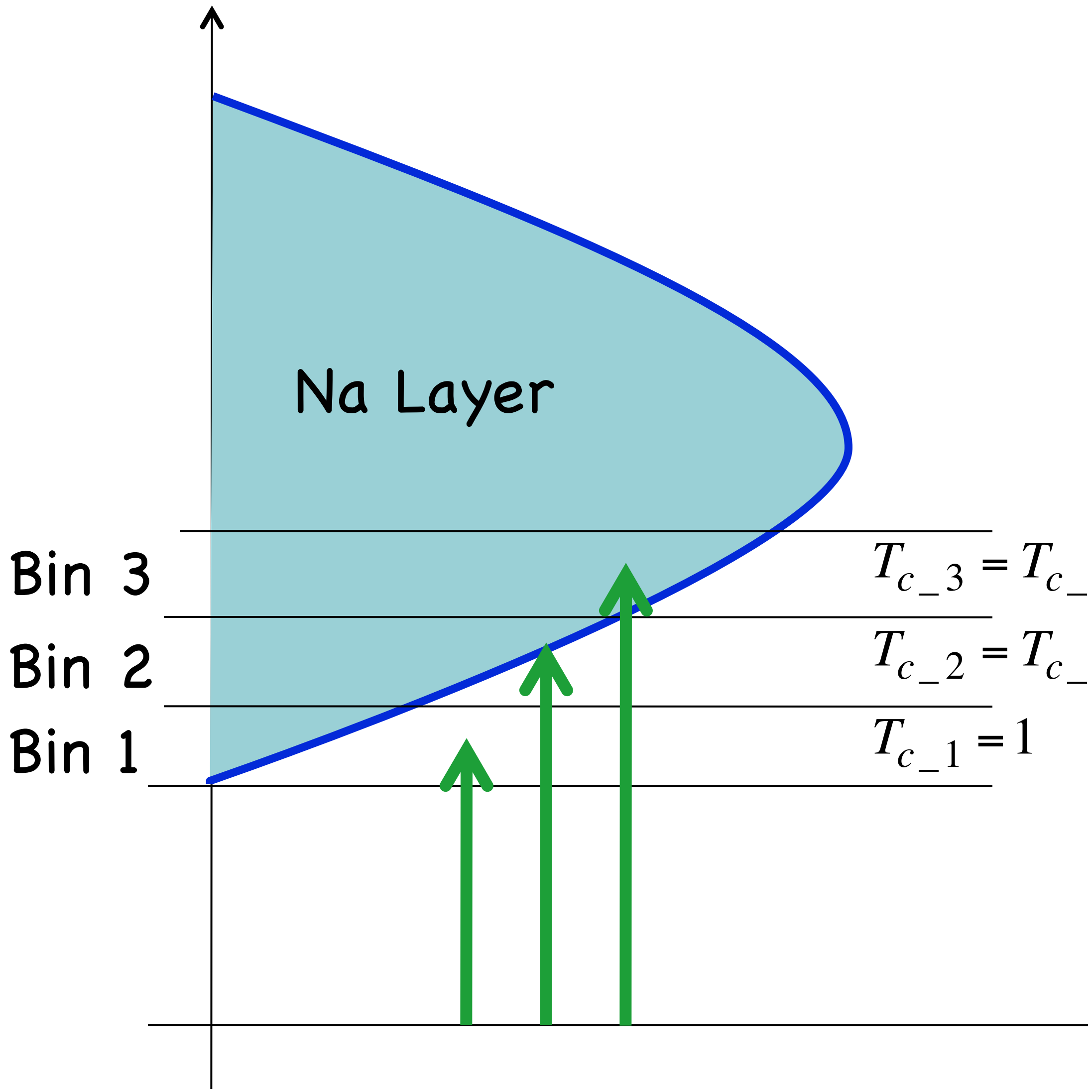
+

$$\boxed{n_c(z) = \left[\frac{N_S(\lambda,z) - N_B}{N_R(\lambda,z_R) - N_B} \cdot \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}\right] \cdot \frac{4\pi\sigma_R(\pi,\lambda) n_R(z_R)}{\sigma_{eff}(\lambda,z) R_B(\lambda)}}$$

