



# Lecture 12. Temperature Lidar (2)

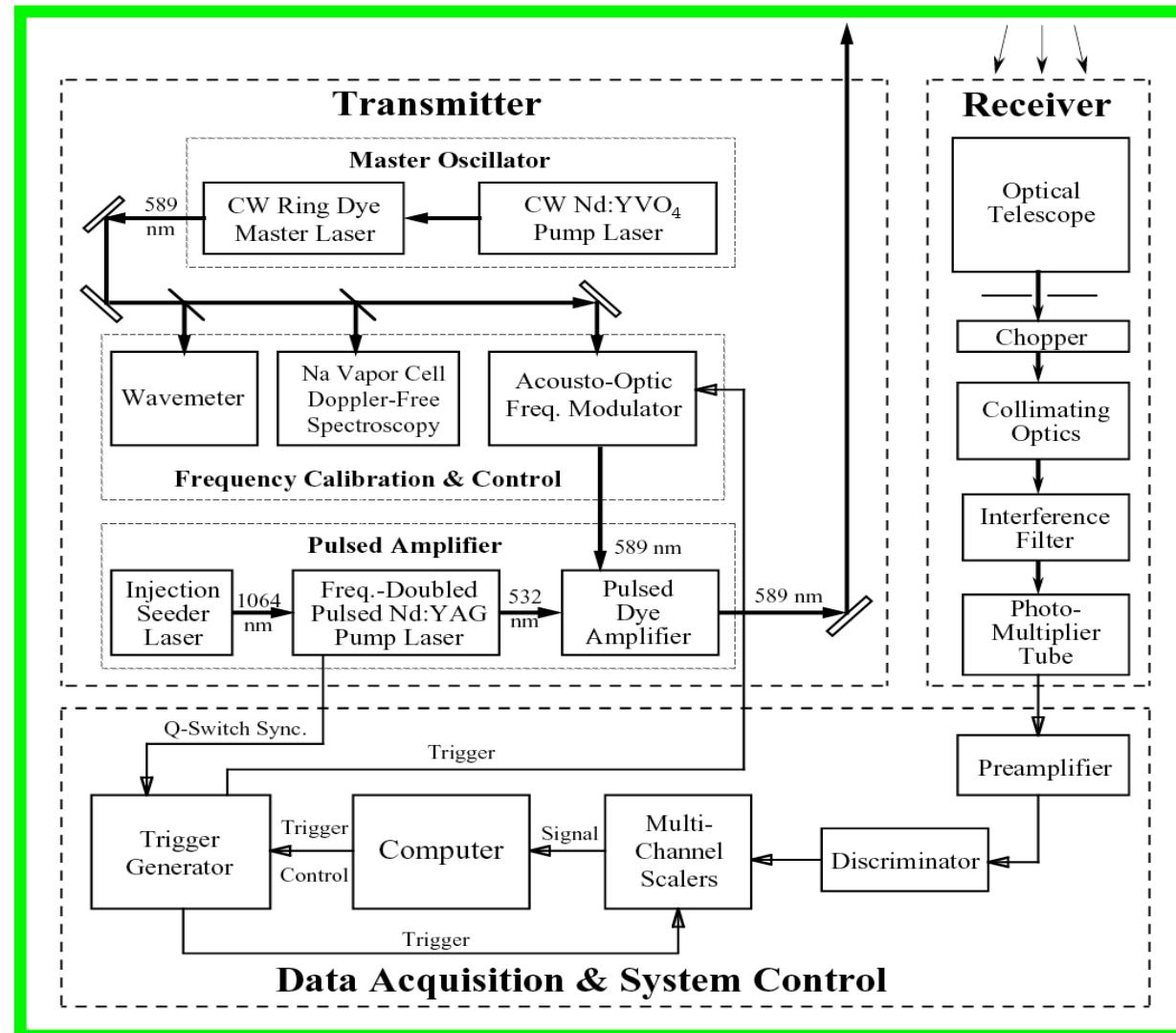
## Resonance Fluorescence Doppler Lidar

- ❑ Resonance Fluorescence Na Doppler Lidar
  - Na Structure and Spectroscopy
  - Scanning Technique versus Ratio Technique
- ❑ Principles of Doppler ratio techniques
  - Two-frequency ratio technique
  - Three-frequency ratio technique
  - Comparison of calibration curves
- ❑ Na Doppler-Free Spectroscopy
  - Summary



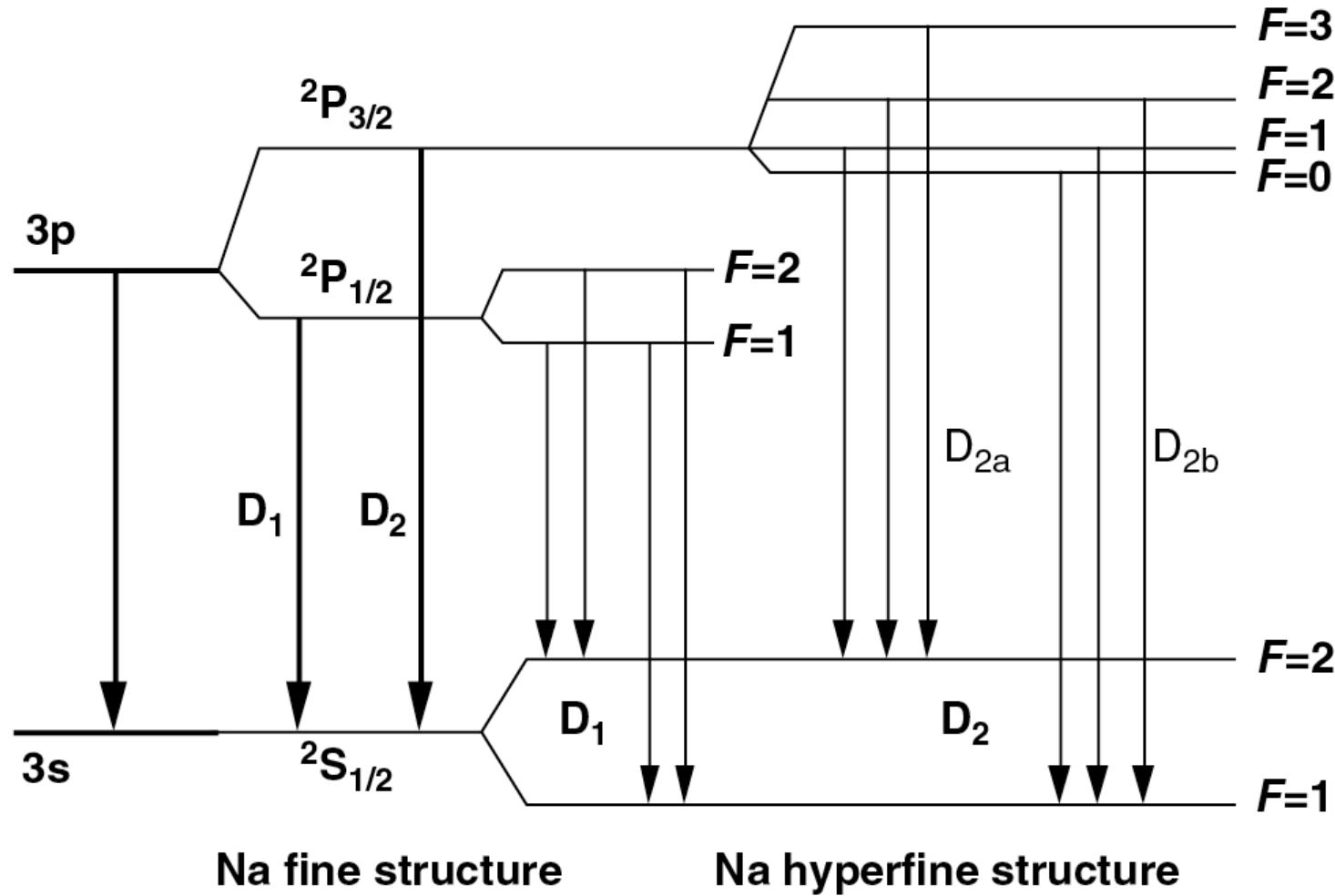
# Na Doppler Wind and Temperature Lidar

- ❑ Na Doppler lidar is one of the most successful lidars.





# Na Atomic Energy Levels





# Na Atomic Parameters

**Table 5.1** Parameters of the Na D<sub>1</sub> and D<sub>2</sub> Transition Lines

Transition Line	Central Wavelength (nm)	Transition Probability ( $10^8 \text{ s}^{-1}$ )	Radiative Lifetime (nsec)	Oscillator Strength $f_{ik}$
D <sub>1</sub> ( ${}^2\text{P}_{1/2} \rightarrow {}^2\text{S}_{1/2}$ )	589.7558	0.614	16.29	0.320
D <sub>2</sub> ( ${}^2\text{P}_{3/2} \rightarrow {}^2\text{S}_{1/2}$ )	589.1583	0.616	16.23	0.641
Group	${}^2\text{S}_{1/2}$	${}^2\text{P}_{3/2}$	Offset (GHz)	Relative Line Strength <sup>a</sup>
D <sub>2b</sub>	$F=1$	$F=2$	1.0911	5/32
		$F=1$	1.0566	5/32
		$F=0$	1.0408	2/32
D <sub>2a</sub>	$F=2$	$F=3$	-0.6216	14/32
		$F=2$	-0.6806	5/32
		$F=1$	-0.7150	1/32

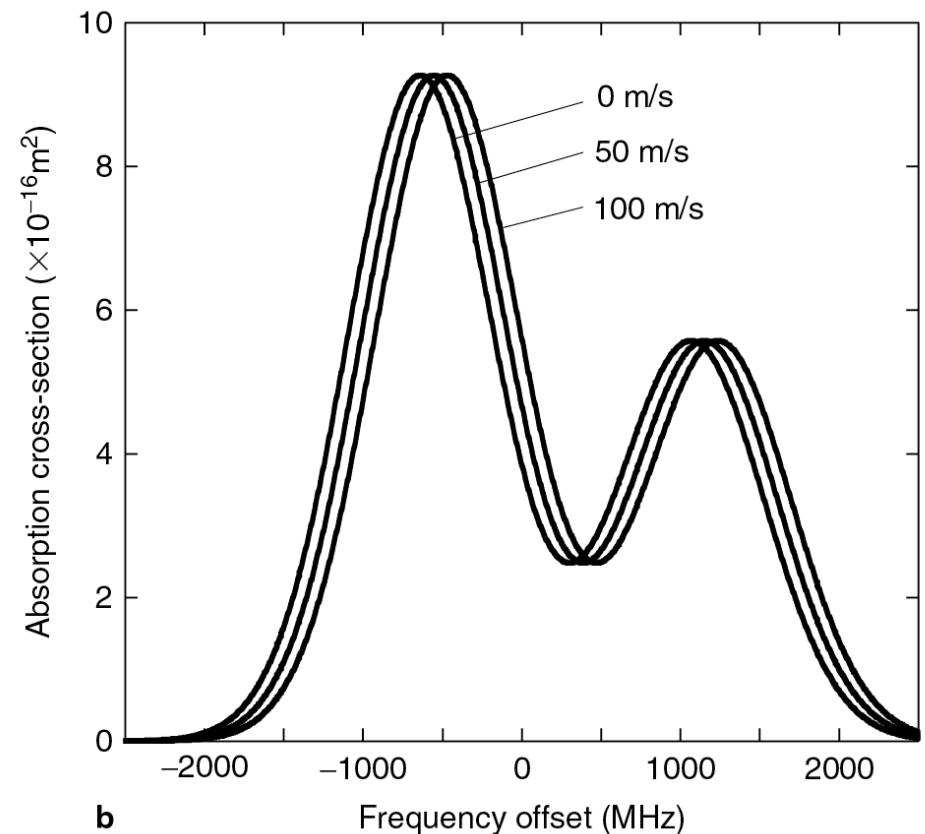
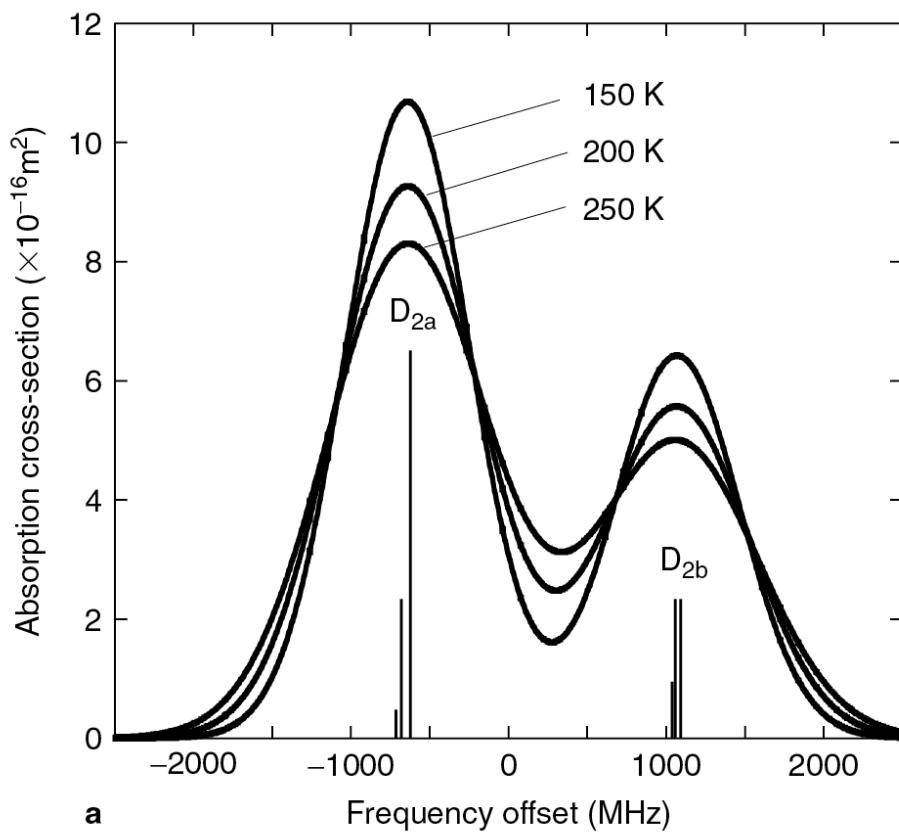
Doppler-Free Saturation–Absorption Features of the Na D<sub>2</sub> Line

$f_a$ (MHz)	$f_c$ (MHz)	$f_b$ (MHz)	$f_+$ (MHz)	$f_-$ (MHz)
-651.4	187.8	1067.8	-21.4	-1281.4

<sup>a</sup>Relative line strengths are in the absence of a magnetic field or the spatial average. When Hanle effect is considered in the atmosphere, the relative line strengths will be modified depending on the geomagnetic field and the laser polarization.



