



Lecture 30. Further Consideration on Lidar Data Inversion

- ❑ Aerosol and Cloud Lidar
 - Finish Remaining of Lecture 29
- ❑ Resonance Doppler lidar data processing
 - Further consideration on T_c
 - Na density derivation



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graph TD
    A[Read Data File] --> B[PMT/Discriminator Saturation Correction]
    B --> C[Chopper/Filter Correction]
    C --> D[Subtract Background]
    D --> E[Remove Range Dependence ( x R^2 )]
    E --> F[Add Base Altitude]
    F --> G[Estimate Rayleigh Signal @ z_R km]
    G --> H[Normalize Profile By Rayleigh Signal @ z_R km]
  
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Read Data File

PMT/Discriminator Saturation Correction

Chopper/Filter Correction

Subtract Background

Remove Range Dependence ($\times R^2$)

Add Base Altitude

Estimate Rayleigh Signal
@ z_R km

Normalize Profile
By Rayleigh Signal @ z_R km

Preprocess Procedure and Profile-Process Procedure for Na/Fe/K Doppler Lidar

- Read data: for each set, and calculate T , W , and n for each set
- PMT/Discriminator saturation correction
- Chopper/Filter correction

Integration

- Background estimate and subtraction
- Range-dependence removal ($\times R^2$, not z^2)
- Base altitude adjustment
- Take Rayleigh signal @ z_R (Rayleigh fit or Rayleigh mean)
- Rayleigh normalization

$$N_N(\lambda, z) = \frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} \frac{z^2}{z_R^2}$$

Main Process

- Subtract Rayleigh signals from Na/Fe/K region after counting in the factor of T_C

Main Process Step 1: Starting Point

1. Set transmission (T_c) at the bottom of Na layer to be 1
2. Calculate the normalized photon count for each frequency

$$N_{Norm}(\lambda, z) = \frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda, z)} - \frac{n_R(z)}{n_R(z_R)}$$

3. Take ratios R_T and R_W from normalized photon counts

$$R_T = \frac{N_{Norm}(f_+, z) + N_{Norm}(f_-, z)}{N_{Norm}(f_a, z)}$$

$$R_W = \frac{N_{Norm}(f_+, z) - N_{Norm}(f_-, z)}{N_{Norm}(f_a, z)}$$

4. Estimate the temperature and wind using the calibration curves computed from physics



Main Process Step 2: Bin-by-Bin Procedure

5. Calculate the effective cross section using temperature and wind derived
6. Using the effective cross-section and $T_c = 1$ (at the bottom), calculate the Na density.

$$n_c(z) = \left[\frac{N_S(\lambda, z) - N_B}{N_R(\lambda, z_R) - N_B} \cdot \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda, z)} - \frac{n_R(z)}{n_R(z_R)} \right] \cdot \frac{4\pi\sigma_R(\pi, \lambda)n_R(z_R)}{\sigma_{eff}(\lambda)R_B(\lambda)}$$

7. From effective cross-section and Na density, calculate the transmission T_c for the next bin.

$$T_c(\lambda, z) = \exp\left(-\int_{z_{bottom}}^z \sigma_{eff}(\lambda, z)n_c(z)dz\right) = \exp\left(-\sum_{z_{bottom}}^z \sigma_{eff}(\lambda, z)n_c(z)\Delta z\right)$$

Main Process

Load Atmosphere n_R , T , P
Profiles from MSIS00

Start from Na layer bottom
 $E(z=z_b) = 1$
Calculate $N_{\text{norm}}(z=z_b)$ from
photon counts and MSIS
number density for each freq

$$N_{\text{Norm}}(\lambda, z) = \frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda, z)} - \frac{n_R(z)}{n_R(z_R)}$$

Calculate R_T and R_W from N_{Norm}

Are ratios reasonable?