$$\boxed{\sigma_e = \sqrt{\sigma_D^2 + \sigma_L^2}}$$

$$\boxed{T_c(\lambda,z) = \exp\left(-\sum_{z_{bottom}}^{z} \left[\frac{N_S(\lambda,z) - N_B}{N_R(\lambda,z_R) - N_B} \cdot \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}\right] \cdot \frac{4\pi\sigma_R(\pi,\lambda) n_R(z_R)}{R_B(\lambda)} \Delta z\right)}$$

A Step in the Main Process: Estimating Transmission (T_C)



$$T_{c\_3} = T_{c\_2} \times \exp(-\sigma_{eff\_2} \cdot n_{c\_2} \cdot \Delta z)$$

$$T_{c\_2} = T_{c\_1} \times \exp(-\sigma_{eff\_1} \cdot n_{c\_1} \cdot \Delta z)$$

$$T_{c\_1} = 1$$

Main Process Step 1: Starting Point

1. Set transmission (T_c) at the bottom of Na layer to be 1
2. Calculate the normalized photon count for each frequency

$$\boxed{N_{Norm}(\lambda,z) = \frac{N_S(\lambda,z) - N_B}{N_S(\lambda,z_R) - N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}}$$

3. Take ratios R_T and R_W from normalized photon counts

$$\boxed{R_T = \frac{N_{Norm}(f_+,z) + N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}} \qquad \boxed{R_W = \frac{N_{Norm}(f_+,z) - N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}}$$

4. Estimate the temperature and wind using the calibration curves computed from physics

Main Process Step 2: Bin-by-Bin Procedure

5. Calculate the effective cross section using temperature and wind derived

6. Using the effective cross-section and $T_c = 1$ (at the bottom), calculate the Na density.

$$\boxed{n_c(z) = \left[\frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^{\ 2}}\frac{1}{T_c^{\ 2}(\lambda,z)}-\frac{n_R(z)}{n_R(z_R)}\right]\cdot\frac{4\pi\sigma_R(\pi,\lambda)n_R(z_R)}{\sigma_{eff}(\lambda)R_B(\lambda)}}$$

7. From effective cross-section and Na density, calculate the transmission T_c for the next bin.

$$\boxed{T_c(\lambda,z) = \exp\left(-\int_{z_{bottom}}^{z}\sigma_{eff}(\lambda,z)n_c(z)\,dz\right) = \exp\left(-\sum_{z_{bottom}}^{z}\sigma_{eff}(\lambda,z)n_c(z)\Delta z\right)}$$

Na Density Derivation

- The Na density can be inferred from the peak freq signal

$$n_{Na}(z) = \frac{N_{norm}(f_a,z)}{\sigma_a} 4\pi n_R(z_R)\sigma_R = \frac{N_{norm}(f_a,z)}{\sigma_a} 4\pi \times 2.938 \times 10^{-32} \frac{P(z_R)}{T(z_R)} \cdot \frac{1}{\lambda^{4.0117}}$$

- The Na density can also be inferred from a weighted average of all three frequency signals.
- The weighted effective cross-section is

$$\sigma_{eff\_wgt} = \sigma_a + \alpha\sigma_+ + \beta\sigma_-$$

where α and β are chosen so that

$$\frac{\partial \sigma_{eff\_wgt}}{\partial T} = 0; \qquad \frac{\partial \sigma_{eff\_wgt}}{\partial v_R} = 0$$

- The Na density is then calculated by

$$n_{Na}(z) = 4\pi n_R(z_R)\sigma_R \frac{N_{norm}(f_a,z) + \alpha N_{norm}(f_+,z) + \beta N_{norm}(f_-,z)}{\sigma_a + \alpha\sigma_+ + \beta\sigma_-}$$