# Doppler Effect in Na D<sub>2</sub> Line Resonance Fluorescence



Na D<sub>2</sub> absorption linewidth  
is temperature dependent

Na D<sub>2</sub> absorption peak freq  
is wind dependent



# Doppler-Limited Na Spectroscopy

- Doppler-broadened Na absorption cross-section is approximated as a Gaussian with rms width  $\sigma_D$

$$\sigma_{abs}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_D} \frac{e^2 f}{4\epsilon_0 m_e c} \sum_{n=1}^6 A_n \exp\left(-\frac{[\nu_n - \nu(1 - V_R/c)]^2}{2\sigma_D^2}\right)$$

- Assume the laser lineshape is a Gaussian with rms width  $\sigma_L$
- The effective cross-section is the convolution of the atomic absorption cross-section and the laser lineshape

$$\sigma_{eff}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_e} \frac{e^2 f}{4\epsilon_0 m_e c} \sum_{n=1}^6 A_n \exp\left(-\frac{[\nu_n - \nu(1 - V_R/c)]^2}{2\sigma_e^2}\right)$$

where  $\sigma_e = \sqrt{\sigma_D^2 + \sigma_L^2}$  and  $\sigma_D = \sqrt{\frac{k_B T}{M \lambda_0^2}}$

The frequency discriminator/analyzer is in the atmosphere!



# Doppler Scanning Technique

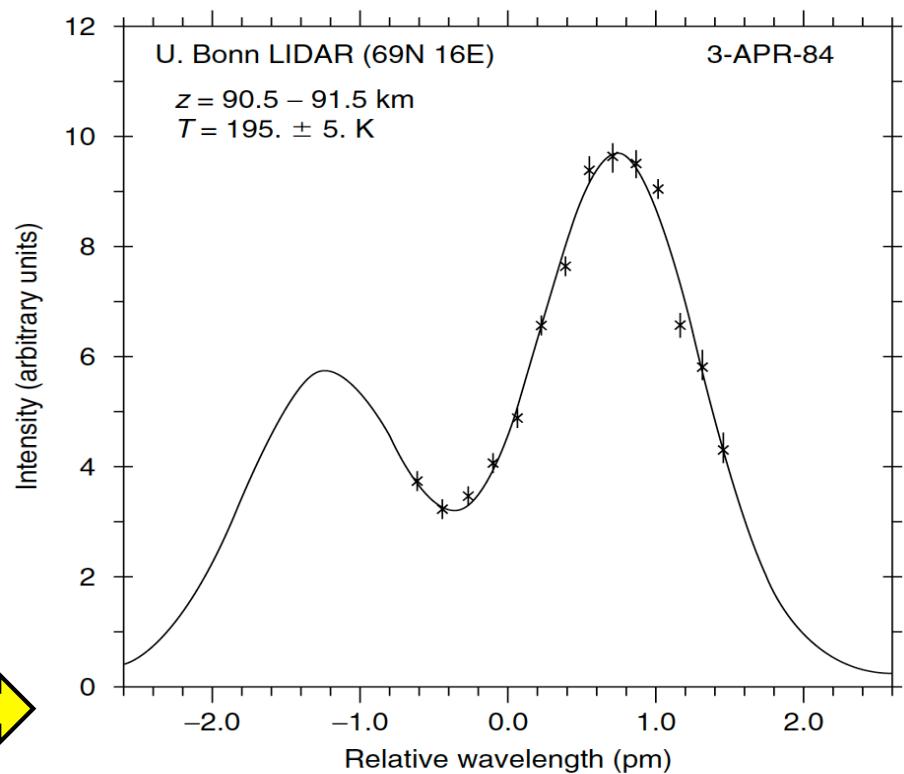
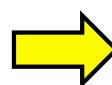
$$N_{Na}(\lambda, z) = \left( \frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) (\sigma_{eff}(\lambda)n_{Na}(z)\Delta z) \left( \frac{A}{4\pi z^2} \right) \left( \eta(\lambda)T_a^2(\lambda)T_c^2(\lambda, z)G(z) \right)$$

$$N_R(\lambda, z_R) = \left( \frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) (\sigma_R(\pi, \lambda)n_R(z_R)\Delta z) \left( \frac{A}{z_R^2} \right) \left( \eta(\lambda)T_a^2(\lambda, z_R)G(z_R) \right)$$

$$\sigma_{eff}(\lambda, z) = \frac{C(z)}{T_c^2(\lambda, z)} \frac{N_{Na}(\lambda, z)}{N_R(\lambda, z_R)}$$

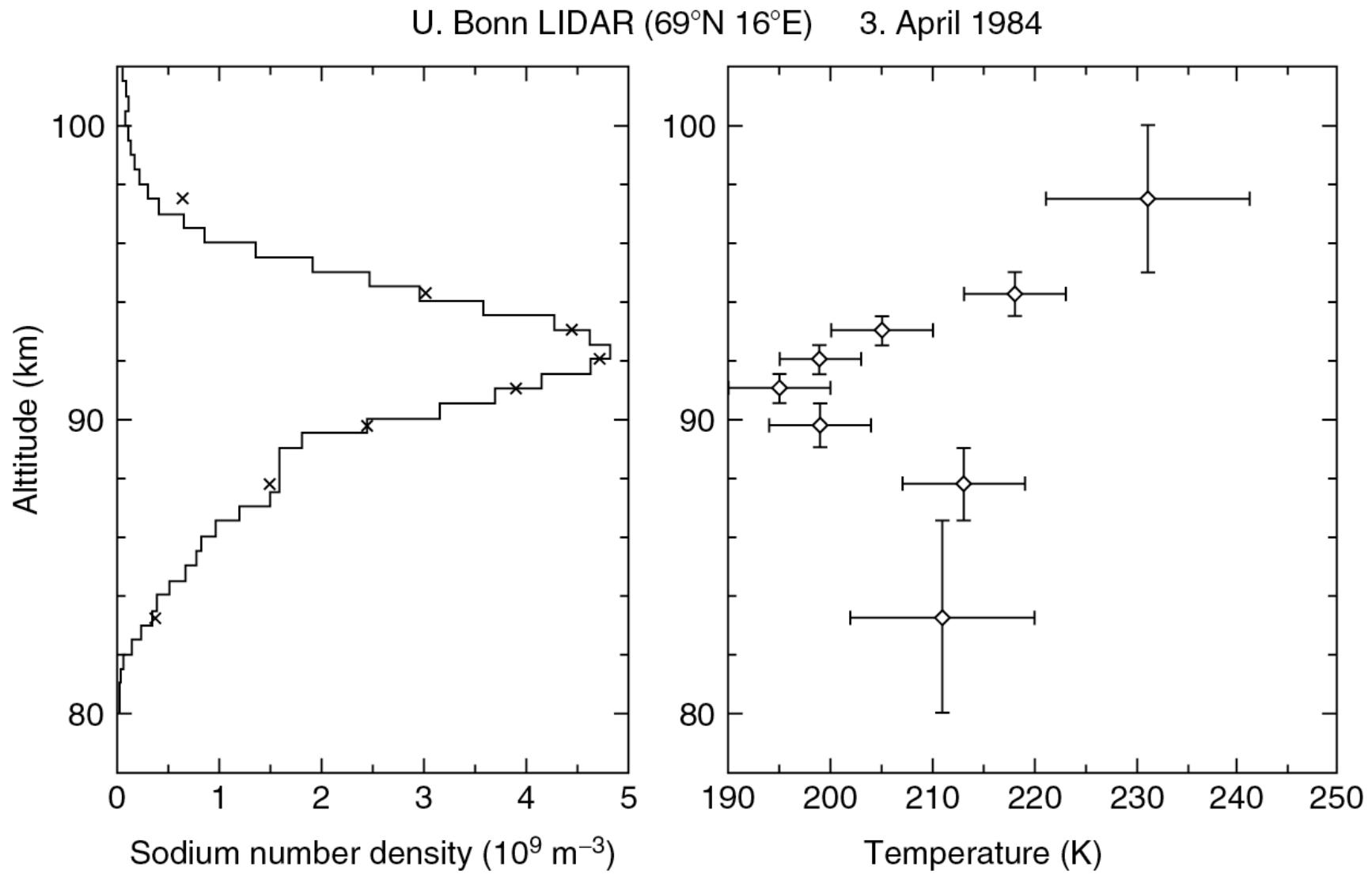
$$\text{where } C(z) = \frac{\sigma_R(\pi, \lambda)n_R(z_R)}{n_{Na}(z)} \frac{4\pi z^2}{z_R^2}$$

Least-square fitting gives temp  
[Fricke and von Zahn, JATP, 1985]





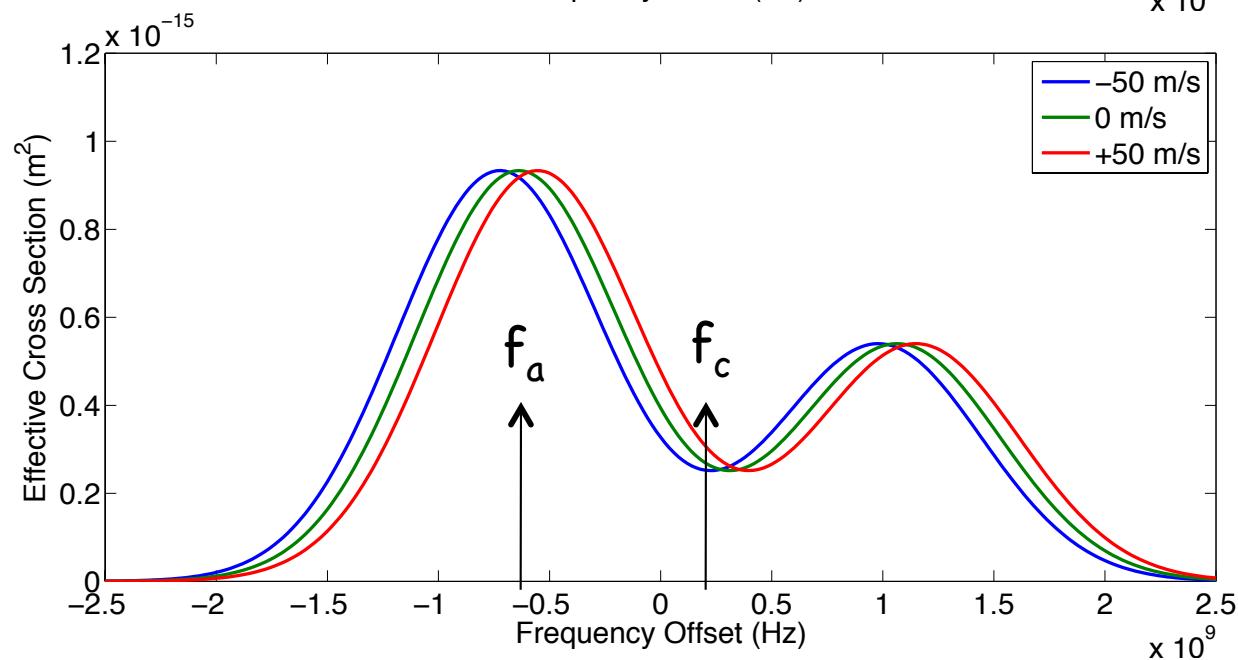
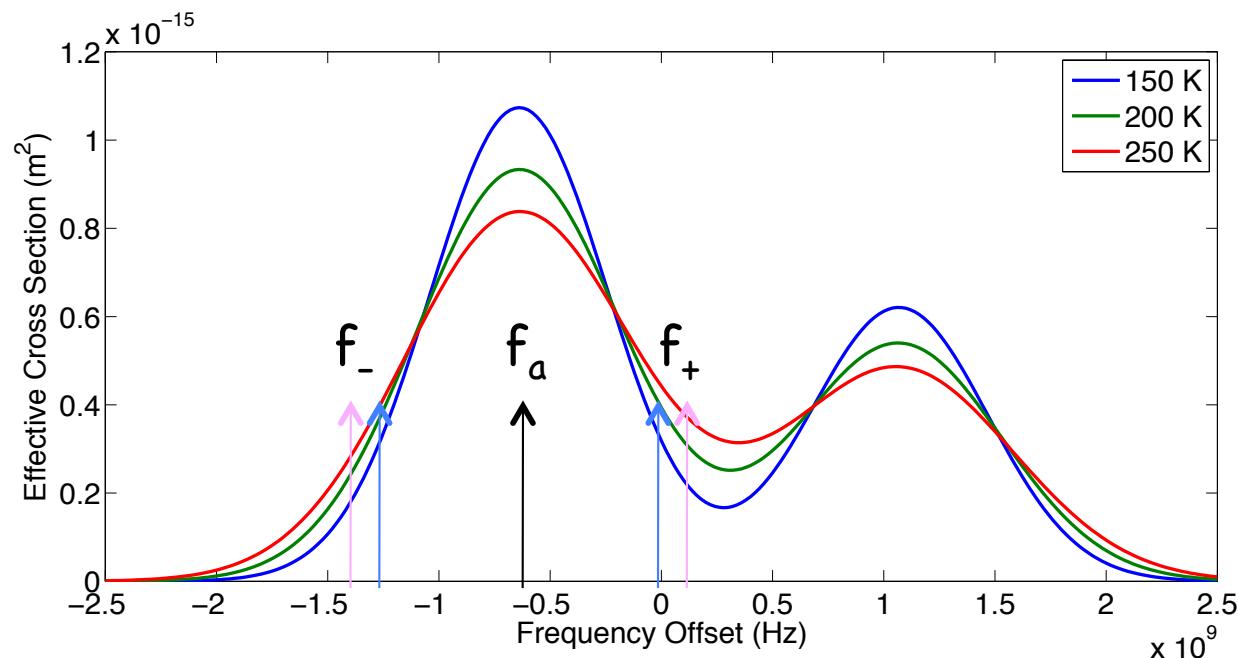
# Scanning Na Lidar Results



