Lecture 12. Temperature Lidar (1)
Overview and Doppler Technique

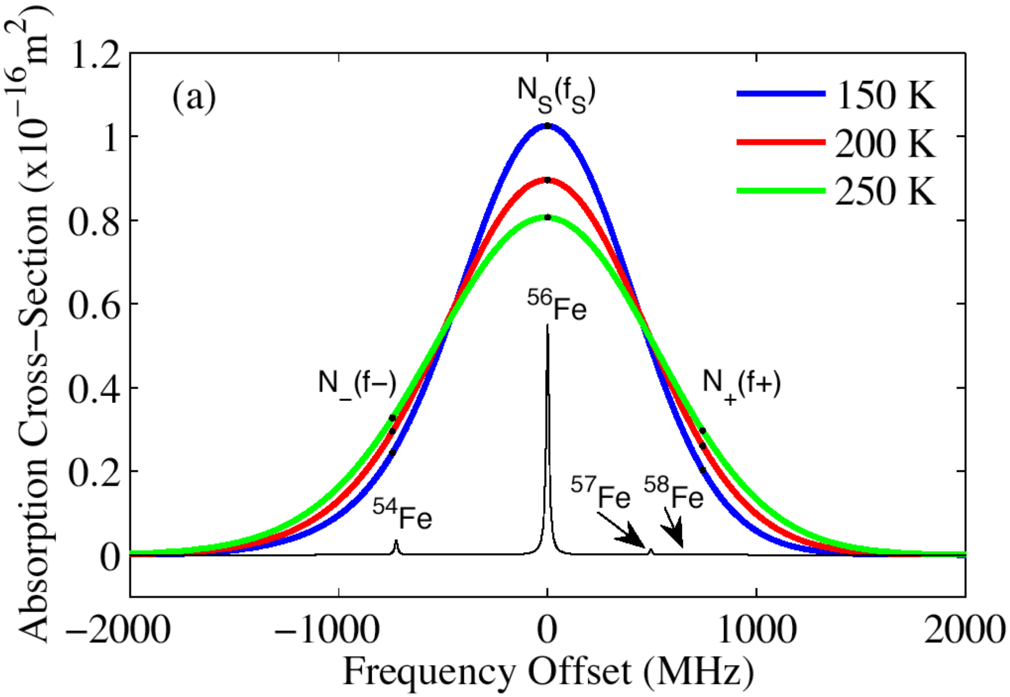
- Overview of Temperature Measurement Techniques
  - Doppler, Boltzmann, Integration and Rotational Raman
- Doppler Technique to Measure Temperature and Wind
  - Doppler Shift and Broadening in Resonance Absorption
  - Doppler Shift and Broadening in Rayleigh Scattering
  - Resonance Fluorescence Doppler vs Rayleigh Doppler
- Resonance Fluorescence Na Doppler Lidar
  - Na Structure and Spectroscopy
  - Scanning versus Ratio Techniques
- Summary

Temperature Measurement Techniques

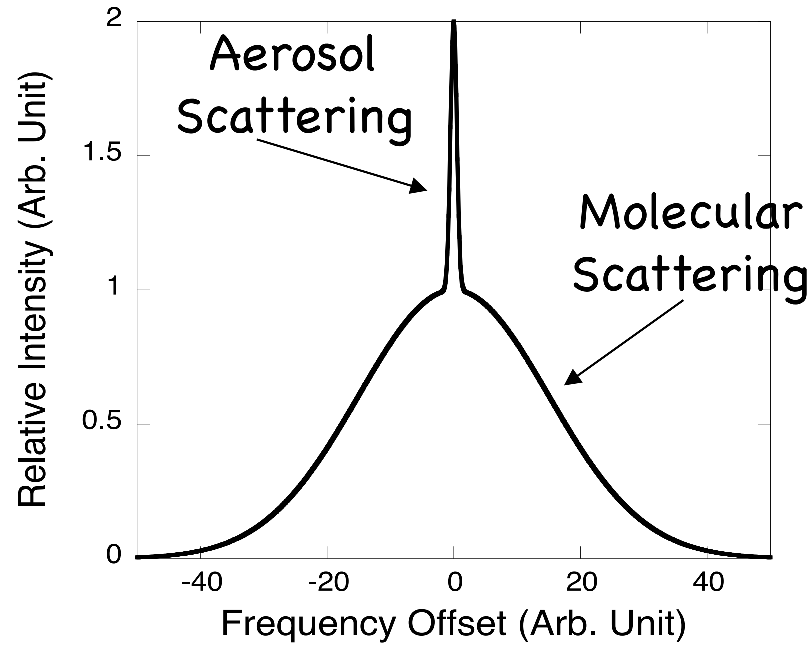
Use temperature-dependent effects or phenomena

- Doppler Technique – Doppler broadening (not only for Na, K, and Fe, but also for Rayleigh scattering, as long as Doppler broadening dominate and can be detected)
- Boltzmann Technique – Boltzmann distribution of atomic populations on different energy levels (not only for Fe, but also for molecular spectroscopy in optical remote sensing)
- Integration Technique (Rayleigh or Raman) – integration lidar technique using ideal gas law and assuming hydrostatic equilibrium (not only for modern lidar, but also for cw searchlight and rocket falling sphere – some way to measure atmosphere number density)
- Rotational Raman Technique – temperature dependence of population ratio, similar to Boltzmann technique