Process Procedure for Deriving β

Read Data File

PMT Saturation Correction

Chopper Correction

Integrate and/or smooth profiles to improve SNR

Subtract Background

Remove Range Dependence (x R^2)

Add Base Altitude

Take Rayleigh Signal @ z_R km

Normalize Profile By Rayleigh Signal @ z_R km

Load Atmosphere n_R, T, P Profiles from MSIS00

Calculate $R(z)$ and $\beta(z)$

Smooth $R(z)$ and $\beta(z)$ By Hamming Window FWHM = 250 m

Calculate z_c, σ_{rms}, β_{total}

Process Procedure for β of PMC

Load Atmosphere n_R, T, P Profiles from MSIS00

↓

Calculate $R(z)$ and $\beta(z)$

↓

Smooth $R(z)$ and $\beta(z)$ By Hamming Window FWHM = 250 m

↓

Calculate z_c, σ_{rms}, β_{total}

$$R = \frac{\left[N_S(z) - N_B\right]\cdot z^2}{\left[N_S(z_{RN}) - N_B\right]\cdot z_{RN}^2}\cdot\frac{n_R(z_{RN})}{n_R(z)}$$

$$\beta_{PMC}(z) = \left[\frac{\left[N_S(z) - N_B\right]\cdot z^2}{\left[N_S(z_{RN}) - N_B\right]\cdot z_{RN}^2} - \frac{n_R(z)}{n_R(z_{RN})}\right]\cdot\beta_R(z_{RN})$$

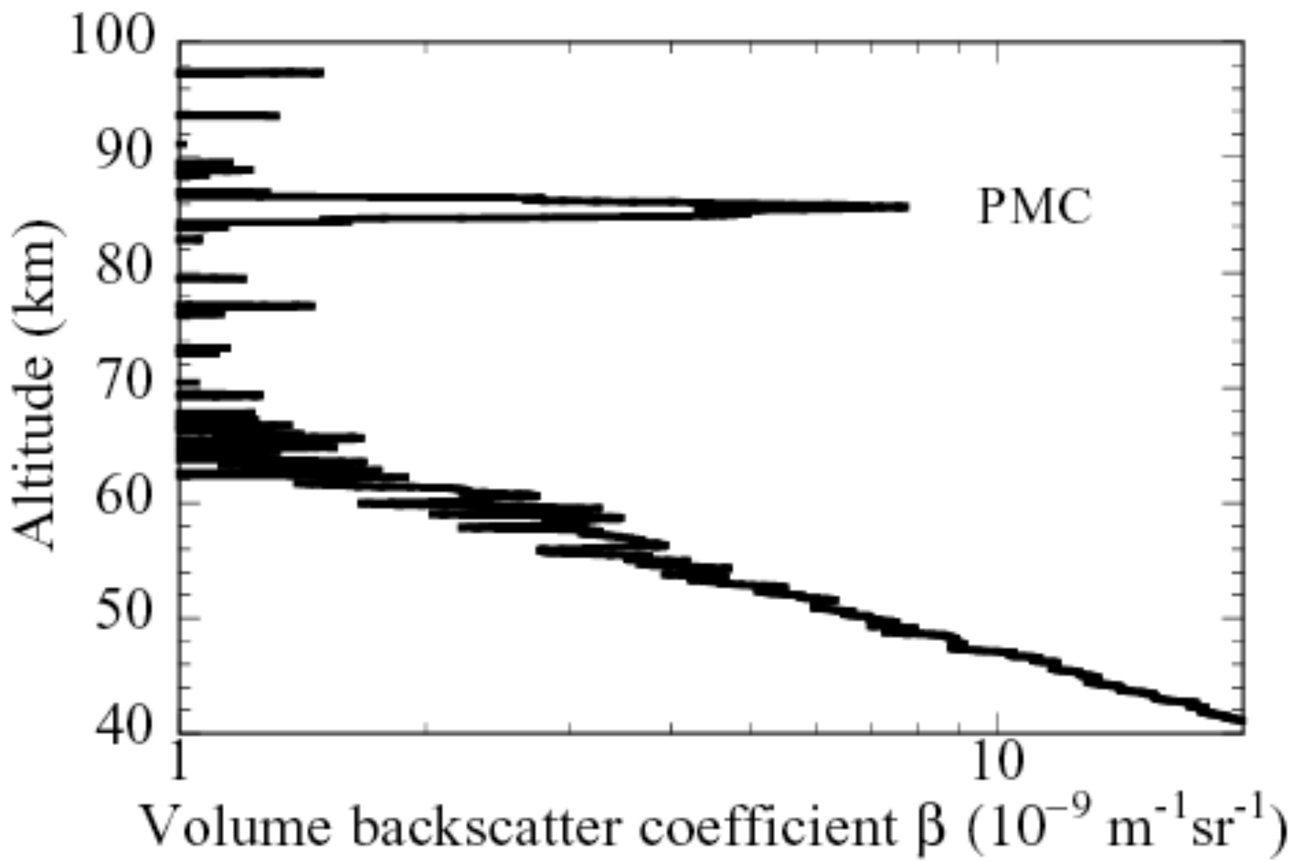
$$\beta_R(z_{RN},\pi) = \frac{\beta}{4\pi}P(\pi) = 2.938\times10^{-32}\frac{P(z_{RN})}{T(z_{RN})}\cdot\frac{1}{\lambda^{4.0117}}$$