[Fricke and von Zahn, JATP, 1985]



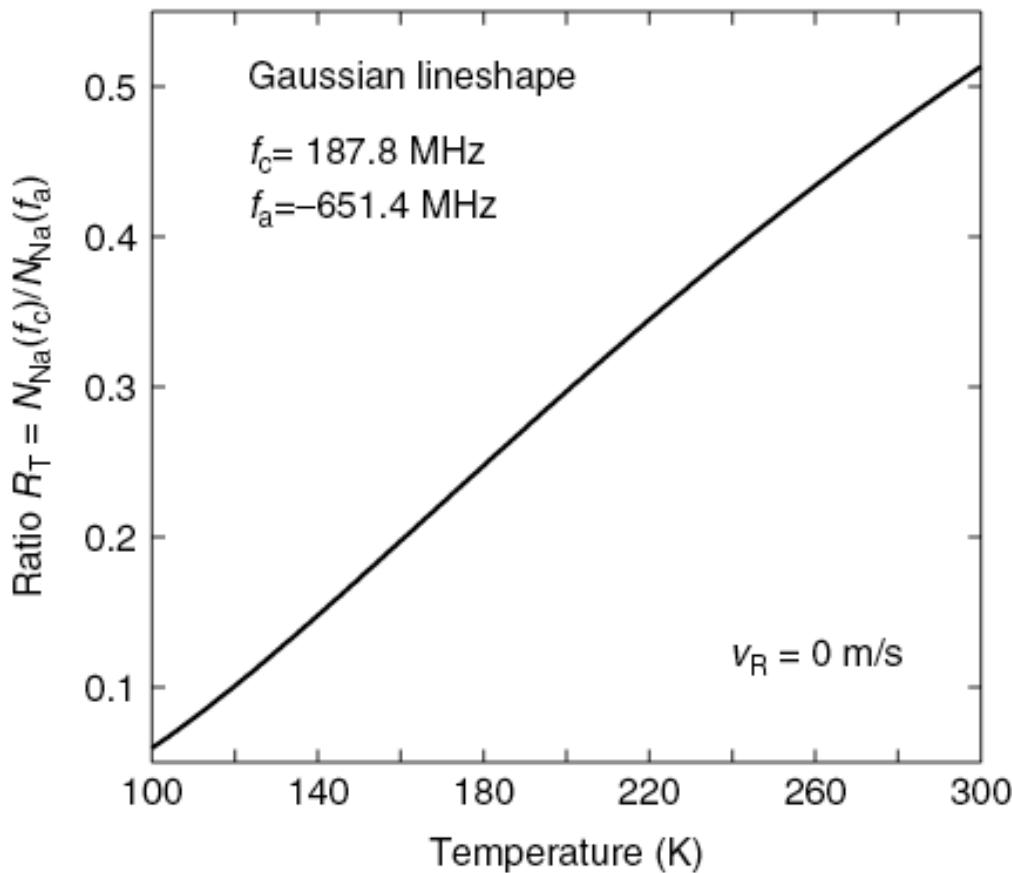
# Doppler Ratio Technique





# 2-Frequency Doppler Ratio Technique

$$R_T(z) = \frac{N_{norm}(f_c, z, t_1)}{N_{norm}(f_a, z, t_2)} = \frac{\sigma_{eff}(f_c, z) n_{Na}(z, t_1)}{\sigma_{eff}(f_a, z) n_{Na}(z, t_2)} \approx \frac{\sigma_{eff}(f_c, z)}{\sigma_{eff}(f_a, z)}$$



$$N_{norm}(f, z, t) \equiv \frac{N_{Na}(f, z, t)}{N_R(f, z, t) T_c^2(f, z)} \frac{z^2}{z_R^2}$$

$$\downarrow$$
$$N_{norm}(f, z, t) = \frac{\sigma_{eff}(f) n_{Na}(z)}{\sigma_R(\pi, f) n_R(z_R)} \frac{1}{4\pi}$$

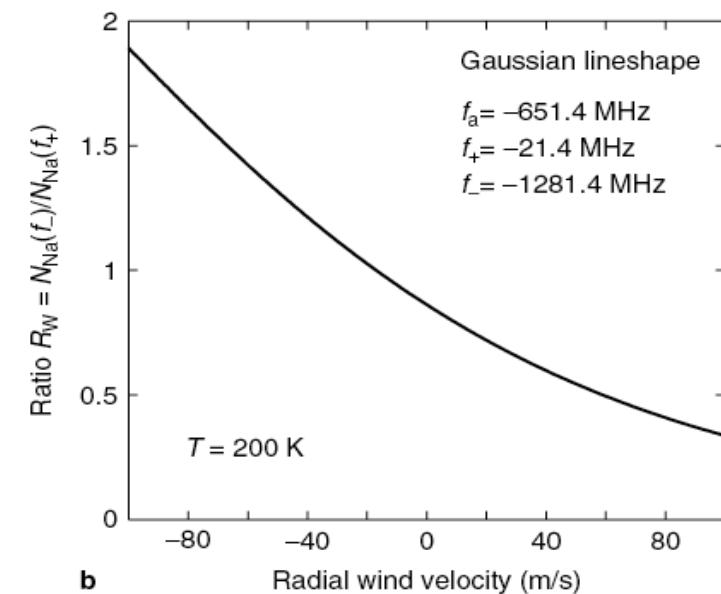
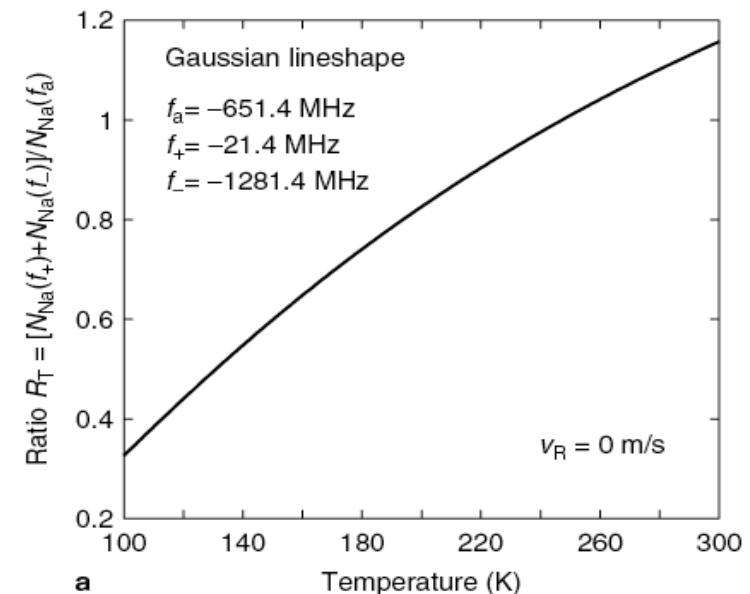
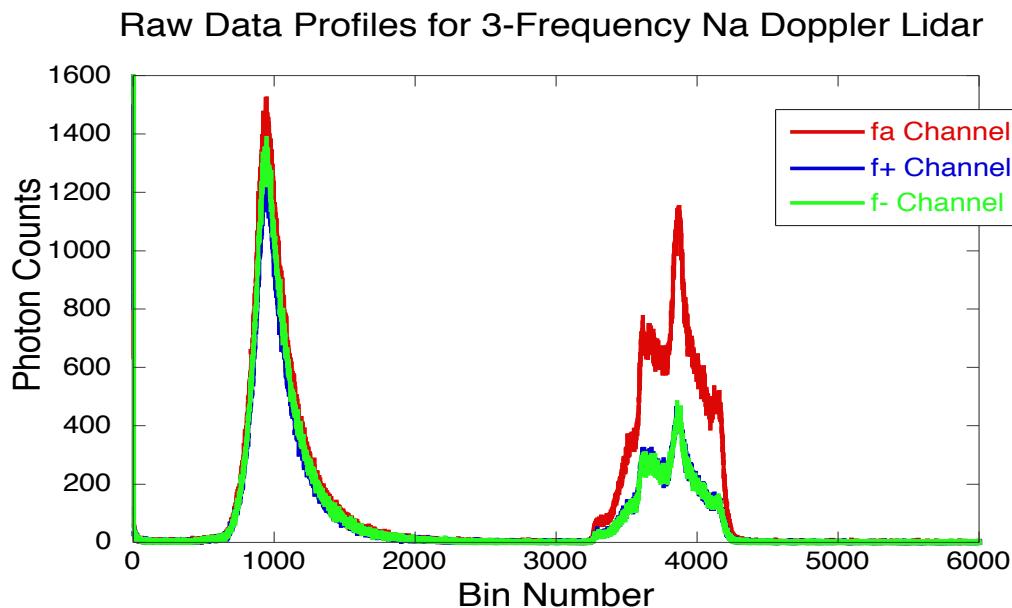
Two frequencies  $f_a$  and  $f_c$  are insensitive to radial wind.



# 3-Frequency Doppler Ratio Technique

$$R_T(z) = \frac{N_{norm}(f_+, z, t_1) + N_{norm}(f_-, z, t_2)}{N_{norm}(f_a, z, t_3)}$$
$$\approx \frac{\sigma_{eff}(f_+, z) + \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

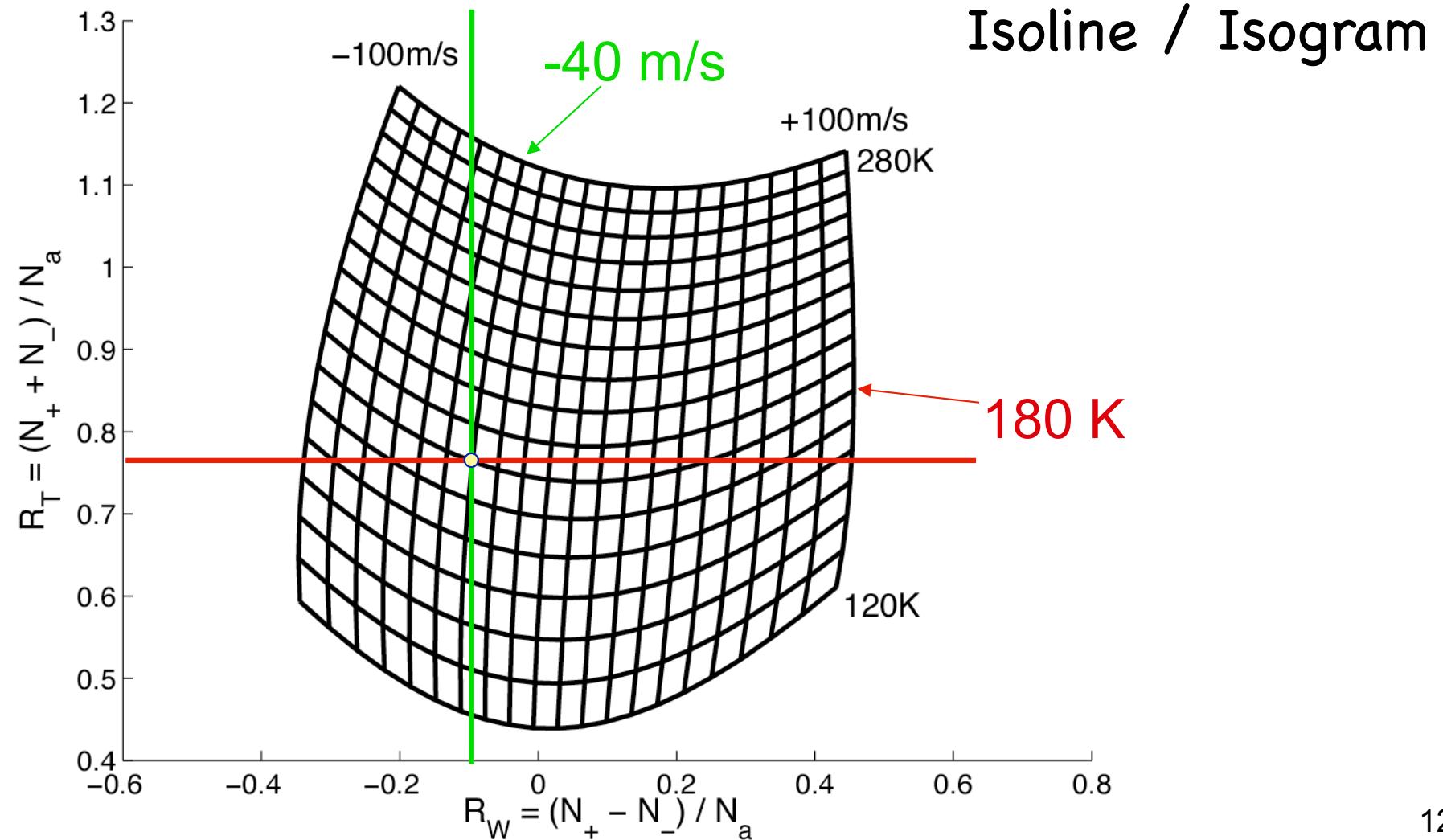
$$R_W(z) = \frac{N_{norm}(f_-, z, t_2)}{N_{norm}(f_+, z, t_1)} \approx \frac{\sigma_{eff}(f_-, z)}{\sigma_{eff}(f_+, z)}$$





# Calibration Curves for 3-Freq Tech

- For given temperatures and winds, we can compute the Doppler lidar calibration curves from atomic physics and lidar physics.