Yes

No

Set to nominal values or MSIS
 $T = 200 \text{ K}$, $W = 0 \text{ m/s}$

Calculate Na density $n_c(z)$

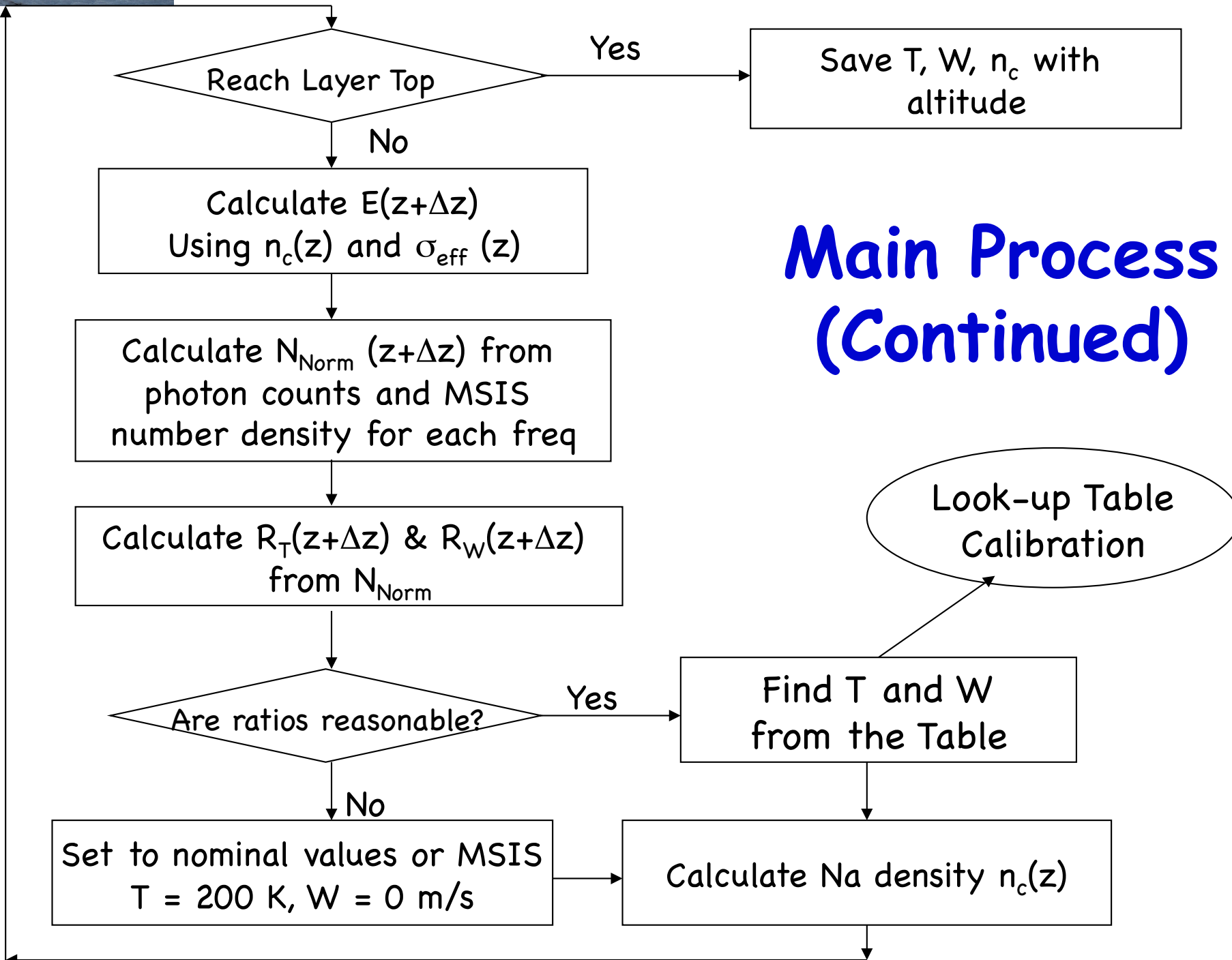
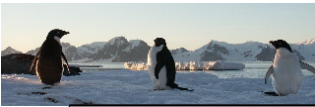
Create look-up table or calibration curves
From physics

$$R_T = \frac{\sigma_{\text{eff}}(f_+, z) + \sigma_{\text{eff}}(f_-, z)}{\sigma_{\text{eff}}(f_a, z)}$$

$$R_W = \frac{\sigma_{\text{eff}}(f_+, z) - \sigma_{\text{eff}}(f_-, z)}{\sigma_{\text{eff}}(f_a, z)}$$