$$z_c = \frac{\sum_i \beta_{PMC}(z_i)\cdot z_i}{\sum_i \beta_{PMC}(z_i)}$$

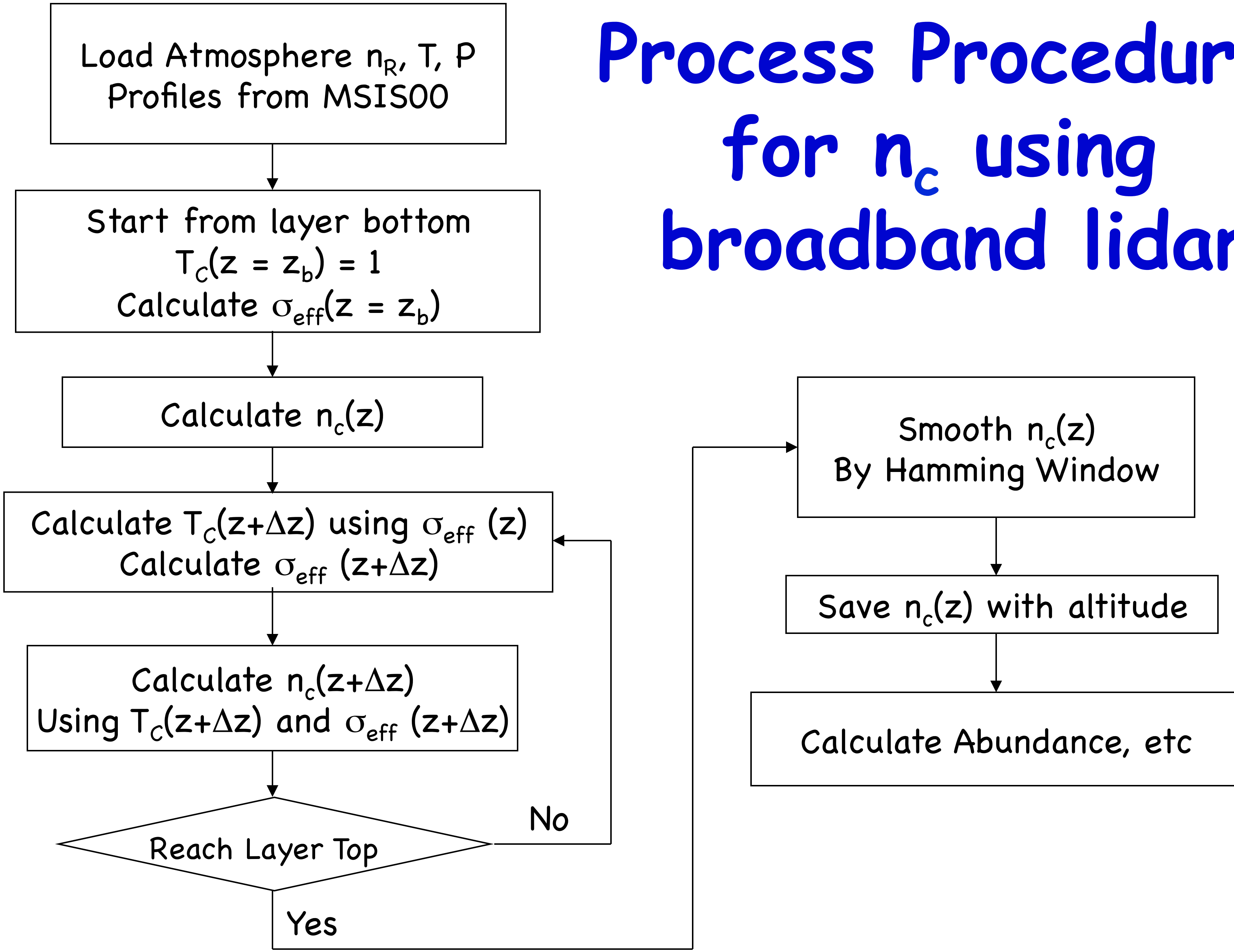
$$\sigma_{rms} = \sqrt{\frac{\sum_i (z_i - z_c)^2\beta_{PMC}(z_i)}{\sum_i \beta_{PMC}(z_i)}}$$