# Main Ideas Behind Ratio Technique

- Three unknown parameters (temperature, radial wind, and Na number density) require 3 lidar equations at 3 frequencies as the minimum  $\Rightarrow$  the highest resolution.
- In the ratio technique, Na number density is cancelled out. So we have two ratios  $R_T$  and  $R_W$  that are independent of Na density but both dependent on T and W.
- The idea is to derive temperature and radial wind from these two ratios first, and then derive Na number density using derived temperature and wind at each altitude bin.
- However, because the Na extinction coefficient is involved, the upper bins are related to lower bins, and extinction coefficient is related to Na density and effective cross-section. The solution is to start from the bottom of the Na layer and then work bin by bin to the layer top.



# Principle of Doppler Ratio Technique

- Lidar equation for resonance fluorescence (Na, K, or Fe)

$$N_S(\lambda, z) = \left( \frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) \left[ \sigma_{eff}(\lambda, z)n_c(z)R_B(\lambda) + \sigma_R(\pi, \lambda)n_R(z) \right] \Delta z \left( \frac{A}{4\pi z^2} \right) \\ \times \left( T_a^2(\lambda)T_c^2(\lambda, z) \right) (\eta(\lambda)G(z)) + N_B$$

$R_B = 1$  for current Na Doppler lidar since return photons at all wavelengths are received by the broadband receiver, so no fluorescence is filtered off.

- Pure Na signal and pure Rayleigh signal in Na region are

$$N_{Na}(\lambda, z) = \left( \frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) \left[ \sigma_{eff}(\lambda, z)n_c(z) \right] \Delta z \left( \frac{A}{4\pi z^2} \right) \left( T_a^2(\lambda)T_c^2(\lambda, z) \right) (\eta(\lambda)G(z))$$

$$N_R(\lambda, z) = \left( \frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) \left[ \sigma_R(\pi, \lambda)n_R(z) \right] \Delta z \left( \frac{A}{z^2} \right) \left( T_a^2(\lambda)T_c^2(\lambda, z) \right) (\eta(\lambda)G(z))$$

- So we have

$$N_S(\lambda, z) = N_{Na}(\lambda, z) + N_R(\lambda, z) + N_B$$



# Principle of Doppler Ratio Technique

- Lidar equation at pure molecular scattering region (35–55km)

$$N_S(\lambda, z_R) = \left( \frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) [\sigma_R(\pi, \lambda)n_R(z_R)] \Delta z \left( \frac{A}{z_R^2} \right) T_a^2(\lambda, z_R) (\eta(\lambda)G(z_R)) + N_B$$

- Pure Rayleigh signal in molecular scattering region is

$$N_R(\lambda, z_R) = \left( \frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) [\sigma_R(\pi, \lambda)n_R(z_R)] \Delta z \left( \frac{A}{z_R^2} \right) T_a^2(\lambda, z_R) (\eta(\lambda)G(z_R))$$

- So we have

$$N_S(\lambda, z_R) = N_R(\lambda, z_R) + N_B$$

- The ratio between Rayleigh signals at  $z$  and  $z_R$  is given by

$$\frac{N_R(\lambda, z)}{N_R(\lambda, z_R)} = \frac{[\sigma_R(\pi, \lambda)n_R(z)]T_a^2(\lambda, z)T_c^2(\lambda, z)G(z)}{[\sigma_R(\pi, \lambda)n_R(z_R)]T_a^2(\lambda, z_R)G(z_R)} \frac{z_R^2}{z^2} = \frac{n_R(z)}{n_R(z_R)} \frac{z_R^2}{z^2} T_c^2(\lambda, z)$$

Where  $n_R$  is the (total) atmospheric number density, usually obtained from atmospheric models like MSIS00.



# Principle of Doppler Ratio Technique

- From above equations, the pure Na and Rayleigh signals are

$$N_{Na}(\lambda, z) = N_S(\lambda, z) - N_B - N_R(\lambda, z)$$

$$N_R(\lambda, z_R) = N_S(\lambda, z_R) - N_B$$

- Normalized Na photon count is defined as

$$N_{Norm}(\lambda, z) = \frac{N_{Na}(\lambda, z)}{N_R(\lambda, z_R) T_c^2(\lambda, z)} \frac{z^2}{z_R^2}$$

- From physics point of view, the normalized Na count is

$$N_{Norm}(\lambda, z) = \frac{N_{Na}(\lambda, z)}{N_R(\lambda, z_R) T_c^2(\lambda, z)} = \frac{\sigma_{eff}(\lambda, z) n_c(z)}{\sigma_R(\pi, \lambda) n_R(z_R)} \frac{1}{4\pi}$$

- From actual photon counts, the normalized Na count is

$$\begin{aligned} N_{Norm}(\lambda, z) &= \frac{N_{Na}(\lambda, z)}{N_R(\lambda, z_R) T_c^2(\lambda, z)} \frac{z^2}{z_R^2} = \frac{N_S(\lambda, z) - N_B - N_R(\lambda, z)}{N_R(\lambda, z_R) T_c^2(\lambda, z)} \frac{z^2}{z_R^2} \\ &= \frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda, z)} - \frac{n_R(z)}{n_R(z_R)} \end{aligned}$$



# Principle of Doppler Ratio Technique

- From physics, the ratios of  $R_T$  and  $R_W$  are then given by

$$R_T = \frac{N_{Norm}(f_+, z) + N_{Norm}(f_-, z)}{N_{Norm}(f_a, z)} = \frac{\frac{\sigma_{eff}(f_+, z)n_c(z)}{\sigma_R(\pi, f_+)n_R(z_R)} + \frac{\sigma_{eff}(f_-, z)n_c(z)}{\sigma_R(\pi, f_-)n_R(z_R)}}{\frac{\sigma_{eff}(f_a, z)n_c(z)}{\sigma_R(\pi, f_a)n_R(z_R)}} = \frac{\sigma_{eff}(f_+, z) + \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

$$R_W = \frac{N_{Norm}(f_+, z) - N_{Norm}(f_-, z)}{N_{Norm}(f_a, z)} = \frac{\frac{\sigma_{eff}(f_+, z)n_c(z)}{\sigma_R(\pi, f_+)n_R(z_R)} - \frac{\sigma_{eff}(f_-, z)n_c(z)}{\sigma_R(\pi, f_-)n_R(z_R)}}{\frac{\sigma_{eff}(f_a, z)n_c(z)}{\sigma_R(\pi, f_a)n_R(z_R)}} = \frac{\sigma_{eff}(f_+, z) - \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

Here, Rayleigh backscatter cross-section is regarded as the same for three frequencies, since the frequency difference is so small. Na number density is also the same for three frequency channels, and so is the atmosphere number density at Rayleigh normalization altitude.



# Principle of Doppler Ratio Technique

- From actual photon counts, the ratios  $R_T$  and  $R_W$  are

$$R_T = \frac{N_{Norm}(f_+, z) + N_{Norm}(f_-, z)}{N_{Norm}(f_a, z)}$$
$$= \frac{\left( \frac{N_S(f_+, z) - N_B}{N_S(f_+, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_+, z)} - \frac{n_R(z)}{n_R(z_R)} \right) + \left( \frac{N_S(f_-, z) - N_B}{N_S(f_-, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_-, z)} - \frac{n_R(z)}{n_R(z_R)} \right)}{\frac{N_S(f_a, z) - N_B}{N_S(f_a, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_a, z)} - \frac{n_R(z)}{n_R(z_R)}}$$

$$R_W = \frac{N_{Norm}(f_+, z) - N_{Norm}(f_-, z)}{N_{Norm}(f_a, z)}$$
$$= \frac{\left( \frac{N_S(f_+, z) - N_B}{N_S(f_+, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_+, z)} - \frac{n_R(z)}{n_R(z_R)} \right) - \left( \frac{N_S(f_-, z) - N_B}{N_S(f_-, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_-, z)} - \frac{n_R(z)}{n_R(z_R)} \right)}{\frac{N_S(f_a, z) - N_B}{N_S(f_a, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_a, z)} - \frac{n_R(z)}{n_R(z_R)}}$$



# How Does Ratio Technique Work?

- From physics, we calculate the ratios of  $R_T$  and  $R_W$  as

$$R_T = \frac{\sigma_{eff}(f_+, z) + \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

$$R_W = \frac{\sigma_{eff}(f_+, z) - \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

- From actual photon counts, we calculate the ratios as

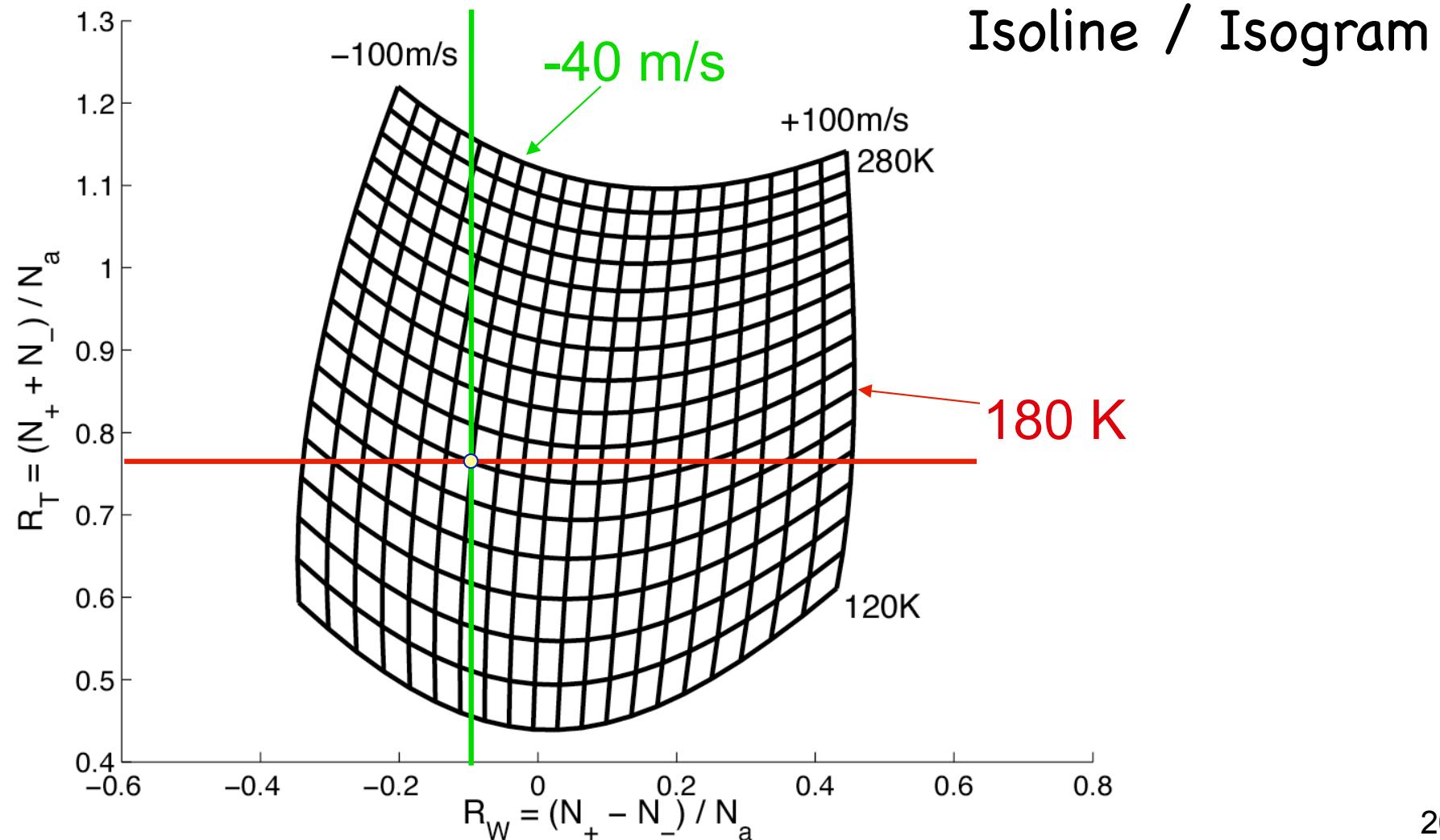
$$\begin{aligned} R_T &= \frac{N_{Norm}(f_+, z) + N_{Norm}(f_-, z)}{N_{Norm}(f_a, z)} \\ &= \frac{\left( \frac{N_S(f_+, z) - N_B}{N_S(f_+, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_+, z)} - \frac{n_R(z)}{n_R(z_R)} \right) + \left( \frac{N_S(f_-, z) - N_B}{N_S(f_-, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_-, z)} - \frac{n_R(z)}{n_R(z_R)} \right)}{\frac{N_S(f_a, z) - N_B}{N_S(f_a, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_a, z)} - \frac{n_R(z)}{n_R(z_R)}} \end{aligned}$$

$$\begin{aligned} R_W &= \frac{N_{Norm}(f_+, z) - N_{Norm}(f_-, z)}{N_{Norm}(f_a, z)} \\ &= \frac{\left( \frac{N_S(f_+, z) - N_B}{N_S(f_+, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_+, z)} - \frac{n_R(z)}{n_R(z_R)} \right) - \left( \frac{N_S(f_-, z) - N_B}{N_S(f_-, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_-, z)} - \frac{n_R(z)}{n_R(z_R)} \right)}{\frac{N_S(f_a, z) - N_B}{N_S(f_a, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_a, z)} - \frac{n_R(z)}{n_R(z_R)}} \end{aligned}$$