Look-up Table
Calibration

Find T and W
from the Table



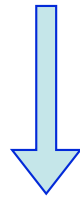
Derivation of T_c (Transmission Caused by Constituent Extinction)

□ T_c (caused by constituent absorption) can be derived from

$$T_c(\lambda, z) = \exp\left(-\int_{z_{bottom}}^z \sigma_{eff}(\lambda, z) n_c(z) dz\right) = \exp\left(-\sum_{z_{bottom}}^z \sigma_{eff}(\lambda, z) n_c(z) \Delta z\right)$$

+

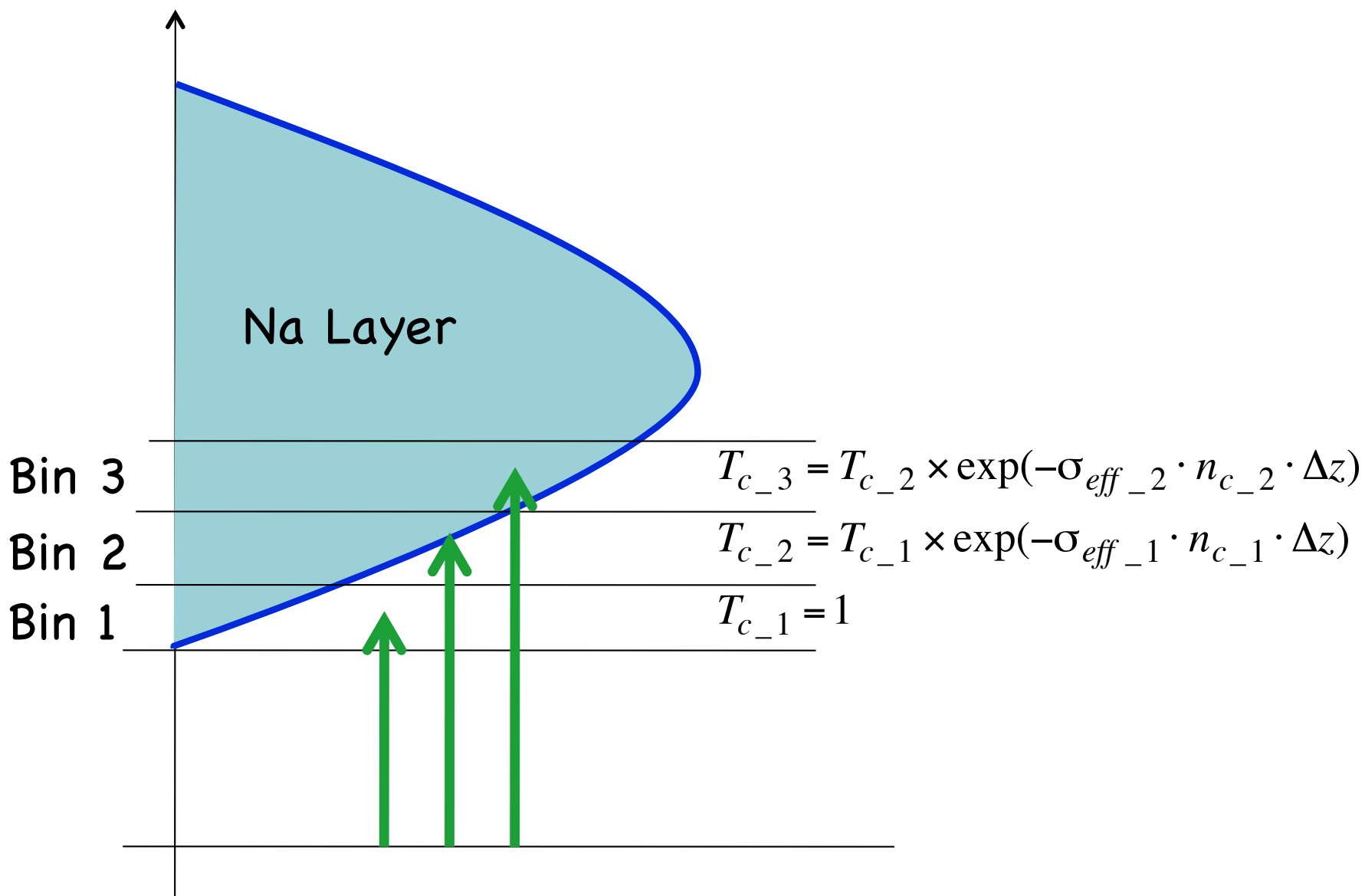
$$n_c(z) = \left[\frac{N_S(\lambda, z) - N_B}{N_R(\lambda, z_R) - N_B} \cdot \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda, z)} - \frac{n_R(z)}{n_R(z_R)} \right] \cdot \frac{4\pi\sigma_R(\pi, \lambda) n_R(z_R)}{\sigma_{eff}(\lambda, z) R_B(\lambda)}$$