Overview: Doppler Technique



Atomic Absorption Line



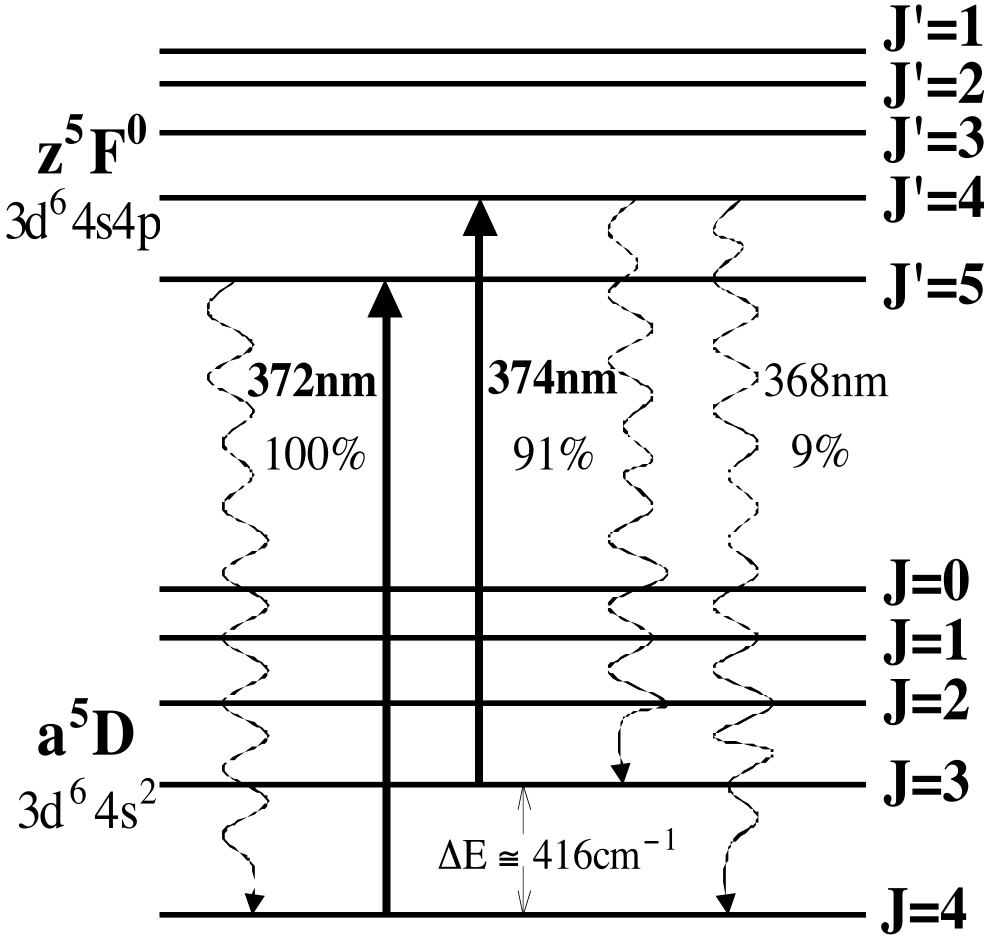
Rayleigh Scattering

$$\sigma_{rms} = \frac{\nu_0}{c}\sqrt{\frac{k_B T}{M}} = \frac{1}{\lambda_0}\sqrt{\frac{k_B T}{M}} = \sqrt{\frac{k_B T}{M\lambda_0^2}} \quad (12.1)$$

$$\sigma_{rms} = 2\nu_0 / c\sqrt{k_B T / M} = \frac{2}{\lambda_0}\sqrt{k_B T / M} \quad (12.2)$$

Doppler Spectrum (Doppler Broadening Width) ⇨ Temperature

Overview: Boltzmann Technique



Atomic Fe Energy Level

[Gelbwachs, 1994; Chu et al., 2002]

Maxwell-Boltzmann Distribution in Thermal-dynamic Equilibrium

$$\frac{P_2(J=3)}{P_1(J=4)} = \frac{\rho_{Fe(374)}}{\rho_{Fe(372)}} = \frac{g_2}{g_1} exp(-\Delta E / k_B T) \quad (12.3)$$

$$T = \frac{\Delta E / k_B}{ln\left(\frac{g_2}{g_1} \cdot \frac{P_1}{P_2}\right)} \quad (12.4)$$

P_1, P_2 -- Fe populations
g_1, g_2 -- Degeneracy
k_B -- Boltzmann constant
T -- Temperature

Population Ratio ⇨ Temperature

Overview: Integration Technique

Hydrostatic Equation

$$dP = -\rho g dz \quad (12.5)$$

+

Ideal Gas Law

$$P = \rho R T \quad (12.6)$$

$$T(z) = T(z_o)\frac{\rho(z_o)}{\rho(z)} + \frac{1}{R}\int_{z}^{z_o} g(r)\frac{\rho(r)}{\rho(z)}dr \quad (12.7)$$

Seeding Temperature

Relative Density

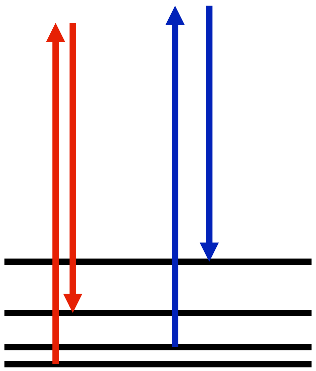
$T(z_0)$ - Seeding Temperature; **ρ - number density**

R - gas constant for dry air; **g - gravitational acceleration**

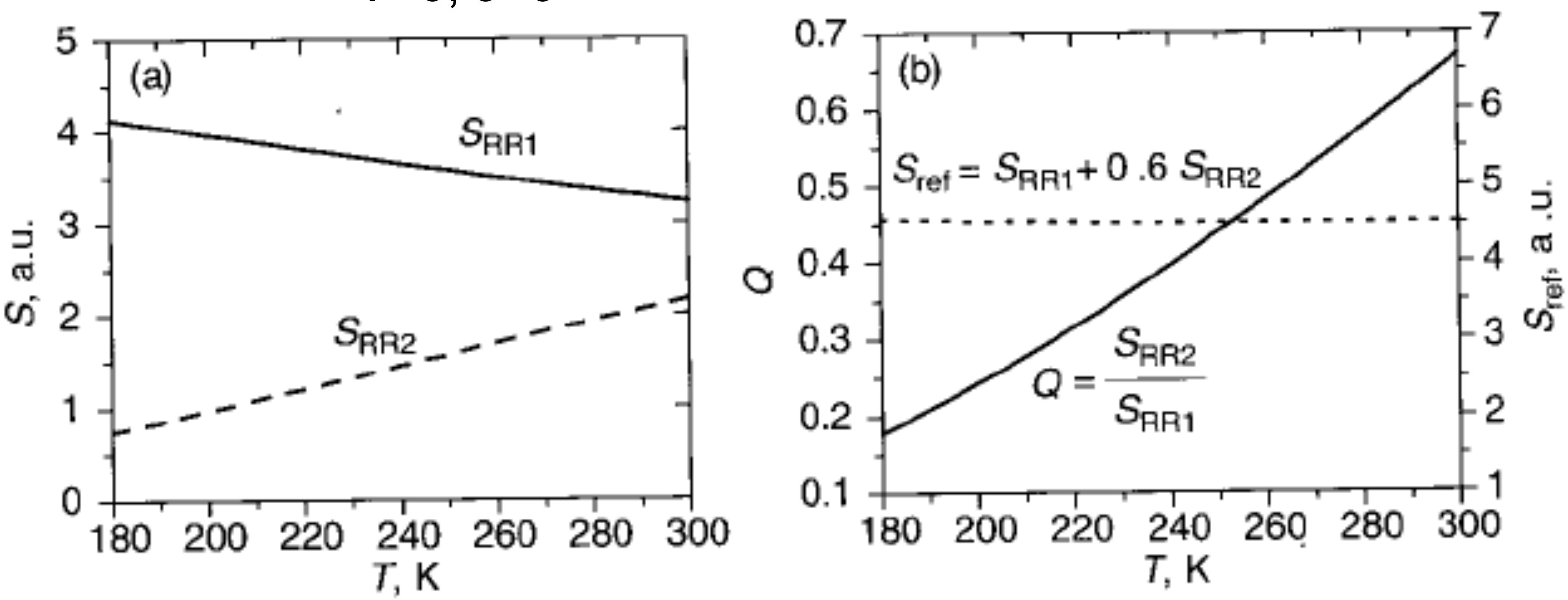
Number Density Ratio ⇒ Temperature

(lidar backscatter ratio at different altitudes)

Overview: Rotation Raman Technique



$$Q(T) = \frac{\sum_{i=O_2,N_2} \sum_{J_i} \tau_{RR2}(J_i)\eta_i \left(\frac{d\sigma}{d\Omega}\right)_{\pi}^{RR,i}(J_i)}{\sum_{i=O_2,N_2} \sum_{J_i} \tau_{RR1}(J_i)\eta_i \left(\frac{d\sigma}{d\Omega}\right)_{\pi}^{RR,i}(J_i)} \quad (12.8)$$



- Temperature can be derived from the ratio of two pure Rotational Raman line intensities. This is essentially the same principle as Boltzmann temperature technique!