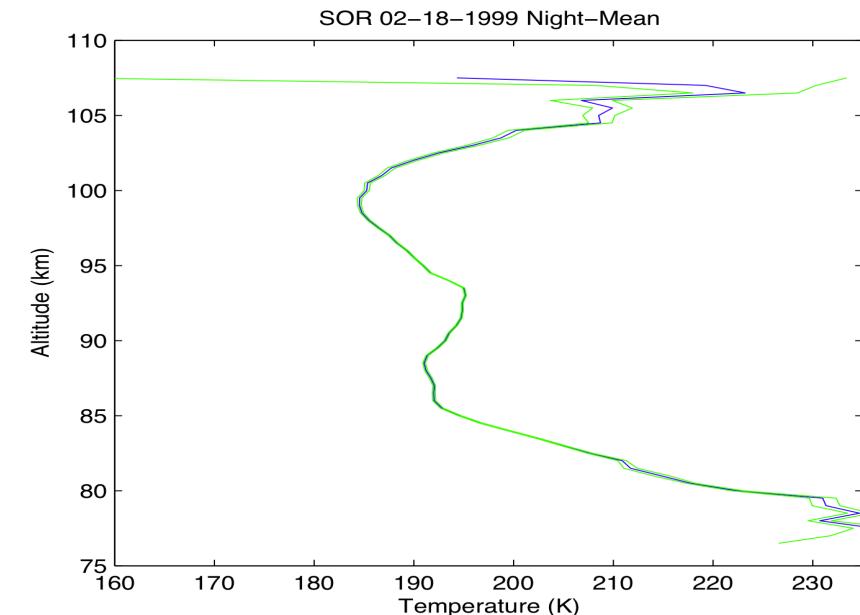
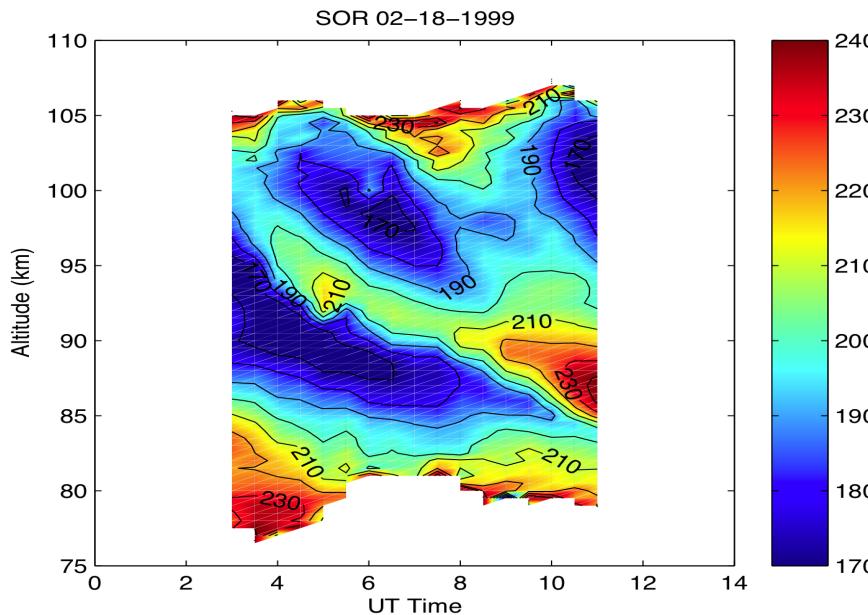
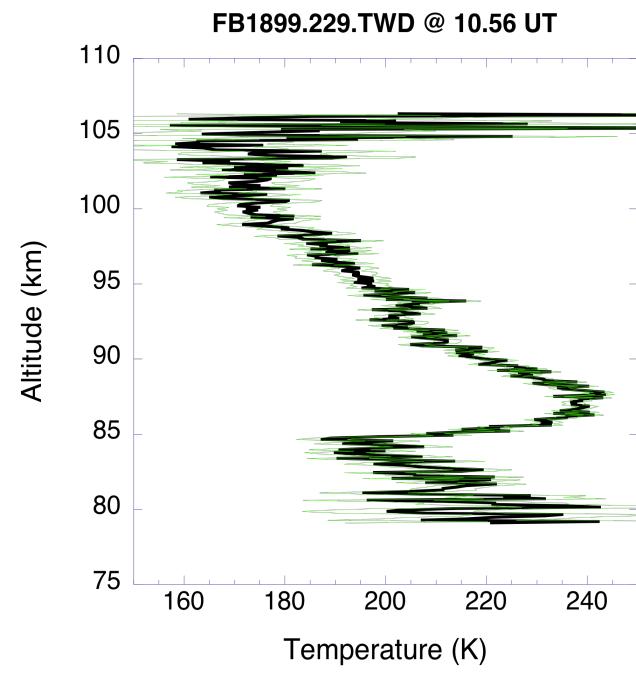
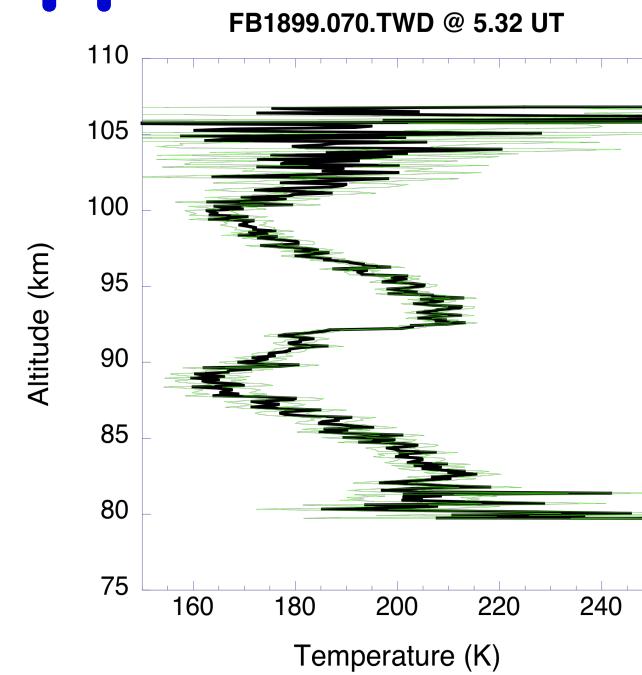
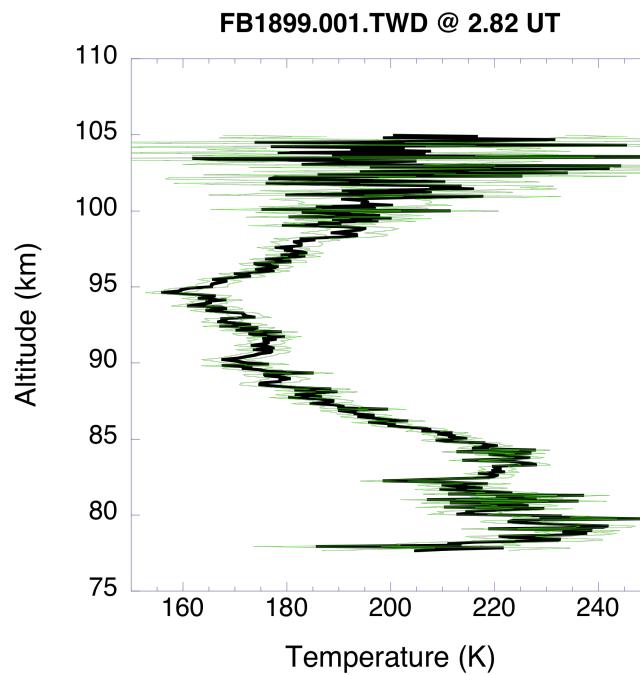
# How Does Ratio Technique Work?

- Compute Doppler calibration curves from physics
- Look up these two ratios on the calibration curves to infer the corresponding Temperature and Wind from isoline/isogram.