$$\beta_{total} = \int \beta_{PMC}(z)dz$$

Example Result: South Pole PMC



Process Procedure for n_c using broadband lidar



Process Procedure for n_c

- Computation of effective cross-section (concerning laser shape, assuming nominal T and W)
- Spatial resolution - binning or smoothing
- temporal resolution – integration or smoothing

-- in order to improve SNR

- Transmission T_C (extinction coefficient)
- Calculate density
- Calculate abundance, peak altitude, etc.

To Improve SNR

- In order to improve signal-to-noise ratio (SNR), we have to sacrifice spatial and/or temporal resolutions.
- Spatial resolution
  - integration (binning)
  - smoothing
- temporal resolution
  - integration
  - smoothing

Analysis of Error Caused by Photon Noise

- Use the temperature error derivation for 3-freq Na lidar as an example to explain the error analysis procedure using a differentiation method. For 3-frequency technique, we have the temperature ratio

$$R_T = \frac{\sigma_{eff}(f_+) + \sigma_{eff}(f_-)}{\sigma_{eff}(f_a)} = \frac{N(f_+) + N(f_-)}{N(f_a)}$$

- Temperature error caused by photon noise is given by (1st order apprx.)

$$\Delta T = \frac{\partial T}{\partial R_T} \Delta R_T = \frac{R_T}{\partial R_T / \partial T} \frac{\Delta R_T}{R_T}$$

- Define the temperature sensitivity as

$$\boxed{S_T = \frac{\partial R_T / \partial T}{R_T}}$$

We have

$$\Delta T = \frac{\partial T}{\partial R_T} \Delta R_T = \frac{R_T}{\partial R_T / \partial T} \frac{\Delta R_T}{R_T} = \frac{1}{S_T} \frac{\Delta R_T}{R_T}$$

Error coefficient $\quad$ R_T relative error

Error Coefficient and $\Delta R_T/R_T$

- The temperature error coefficient can be derived numerically

$$\frac{R_T}{\partial R_T/\partial T} = \frac{R_T}{[R_T(T+\delta T) - R_T(T)]/\delta T}$$