Doppler Technique to Measure Temperature and Wind

- Doppler effect is commonly experienced by moving particles, such as atoms, molecules, and aerosols. It is the apparent frequency change of radiation that is perceived by the particles moving relative to the source of the radiation. This is called Doppler shift.
- Doppler frequency shift is proportional to the radial velocity along the line of sight (LOS) of the radiation –

$$\boxed{\omega = \omega_0 - \vec{k} \cdot \vec{v}} \quad (12.9)$$

$$\boxed{\begin{aligned} \Delta\omega &= \omega - \omega_0 = -\vec{k} \cdot \vec{v} = -\omega_0 (v/c) \cos\theta \\ \Delta\nu &= -\nu_0 (v/c) \cos\theta = -(v/\lambda_0) \cos\theta \end{aligned}} \quad (12.10)$$

where ω_0 is the radiation frequency at rest, ω is the shifted frequency, k is the wave vector of the radiation ($k=2\pi/\lambda$), and v is the particle velocity.

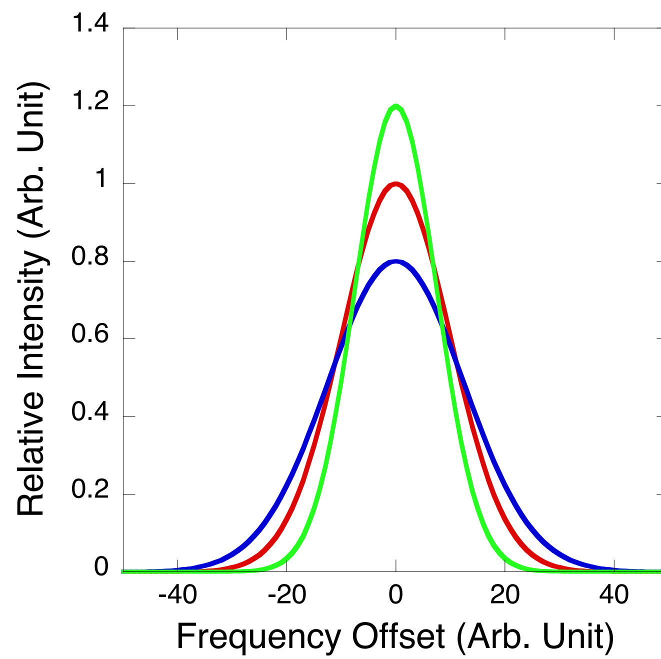
Doppler Technique to Measure Temperature and Wind

- Due to particles' thermal motions in the atmosphere, the distribution of perceived frequencies for all particles mirrors their velocity distribution. According to the Maxwellian velocity distribution, the perceived frequencies by moving particles has a Gaussian lineshape, given by

$$\boxed{\exp\left(-\frac{Mv_z^2}{2k_B T}\right)dv_z = \exp\left\{-\frac{Mc^2(\nu-\nu_0)^2}{2\nu_0^2 k_B T}\right\}\frac{c}{\nu_0}d\nu} \quad (12.11)$$

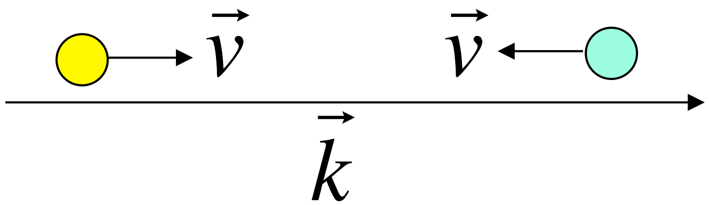
- The peak is at $\omega = \omega_0$ and the rms width is give by

$$\boxed{\sigma_{rms} = \frac{\nu_0}{c}\sqrt{\frac{k_B T}{M}} = \frac{1}{\lambda_0}\sqrt{\frac{k_B T}{M}}} \quad (12.12)$$



Doppler Shift in Resonance Absorption

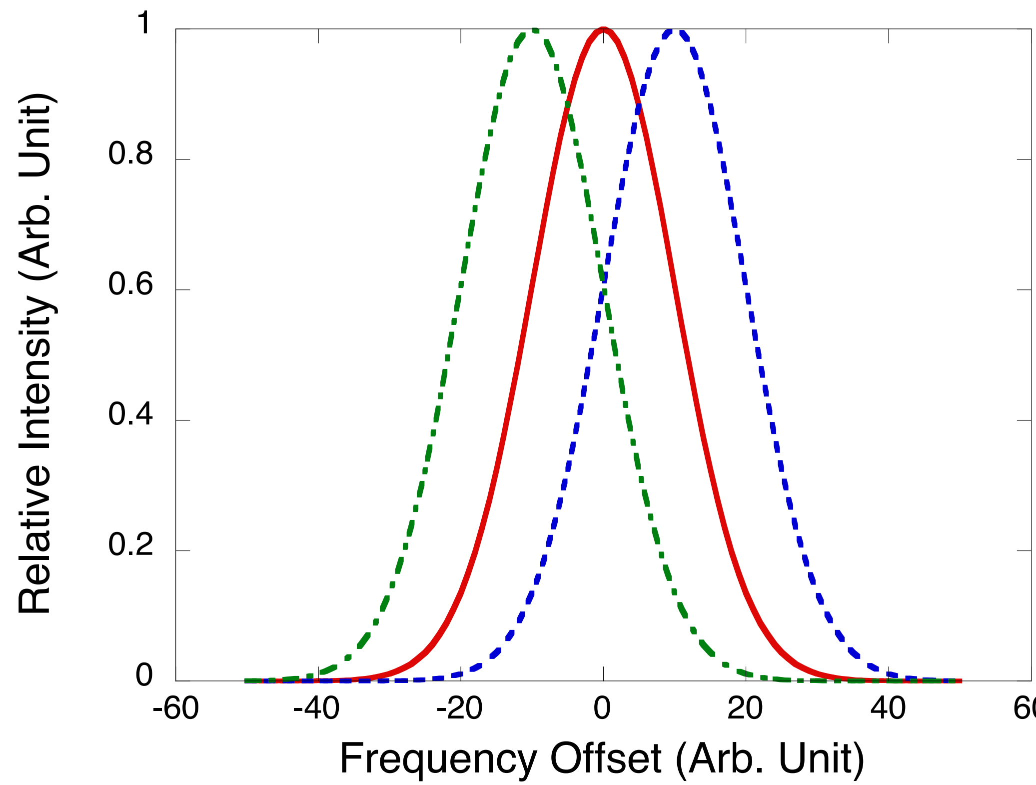
$$\boxed{\Delta\omega = \omega - \omega_0 = -\vec{k}\cdot\vec{v} = -\omega_0 \frac{v\cos\theta}{c}} \qquad (12.13)$$



Emitter and receiver move towards each other:

-Blue shift in perceived radiation frequency

-Red shift in absorption peak frequency



❑ The velocity measurements of lidar, radar, and sodar all base on the Doppler shift principle !