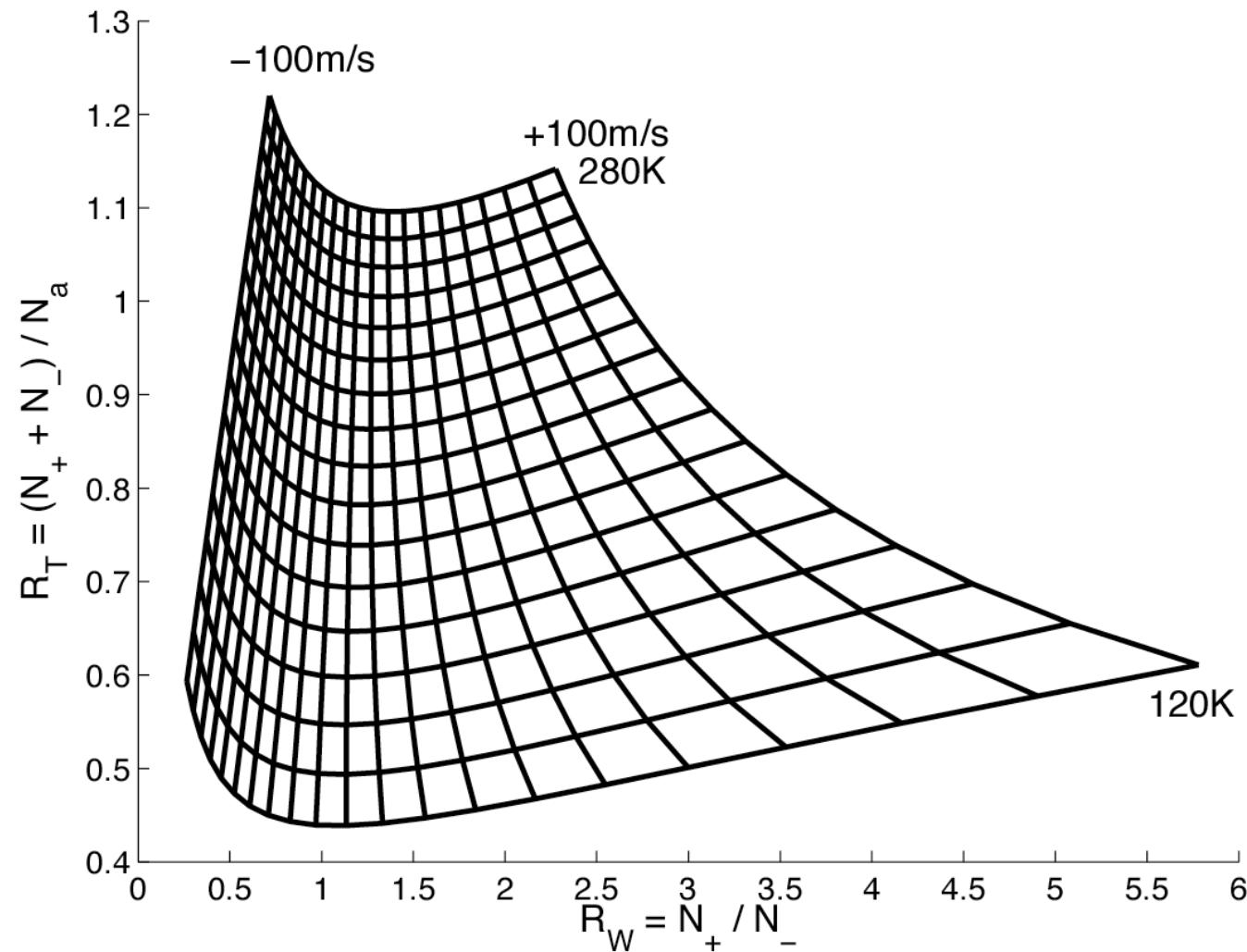
# UIUC Na Doppler Lidar Data @ SOR





# Comparison of Calibration Curves

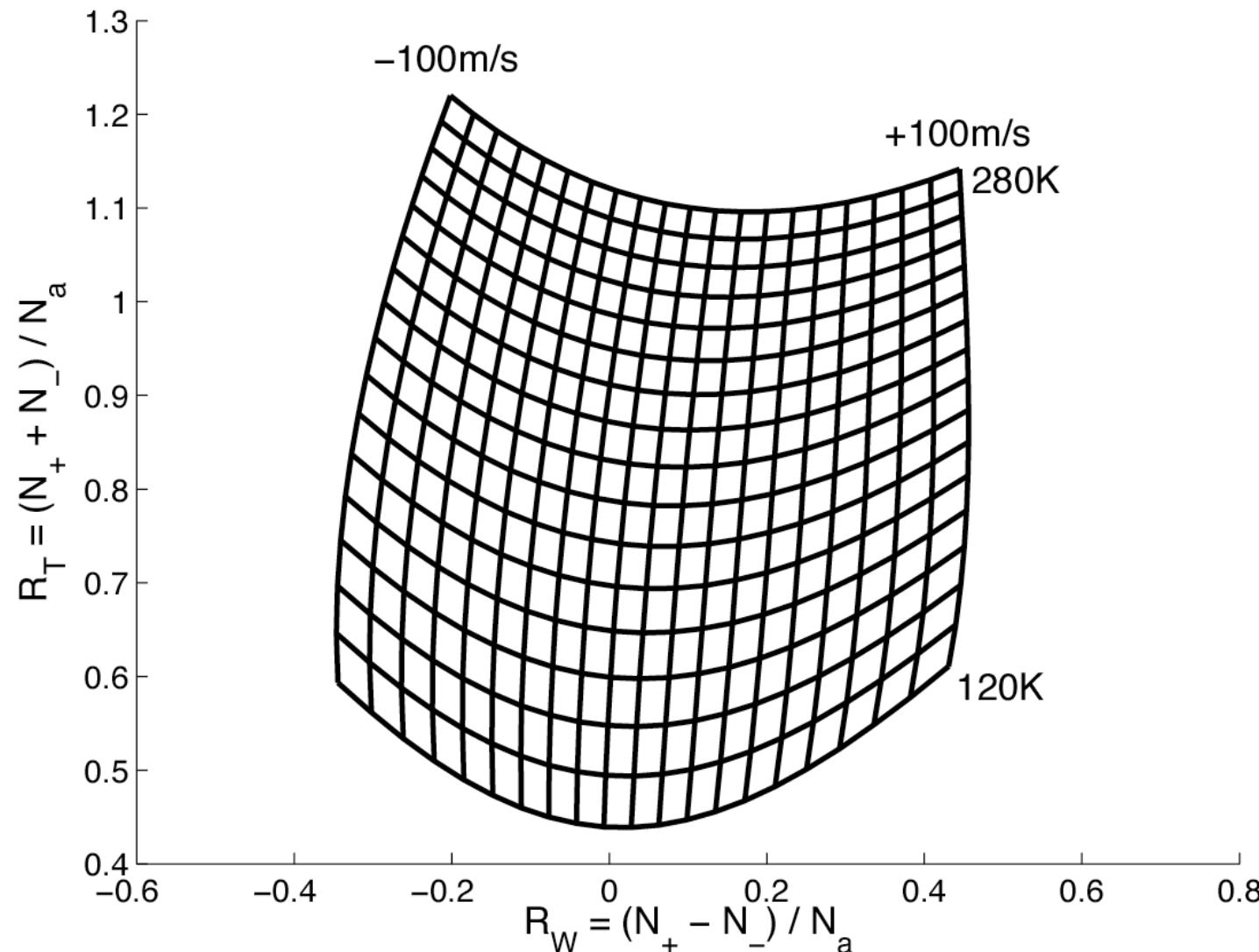
- Different metrics of  $R_w$  result in different wind sensitivities
- The ratio  $R_w = N_+ / N_-$  has inhomogeneous sensitivity





# Comparison of Calibration Curves

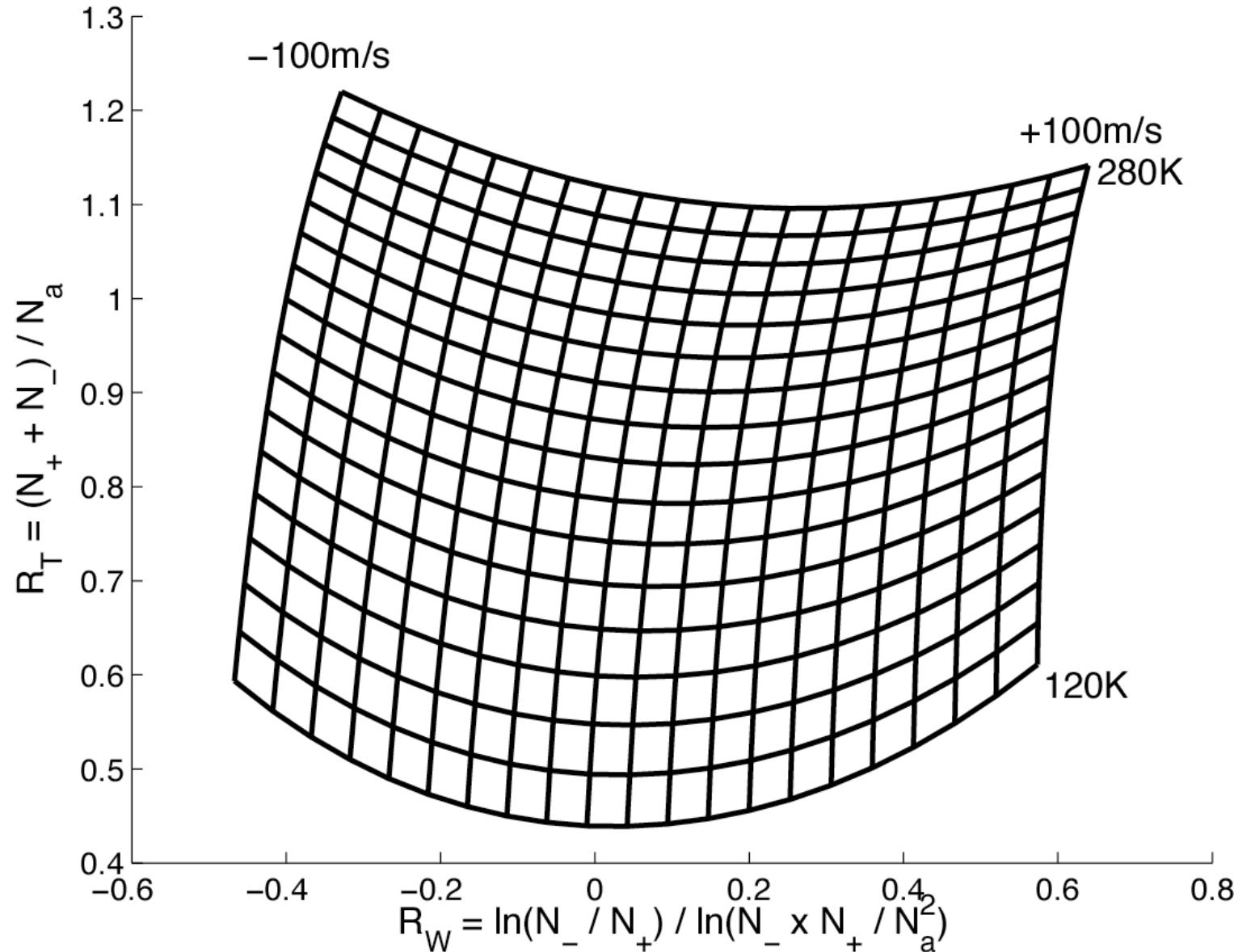
- The ratio  $R_W = (N_+ - N_-)/N_a$  has much better uniformity than the simplest ratio





# Comparison of Calibration Curves

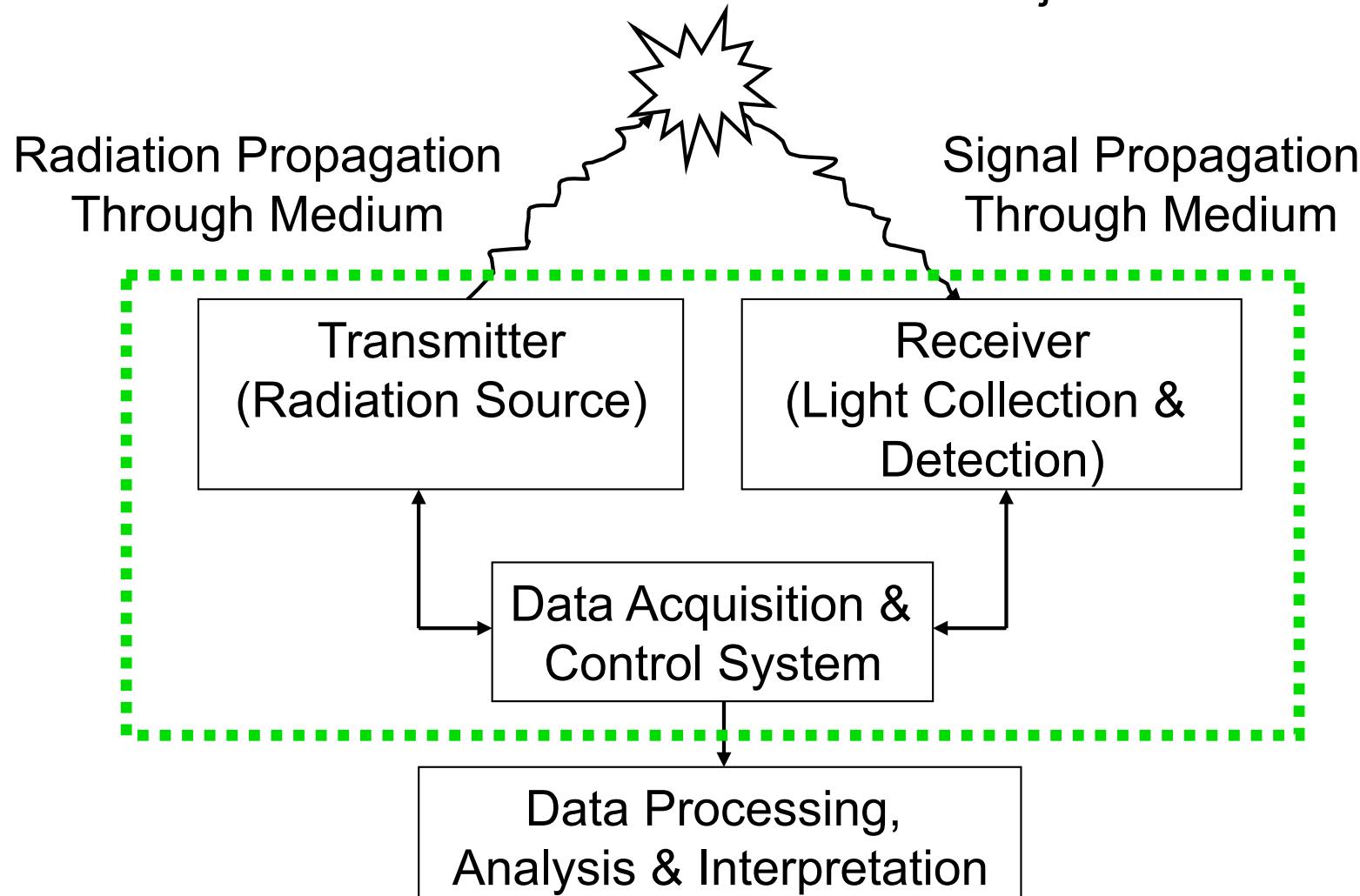
- The ratio  $R_w = \ln(N_- / N_+) / \ln(N_- \times N_+ / N_a^2)$  has good uniformity