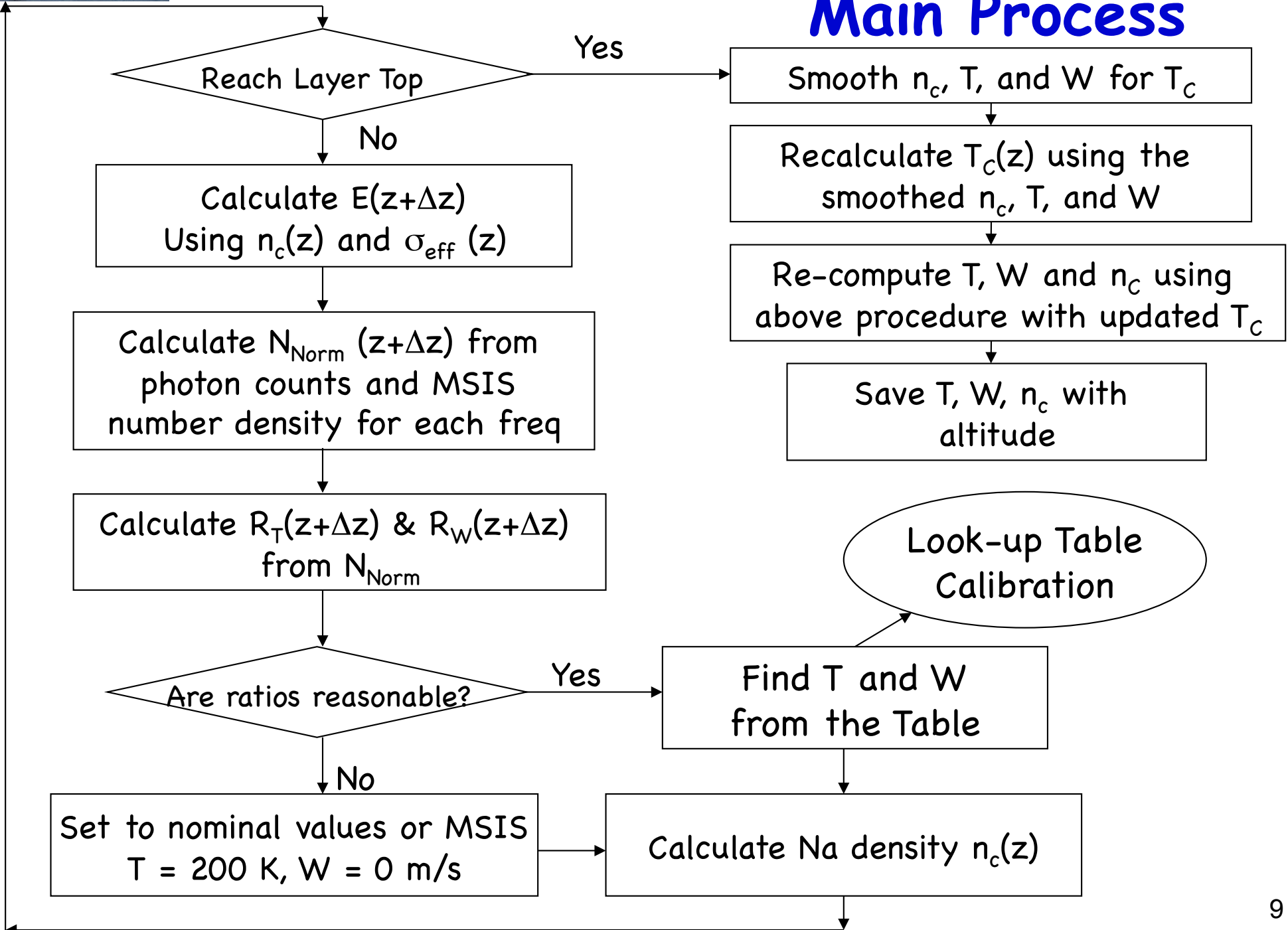
$$\sigma_e = \sqrt{\sigma_D^2 + \sigma_L^2}$$