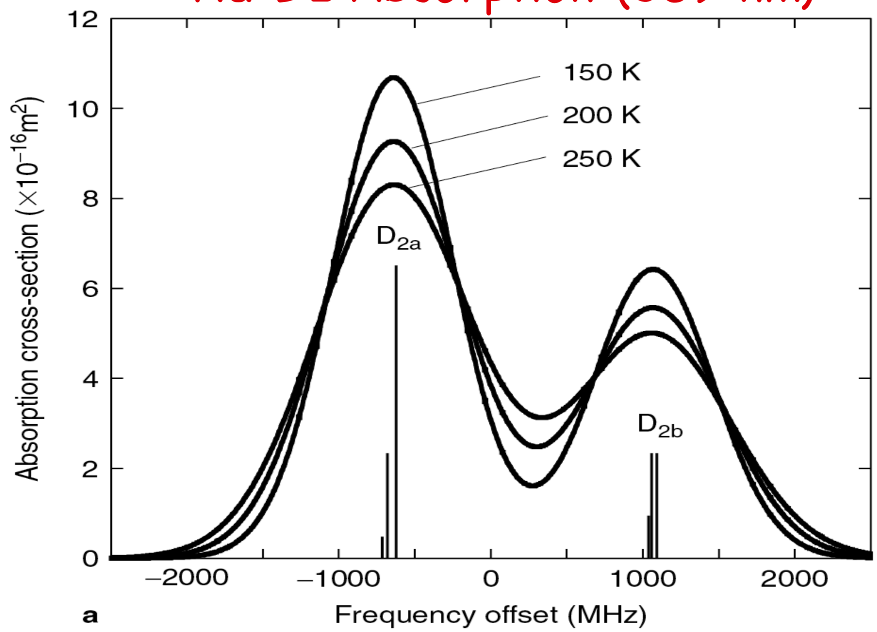
Doppler Broadening in Resonance Absorption Lines

$$\boxed{\sigma_{rms} = \frac{\nu_0}{c}\sqrt{\frac{k_B T}{M}} = \frac{1}{\lambda_0}\sqrt{\frac{k_B T}{M}}}$$

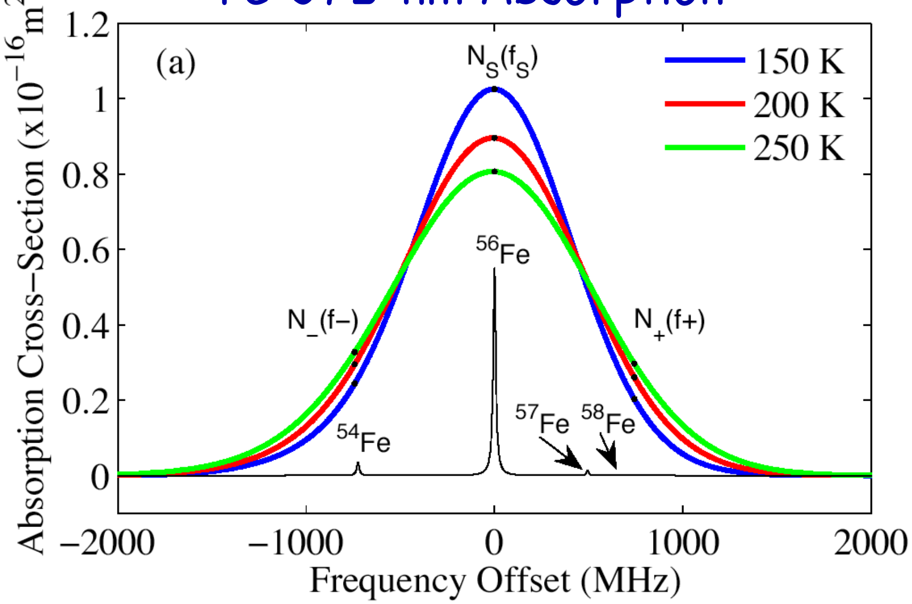
$T \nearrow \;\Rightarrow\; \sigma_{rms} \nearrow$

$M \nearrow \;\Rightarrow\; \sigma_{rms} \searrow$

Na D2 Absorption (589 nm)



Fe 372-nm Absorption



Doppler Shift and Broadening in Resonance Fluorescence

❑ When an atom emits a resonance fluorescence photon, the photon has Doppler shift relative to the center freq. of the atomic absorption line as

$$\omega = \omega_0 + \vec{k} \cdot \vec{v} = \omega_0 \left(1 + \frac{v_R}{c} \right) \quad (12.14) \quad \Rightarrow \quad v_R = \frac{\omega - \omega_0}{\omega_0 / c} = \frac{\nu - \nu_0}{\nu_0 / c} \quad (12.15)$$

❑ According to the Maxwellian velocity distribution, the relative probability that an atom/molecule in a gas at temperature T has its velocity component along the line of sight between v_R and v_R+dv_R is

$$P(v_R \to v_R + dv_R) \propto \exp\left(-M v_R^2 / 2k_B T\right) dv_R \qquad (12.16)$$

❑ Substitute the v_R equation into the Maxwellian distribution,

$$I \propto \exp\left(-\frac{M(\nu - \nu_0)^2}{2k_B T(\nu_0 / c)^2} \right) (c/\nu_0) d\nu \qquad (12.17)$$

❑ Therefore, the rms width of the Doppler broadening is

$$\sigma_{rms} = \nu_0 / c \sqrt{k_B T / M} = \frac{1}{\lambda_0} \sqrt{k_B T / M} \qquad (12.18)$$

1 time

Doppler Shift in Rayleigh Scattering

- Refer to textbook 5.2.2.4 Lidar wind vs radar wind measurements

Momentum Conservation
$$m\vec{v}_1 + \hbar\vec{k}_1 = m\vec{v}_2 + \hbar\vec{k}_2 \qquad (12.19)$$

Energy Conservation
$$\frac{1}{2}mv_1^2 + \hbar\omega_1 = \frac{1}{2}mv_2^2 + \hbar\omega_2 \qquad (12.20)$$

$$\boxed{\omega_1 = \omega_2 + \vec{k}_1\cdot\vec{v}_1 - \vec{k}_2\cdot\vec{v}_2 + \frac{\hbar k_1^2}{2m} - \frac{\hbar k_2^2}{2m}} \qquad (12.21)$$

- For Rayleigh or radar backscatter signals, we have

(12.22) $\vec{k}_2 \approx -\vec{k}_1$ $\qquad$ $\vec{v}_2 \approx \vec{v}_1$ (12.23)

- The frequency shift for Rayleigh or radar backscattering is

$$\boxed{\Delta\omega_{Rayleigh,backscatter} = \omega_2 - \omega_1 = -2\vec{k}_1\cdot\vec{v}_1} \qquad (12.24)$$