# Na Doppler Lidar Instrumentation

Interaction between radiation and objects

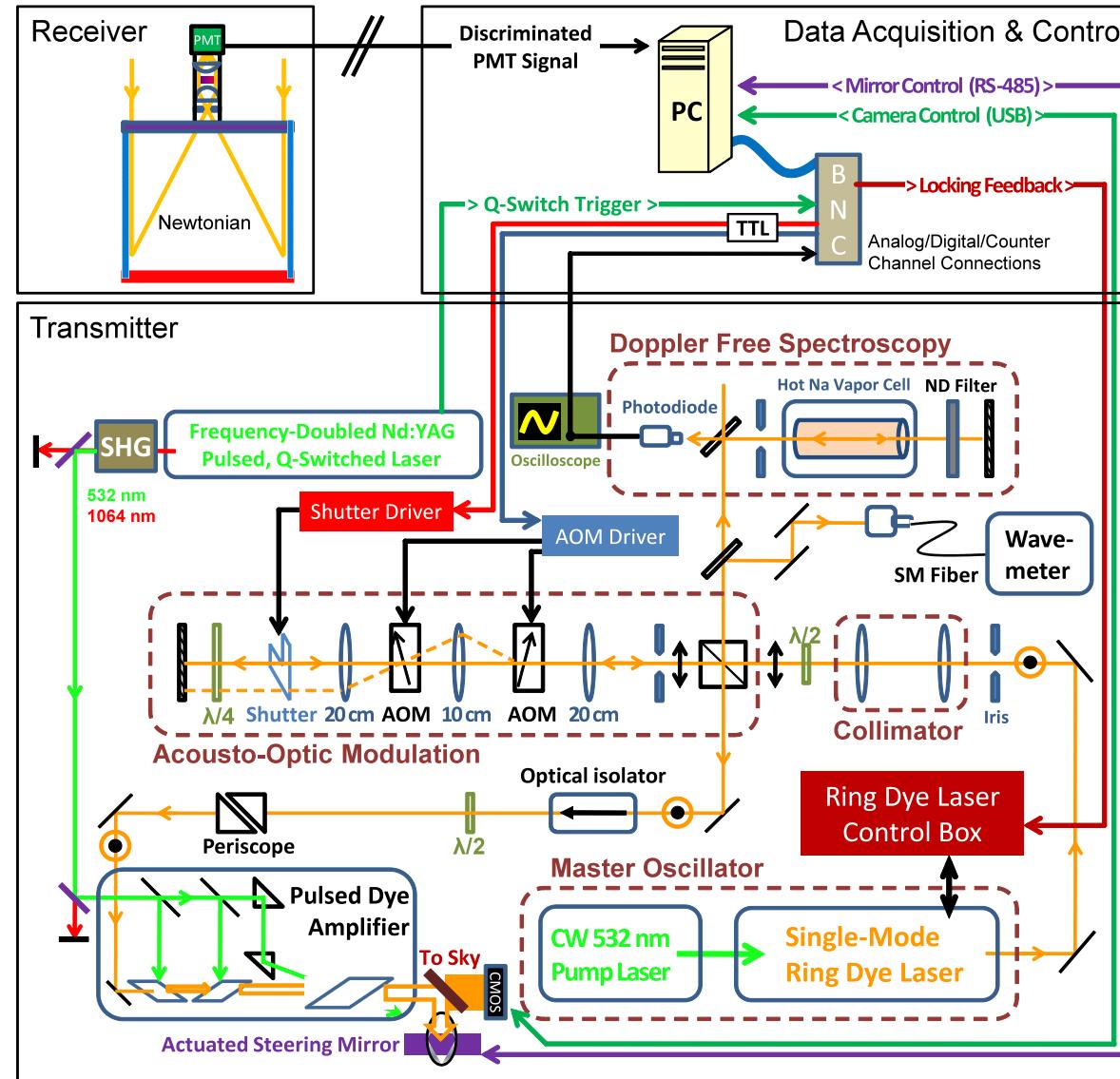


- ☐ Resonance Doppler lidar has the frequency discriminator in atmosphere - atomic absorption lines!  $\Rightarrow$  Narrowband transmitter, broadband receiver.  $\Rightarrow$  High signal levels and accurate knowledge on the frequency discriminator!<sup>25</sup>