- We can derive the error of 3-freq R_T caused by photon noise

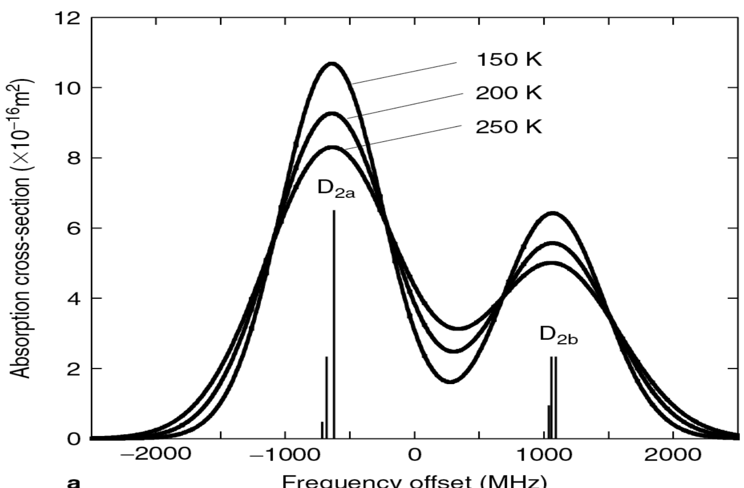
$$\frac{\Delta R_T}{R_T} = \frac{\left(1+\frac{1}{R_T}\right)^{1/2}}{\left(N_{f_a}\right)^{1/2}} \left[1 + \frac{B}{N_{f_a}} \frac{\left(1+\frac{2}{R_T^2}\right)}{\left(1+\frac{1}{R_T}\right)}\right]^{1/2}$$

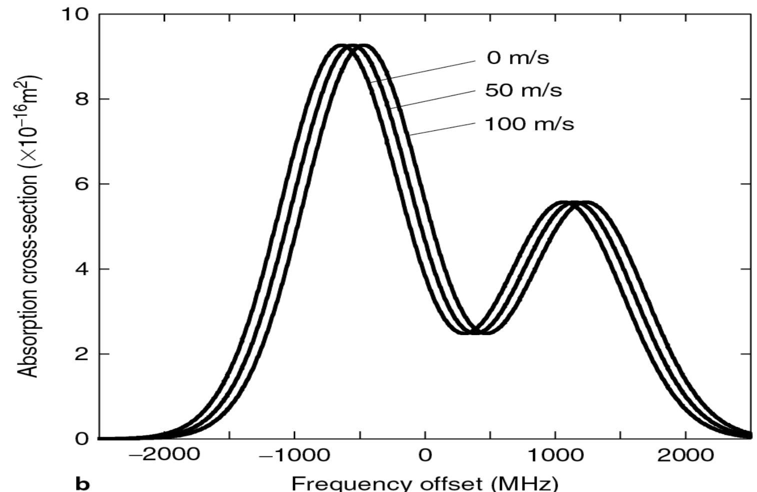
Hint: An example for 2-freq ratio technique, $R_T = N_{fc} / N_{fa}$

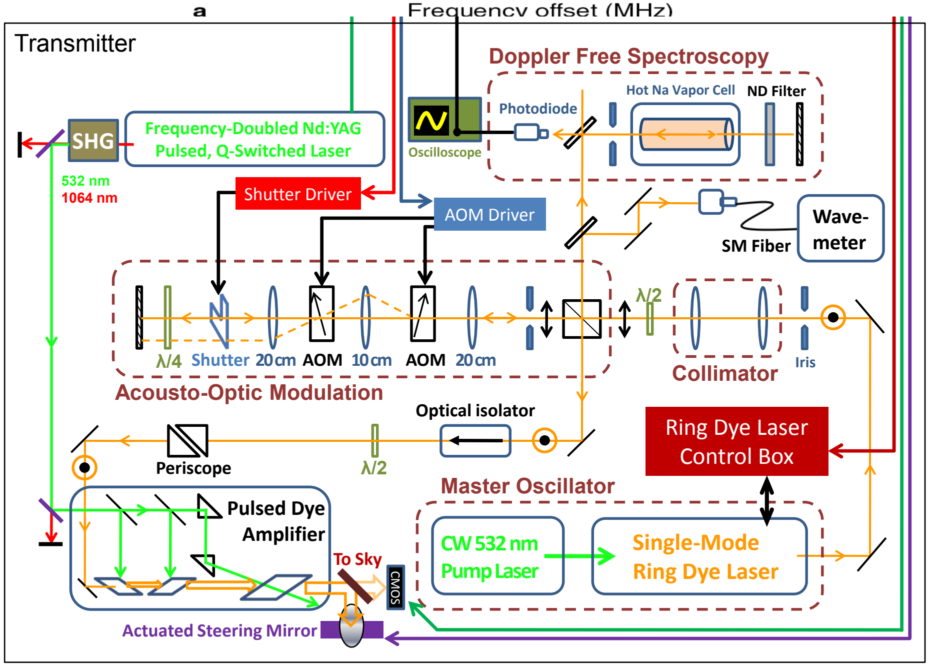
$$\frac{\Delta R_T}{R_T} = \frac{\Delta N_{f_c}}{N_{f_c}} - \frac{\Delta N_{f_a}}{N_{f_a}} \qquad \overline{\left(\Delta N_{f_c}\right)^2} = N_{f_c} + B, \quad \overline{\left(\Delta N_{f_a}\right)^2} = N_{f_a} + B$$