Doppler Broadening in Rayleigh Scatter

- To derive the Doppler broadening, let's write the Doppler shift as

$$\omega = \omega_0\left(1 - \frac{2v_R}{c}\right) \quad (12.25) \quad \Rightarrow \quad v_R = \frac{\omega_0 - \omega}{2\omega_0 / c} = \frac{\nu_0 - \nu}{2\nu_0 / c} \quad (12.26)$$

- According to the Maxwellian velocity distribution, the relative probability that an atom/molecule in a gas at temperature T has its velocity component along the line of sight between v_R and v_R+dv_R is

$$P(v_R \to v_R + dv_R) \propto \exp\left(-Mv_R^2 / 2k_B T\right) dv_R \qquad (12.27)$$

- Substitute the v_R equation into the Maxwellian distribution,

$$I \propto \exp\left(-\frac{M(\nu_0 - \nu)^2}{2k_B T(2\nu_0 / c)^2}\right)(c / 2\nu_0) d\nu \qquad (12.28)$$

- Therefore, the rms width of the Doppler broadening is

$$\sigma_{rms} = 2\nu_0 / c\sqrt{k_B T / M} = \frac{2}{\lambda_0}\sqrt{k_B T / M} \qquad (12.29)$$

2 times !

Doppler Effect in Rayleigh Scattering

- In the atmosphere when aerosols present, the lidar returns contains a narrow spike near the laser frequency caused by aerosol scattering riding on a Doppler broadened molecular scattering profile.

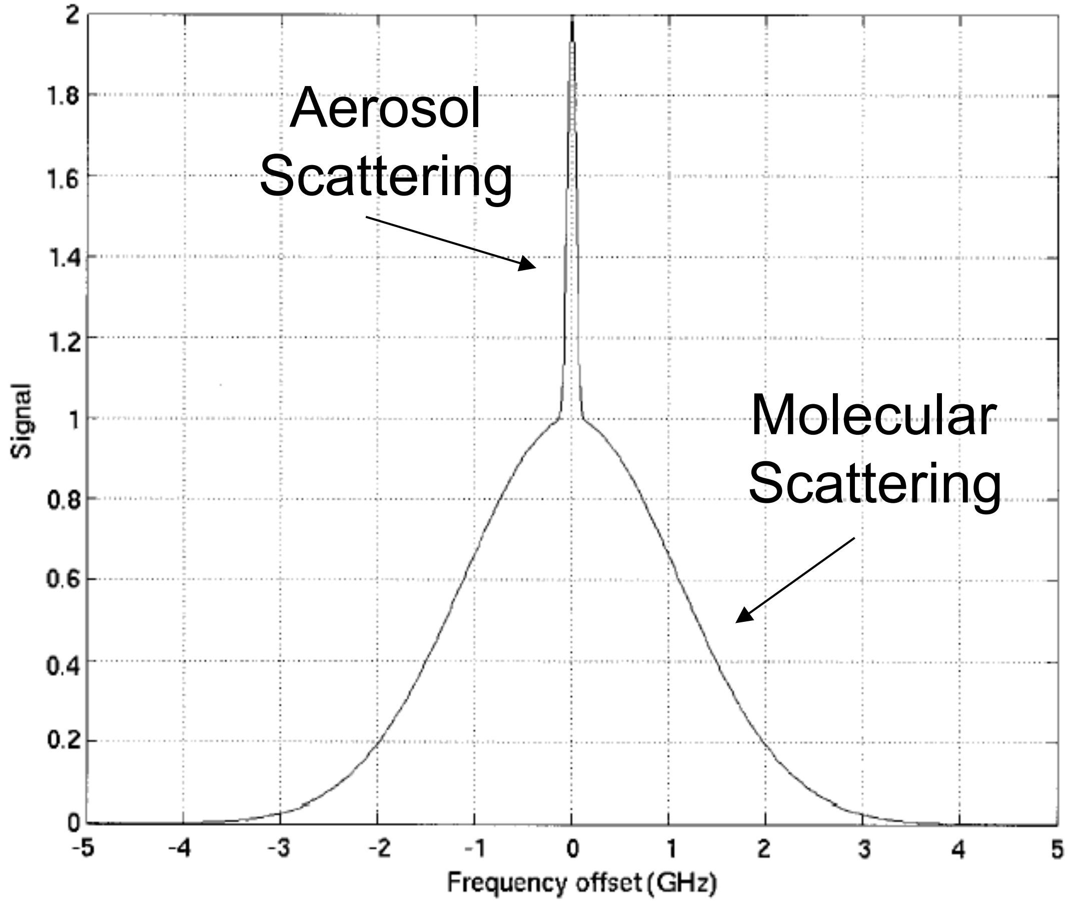


Fig. 5.1. Spectral profile of backscattering from a mixture of molecules and aerosols for a temperature of 300 K. The spectral width of the narrow aerosol return is normally determined by the line width of the transmitting laser.

At T = 300 K, the Doppler broadened FWHM for Rayleigh scattering is 2.58GHz, not 1.29GHz. Why?

Because Rayleigh backscatter signals have 2 times of Doppler shift!

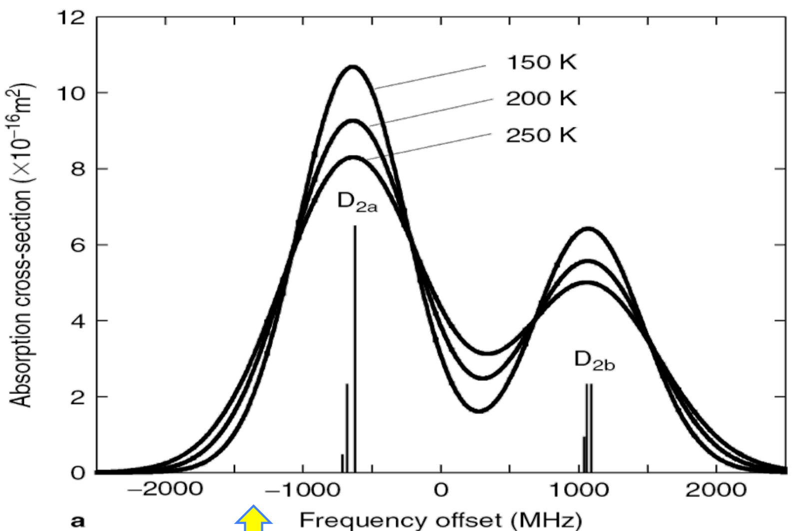
Courtesy of Dr. Ed Eloranta University of Wisconsin

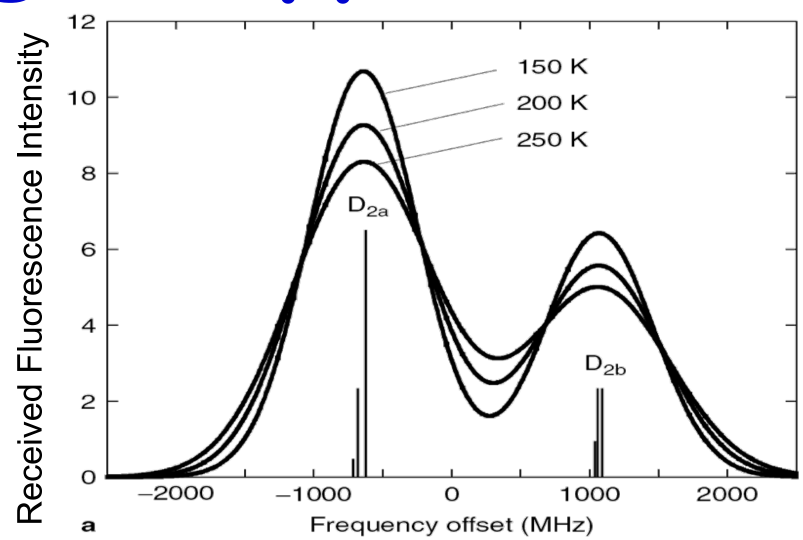
Resonance Fluorescence Doppler versus Rayleigh Doppler