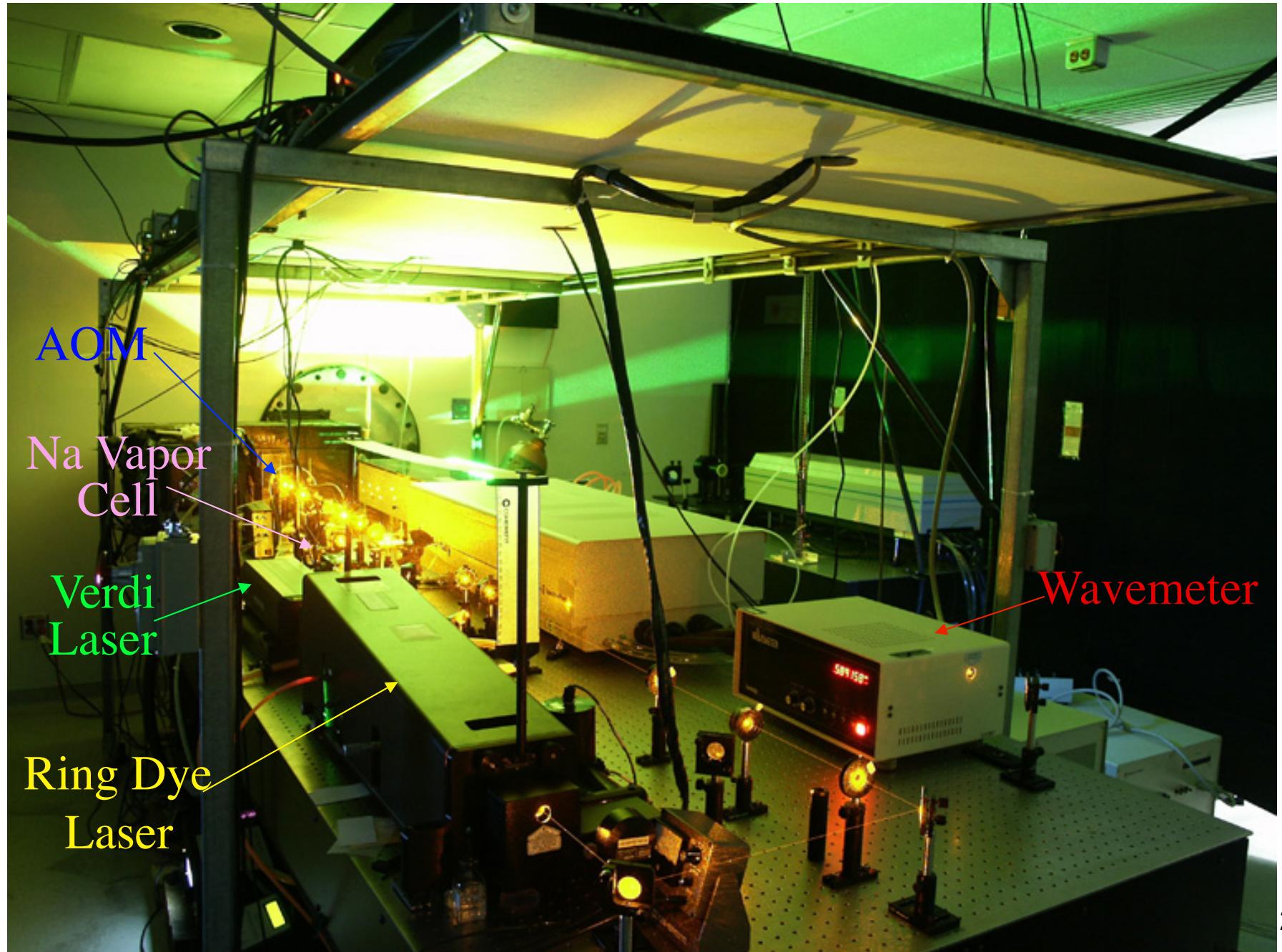
# STAR Na LIDAR

## Modernized DAQ, System Control and Receiver



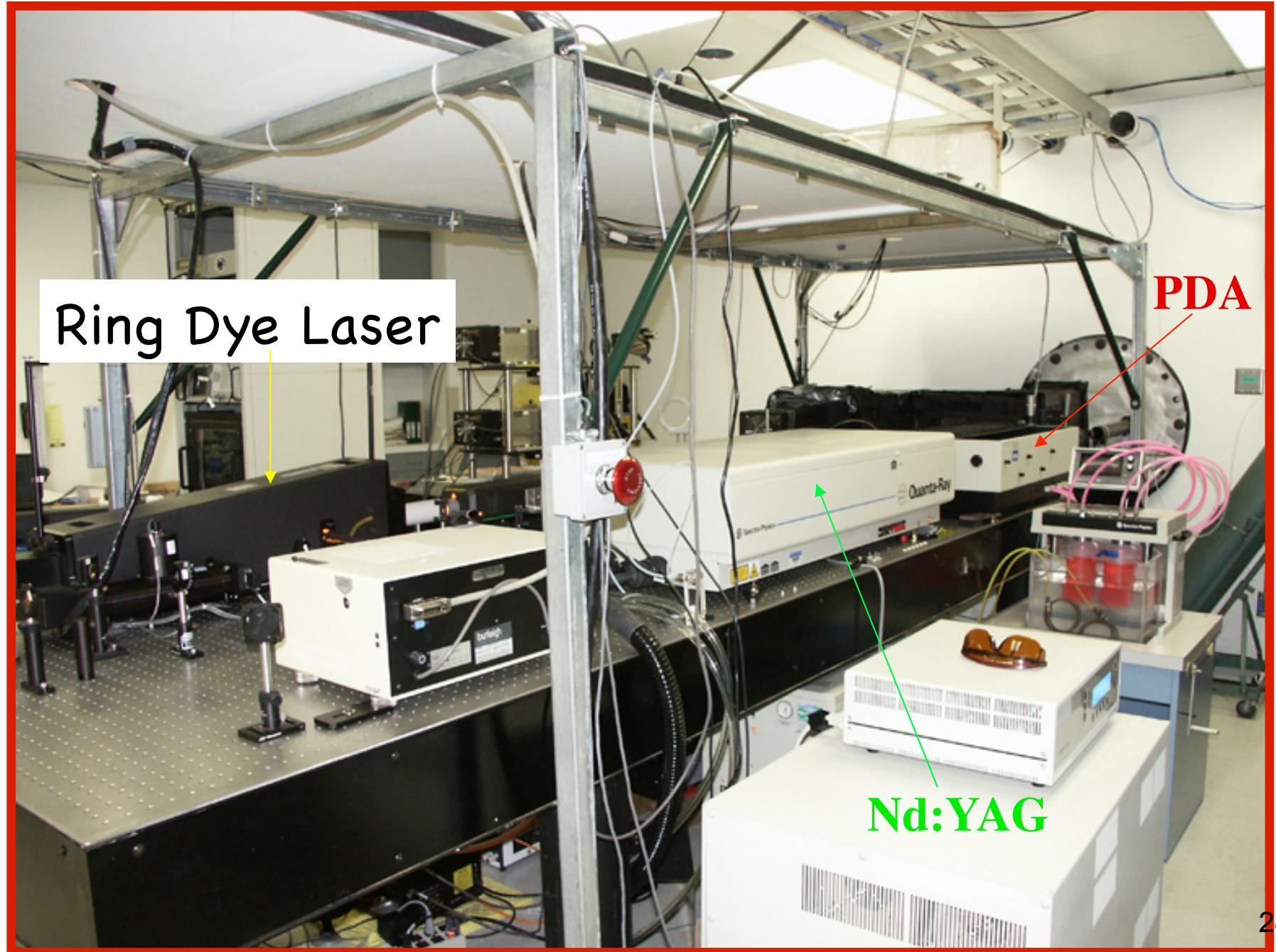


# Na Lidar Transmitter Photo 1



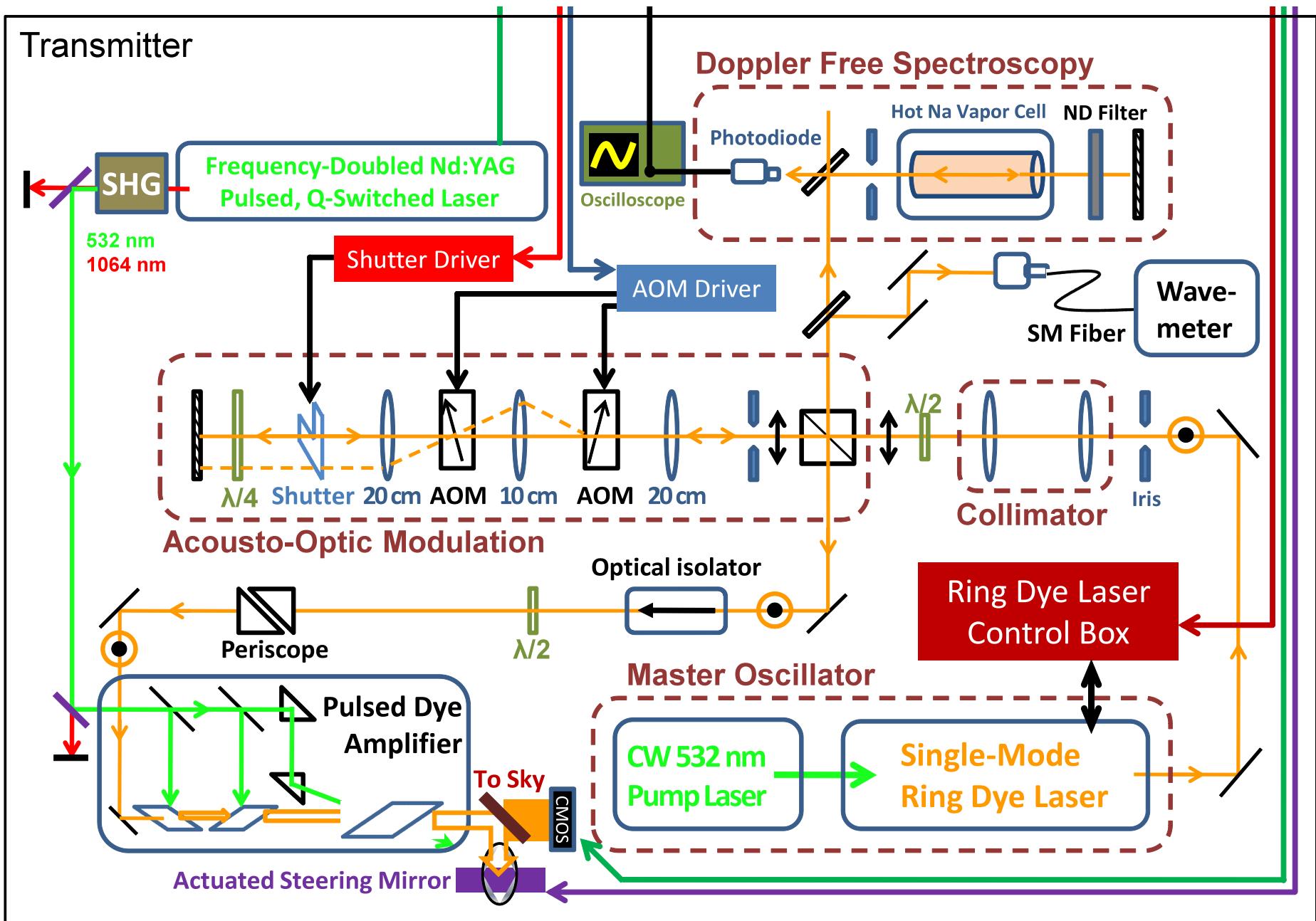


# Na Lidar Transmitter Photo 2



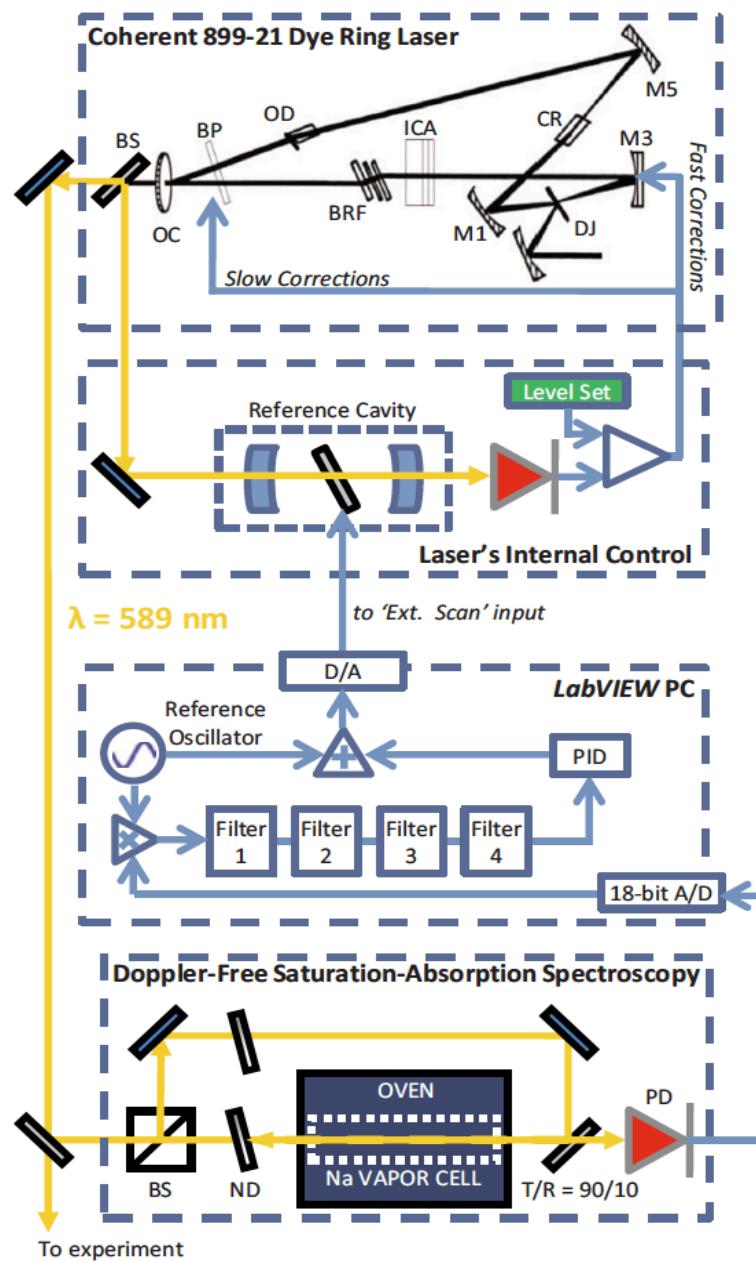


# STAR Na Doppler LIDAR Transmitter

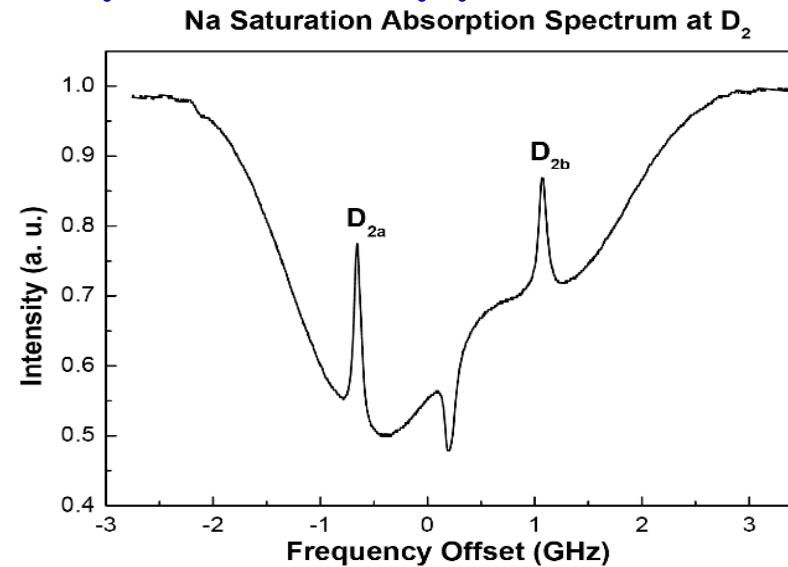




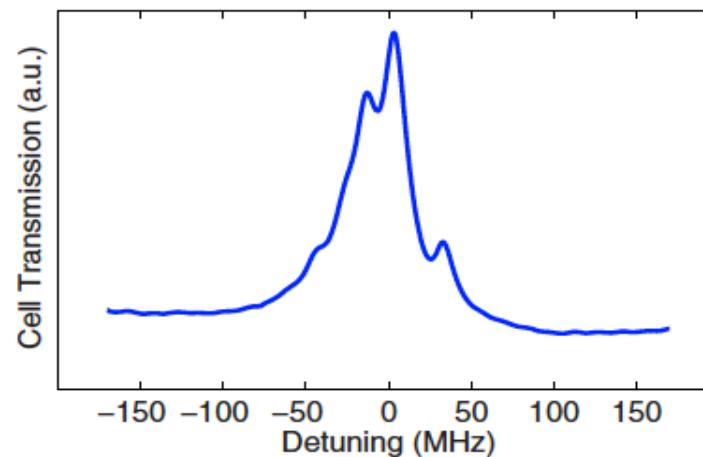
# Master Oscillator and Freq Locking with Doppler-Free Spectroscopy



[Smith et al., OE, 2009]



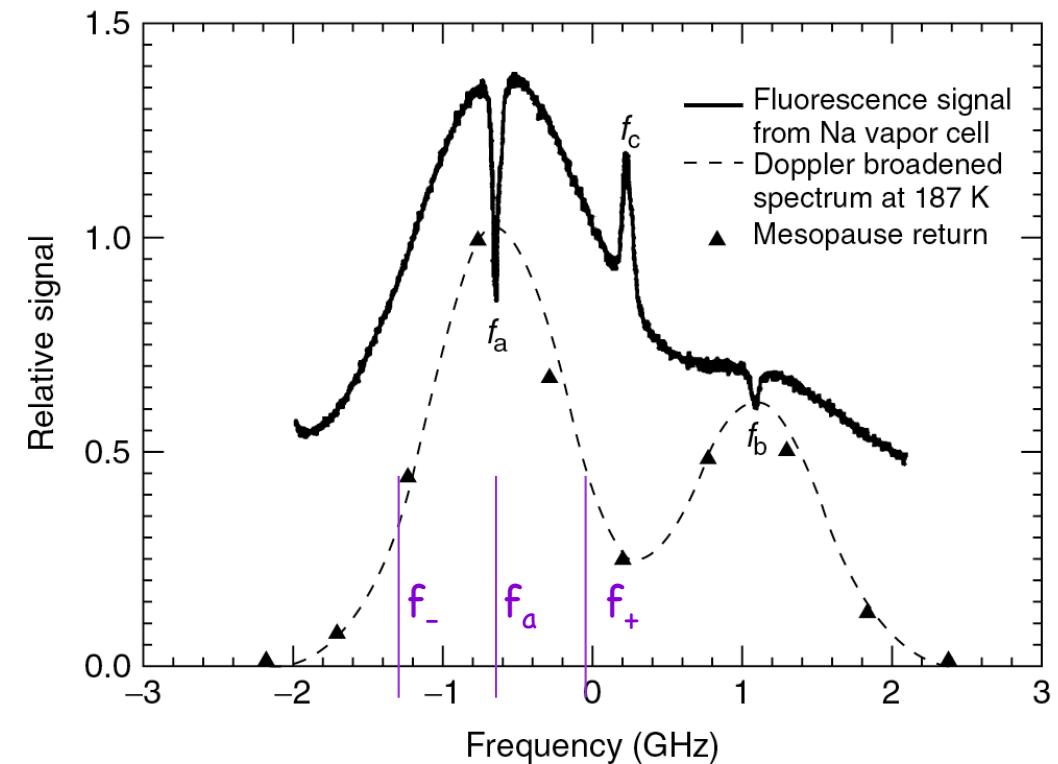
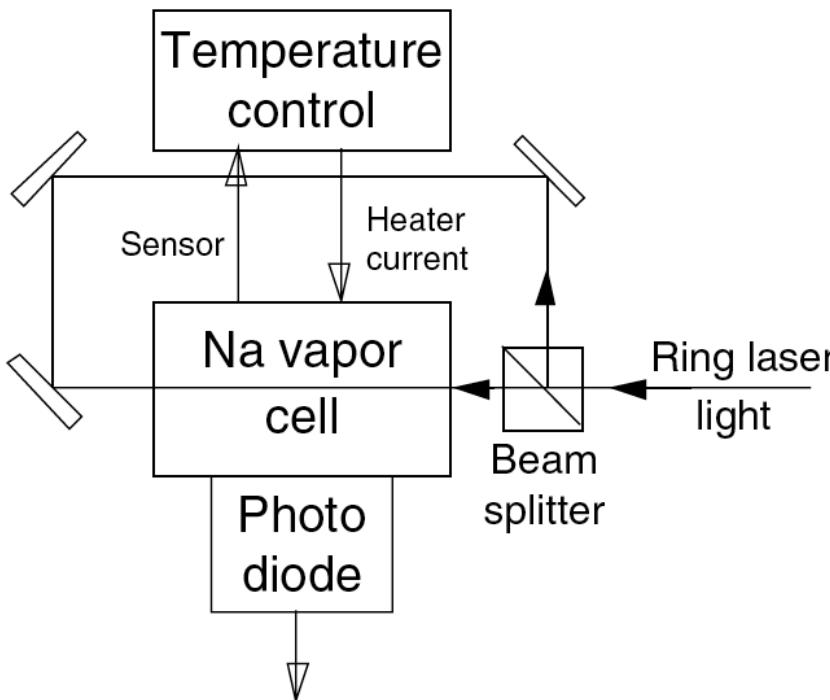
(a) Sodium D<sub>2a</sub> Doppler-Free Peak



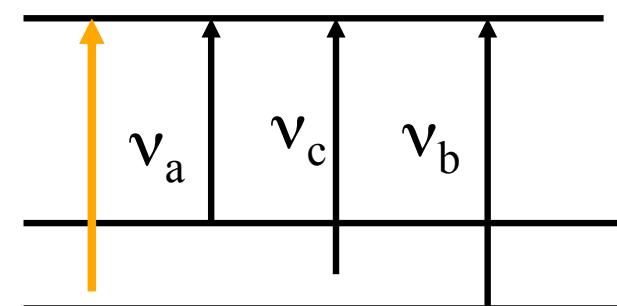
See detailed explanations on Na Doppler-free saturation-fluorescence spectroscopy in Textbook Chapter 5.2.2.3.2



# Doppler-Free Na Spectroscopy



See detailed explanation  
on Na Doppler-free  
saturation-fluorescence  
spectroscopy in Textbook  
Chapter 5.2.2.3.2



$$v_c = (v_a + v_b)/2$$



# Summary (1)

- ❑ Resonance fluorescence Doppler lidars apply Doppler technique to infer temperature and wind from the Doppler-broadened and Doppler-shifted atomic absorption spectroscopy. The Doppler-limited atomic absorption spectroscopy is inferred from the returned fluorescence intensity ratios at different frequencies.
- ❑ Both scanning and ratio techniques can work for the Doppler lidar. With scanning technique, the laser will be operated at many different frequencies, and then a least-square fit derives the width of the atomic absorption line, thus deriving the temperature. Its advantage is to provide more than 3 frequency information, so providing checks on more system parameters. But it requires longer integration time.
- ❑ Doppler ratio technique takes advantage of the high temporal resolution feature by limiting the lidar detection to 3 preset frequencies (usually one peak and two wing frequencies) for 3 unknown parameters ( $T$ ,  $W$ , and density).
- ❑ By taking the ratios among signals at these three frequencies,  $R_T$  and  $R_W$  are sensitive functions of temperature and radial wind, respectively.



# Summary (2)

- We compute the ratios  $R_T$  and  $R_W$  from atomic physics first to form the lidar calibration curves, and then look up the two ratios calculated from actual photon counts on the calibration curves to infer the corresponding temperature T and radial wind W.
- Different metrics exhibit different inhomogeneity, resulting in different crosstalk between T and W errors.
- There are several different atomic species (Na, K, Fe, Ca, Ca<sup>+</sup>, etc.) originating from meteor ablation in the mesosphere and lower thermosphere (MLT) region. They all have the potentials to be tracers for resonance fluorescence Doppler lidars to measure the temperature and wind in the MLT region.
- Na and K Doppler lidars are currently near mature status and making great contributions to MLT science.
- Fe Doppler lidar has very high future potential due to the high Fe abundance, advanced alexandrite laser technology, Doppler-free Fe spectroscopy, and bias-free measurements, etc.