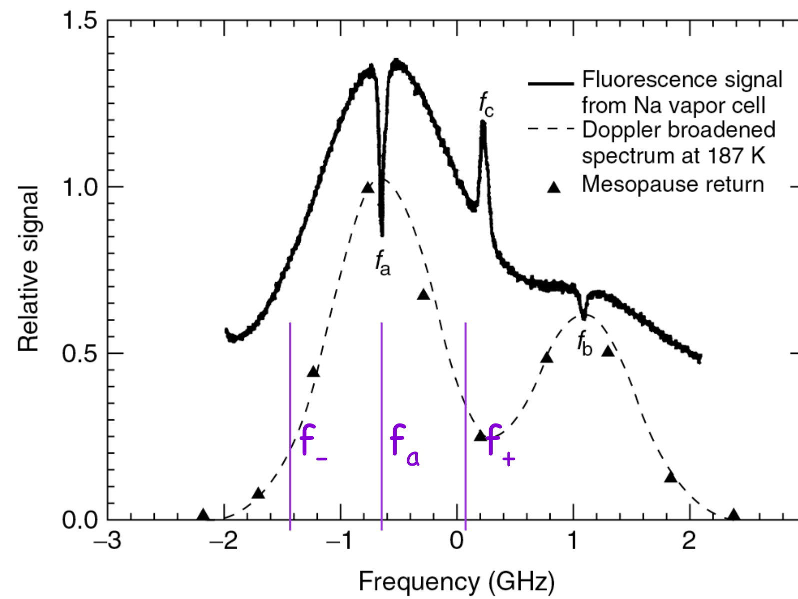
$$\left(\frac{\Delta R_T}{R_T}\right)_{rms} = \sqrt{\overline{\left(\frac{\Delta N_{f_c}}{N_{f_c}} - \frac{\Delta N_{f_a}}{N_{f_a}}\right)^2}} = \sqrt{\overline{\left(\frac{\Delta N_{f_c}}{N_{f_c}}\right)^2} + \overline{\left(\frac{\Delta N_{f_a}}{N_{f_a}}\right)^2}}$$

Other Possible Errors or Biases









1) Laser freq locking error, 2) pulsed laser freq chirp, 3) laser line shape and linewidth, 4) Hanle effect, 5) metal layer saturation, etc. …

Summary

- The pre-process and profile-process are to convert the raw photon counts to corrected and normalized photon counts in consideration of hardware properties and limitations.
- The main process of T and V_R is to convert the normalized photon counts to T and V_R through iteration or looking-up table methods.
- The main process of n_c or β is to convert the normalized photon counts to number density or volume backscatter coefficient, in combination with prior acquired knowledge or model knowledge of certain atmosphere information or atomic/molecular spectroscopy.
- Data inversion procedure consists of the following processes:

  (1) pre- and profile-process,

  (2) process of T and V_R,

  (3) process of n_c and β, etc.