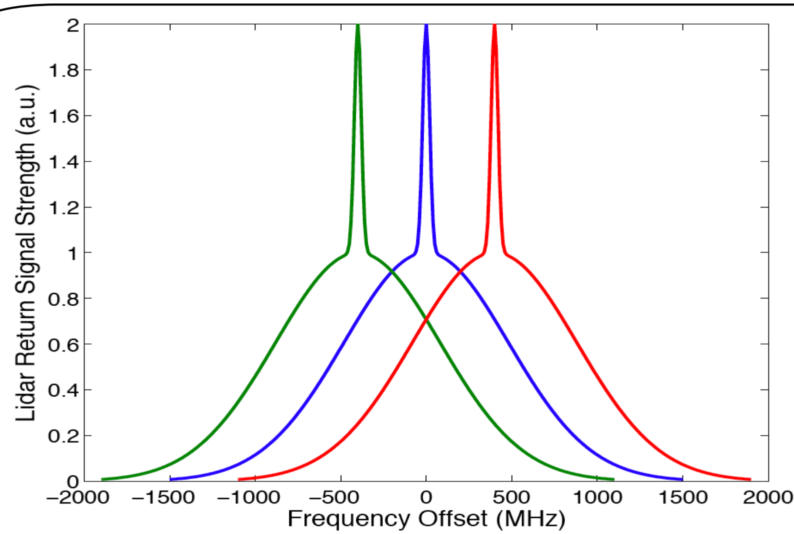
- Atomic absorption lines provide a natural frequency analyzer or frequency discrimination. This is because the absorption cross section undergoes Doppler shift and Doppler broadening. Thus, when a narrowband laser scans through the absorption lines, different absorption and fluorescence strength will be resulted at different laser frequencies. By using a broadband receiver to collect the returned resonance fluorescence, we can easily obtain the line shape of the absorption cross section so that we can infer wind and temperature. There is no need to measure the fluorescence spectrum. - Resonance fluorescence Doppler technique

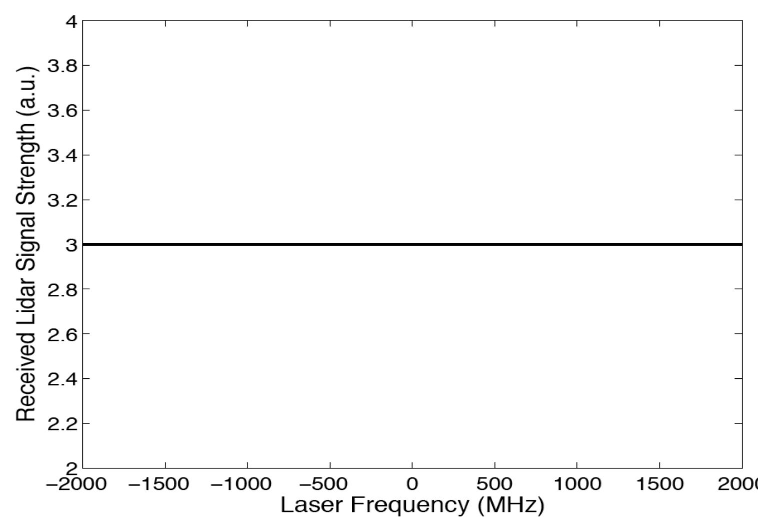
- Rayleigh scattering also undergoes Doppler shift and broadening, however, it is not frequency discriminated. In other words, when scanning a laser frequency, the backscattered Rayleigh signal gives nearly the same Doppler broadened line width, independent of laser frequency. Thus, the atmosphere molecule scattering does not provide frequency discrimination. A frequency analyzer must be implemented into the lidar receiver to discriminate the return light frequency, i.e., analyze Rayleigh scattering spectrum to infer wind and temperature. - Rayleigh Doppler technique