$$T_c(\lambda, z) = \exp\left(-\sum_{z_{bottom}}^z \left[\frac{N_S(\lambda, z) - N_B}{N_R(\lambda, z_R) - N_B} \cdot \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda, z)} - \frac{n_R(z)}{n_R(z_R)} \right] \cdot \frac{4\pi\sigma_R(\pi, \lambda) n_R(z_R)}{R_B(\lambda)} \Delta z\right)$$

A Step in the Main Process: Estimating Transmission (T_c)



Main Process





Na Density Derivation

- The Na density can be inferred from the peak freq signal

$$n_{Na}(z) = \frac{N_{norm}(f_a, z)}{\sigma_a} 4\pi n_R(z_R) \sigma_R = \frac{N_{norm}(f_a, z)}{\sigma_a} 4\pi \times 2.938 \times 10^{-32} \frac{P(z_R)}{T(z_R)} \cdot \frac{1}{\lambda^{4.0117}}$$

- The Na density can also be inferred from a weighted average of all three frequency signals.

- The weighted effective cross-section is

$$\sigma_{eff_wgt} = \sigma_a + \alpha\sigma_+ + \beta\sigma_-$$

where α and β are chosen so that

$$\frac{\partial \sigma_{eff_wgt}}{\partial T} = 0; \quad \frac{\partial \sigma_{eff_wgt}}{\partial \nu_R} = 0$$

- The Na density is then calculated by

$$n_{Na}(z) = 4\pi n_R(z_R) \sigma_R \frac{N_{norm}(f_a, z) + \alpha N_{norm}(f_+, z) + \beta N_{norm}(f_-, z)}{\sigma_a + \alpha\sigma_+ + \beta\sigma_-}$$



Na Density Derivation

- The Na density can be inferred from the peak freq signal

$$n_{Na}(z) = \frac{N_{norm}(f_a, z)}{\sigma_a} 4\pi n_R(z_R) \sigma_R = \frac{N_{norm}(f_a, z)}{\sigma_a} 4\pi \times 2.938 \times 10^{-32} \frac{P(z_R)}{T(z_R)} \cdot \frac{1}{\lambda^{4.0117}}$$

- The Na density can also be inferred from a weighted average of all three frequency signals as explained above.

- The Na density can also be inferred from each individual frequency, and in principle, all three frequencies should give identical results of Na density.

$$n_{Na}(z) = \frac{N_{norm}(f_{\pm}, z)}{\sigma_{\pm}} 4\pi n_R(z_R) \sigma_R = \frac{N_{norm}(f_{\pm}, z)}{\sigma_{\pm}} 4\pi \times 2.938 \times 10^{-32} \frac{P(z_R)}{T(z_R)} \cdot \frac{1}{\lambda^{4.0117}}$$