Resonance Fluorescence Doppler versus Rayleigh Doppler



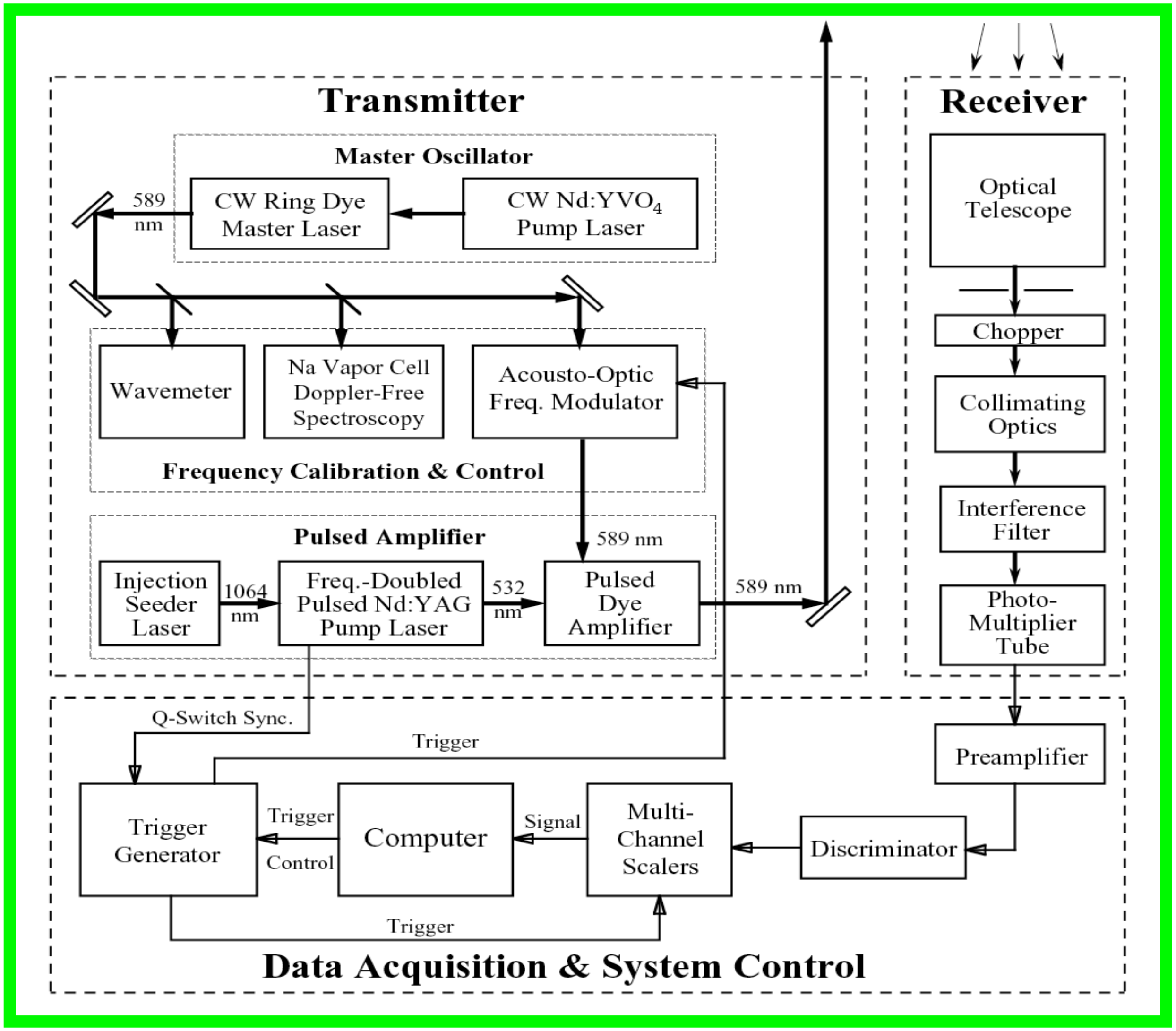






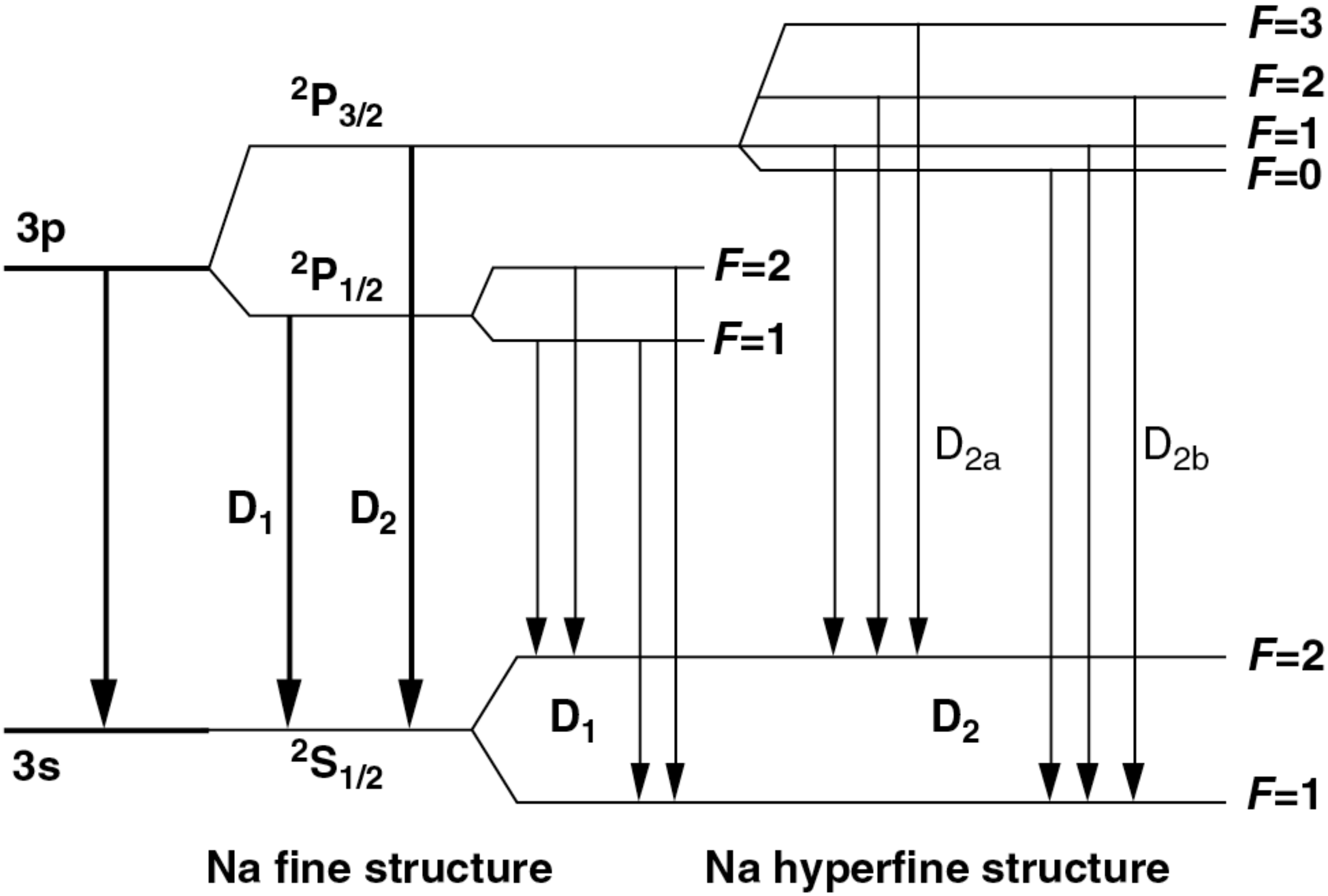
Na Doppler Wind and Temperature Lidar

- Na Doppler lidar is one of the most successful lidars.

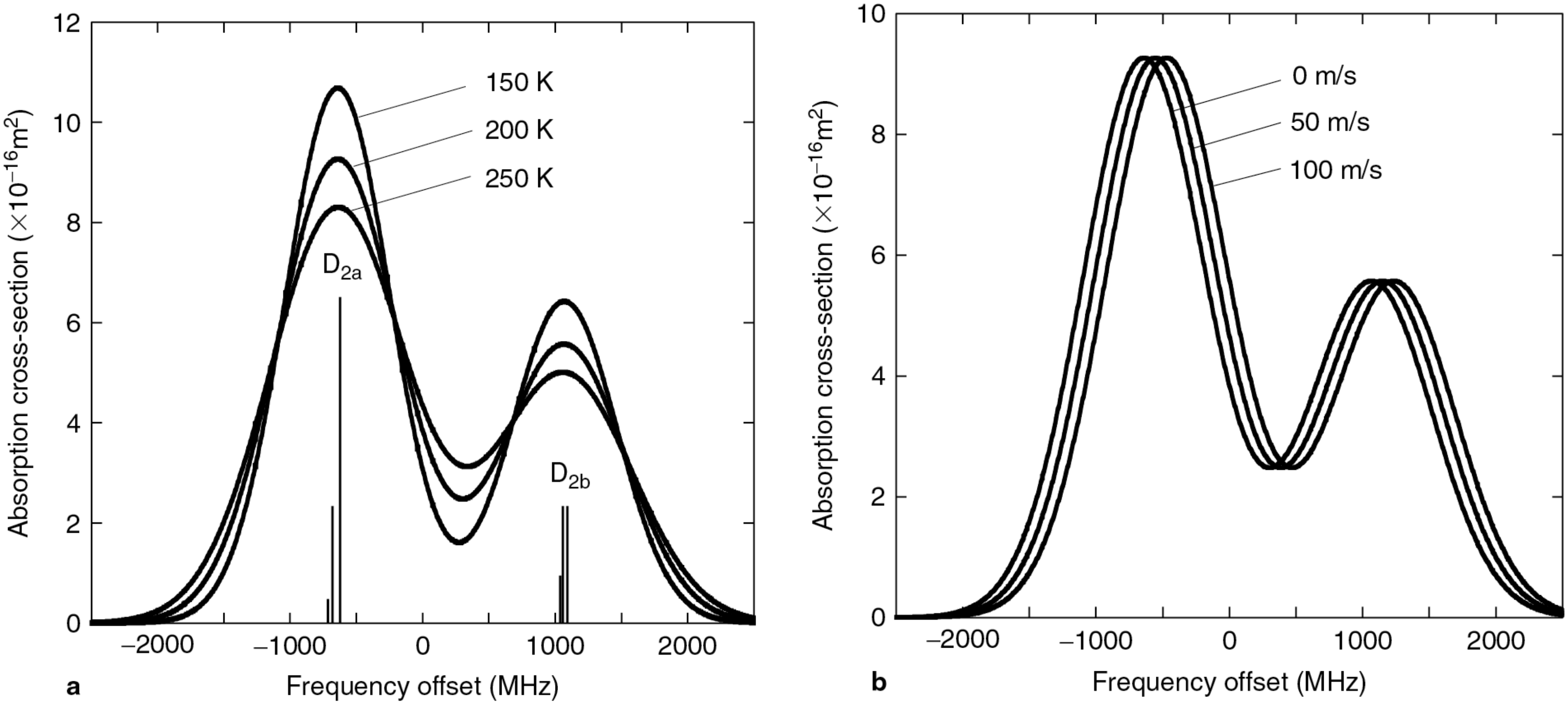


Textbook Chapter 5 by Chu and Papen

Na Atomic Energy Levels



Doppler Effect in Na D_2 Line Resonance Fluorescence



Na D_2 absorption linewidth is temperature dependent

Na D_2 absorption peak freq is wind dependent

Na Atomic Parameters

Table 5.1 Parameters of the Na D_1 and D_2 Transition Lines

Transition Line	Central Wavelength (nm)	Transition Probability ($10^8\ s^{-1}$)	Radiative Lifetime (nsec)	Oscillator Strength f_{ik}
D_1 ($^2P_{1/2} \rightarrow {}^2S_{1/2}$)	589.7558	0.614	16.29	0.320
D_2 ($^2P_{3/2} \rightarrow {}^2S_{1/2}$)	589.1583	0.616	16.23	0.641

Group	$^2S_{1/2}$	$^2P_{3/2}$	Offset (GHz)	Relative Line Strength[a]
D_{2b}	$F=1$	$F=2$	1.0911	5/32
		$F=1$	1.0566	5/32
		$F=0$	1.0408	2/32
D_{2a}	$F=2$	$F=3$	−0.6216	14/32
		$F=2$	−0.6806	5/32
		$F=1$	−0.7150	1/32

Doppler-Free Saturation–Absorption Features of the Na D_2 Line

f_a (MHz)	f_c (MHz)	f_b (MHz)	f_+ (MHz)	f_- (MHz)
−651.4	187.8	1067.8	−21.4	−1281.4

[a]Relative line strengths are in the absence of a magnetic field or the spatial average. When Hanle effect is considered in the atmosphere, the relative line strengths will be modified depending on the geomagnetic field and the laser polarization.

Doppler-Limited Na Spectroscopy

- Doppler-broadened Na absorption cross-section is approximated as a Gaussian with rms width σ_D

$$\sigma_{abs}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_D}\frac{e^2 f}{4\varepsilon_0 m_e c}\sum_{n=1}^{6} A_n \exp\left(-\frac{[\nu_n - \nu(1 - V_R/c)]^2}{2\sigma_D^2}\right) \quad (12.30)$$

- Assume the laser lineshape is a Gaussian with rms width σ_L
- The effective cross-section is the convolution of the atomic absorption cross-section and the laser lineshape

$$\sigma_{eff}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_e}\frac{e^2 f}{4\varepsilon_0 m_e c}\sum_{n=1}^{6} A_n \exp\left(-\frac{[\nu_n - \nu(1 - V_R/c)]^2}{2\sigma_e^2}\right) \quad (12.31)$$

where (12.32) $\sigma_e = \sqrt{\sigma_D^2 + \sigma_L^2}$ and $\sigma_D = \sqrt{\frac{k_B T}{M\lambda_0^2}}$ (12.33)

The frequency discriminator/analyzer is in the atmosphere!!!

Doppler Scanning Technique

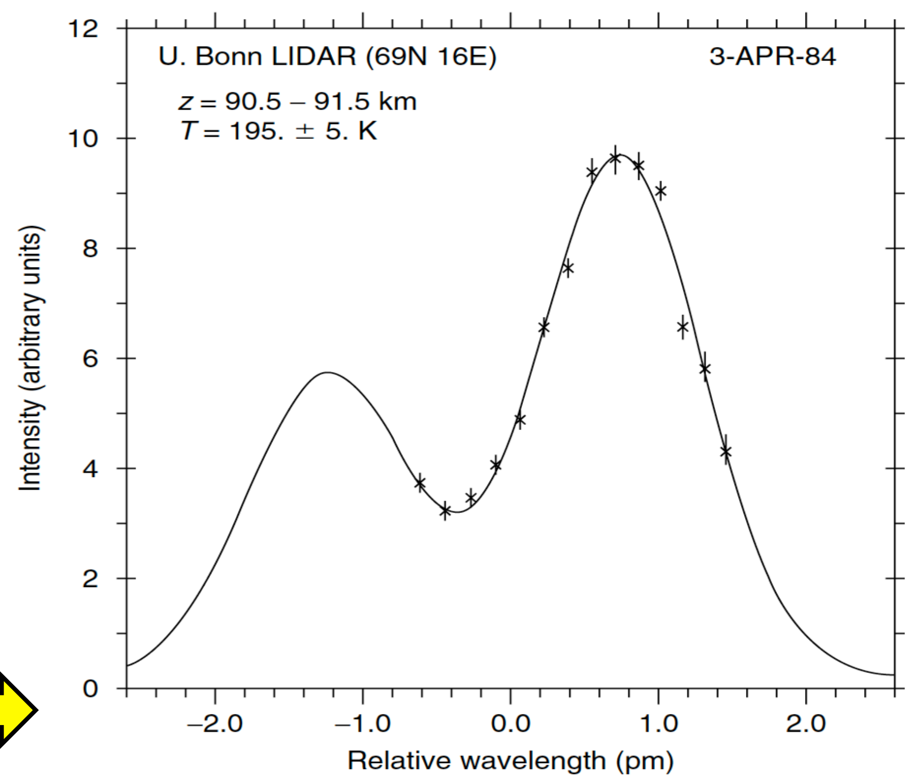
$$N_{Na}(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda)n_{Na}(z)\Delta z\right)\left(\frac{A}{4\pi z^2}\right)\left(\eta(\lambda)T_a^{\,2}(\lambda)T_c^{\,2}(\lambda,z)G(z)\right) \tag{12.34}$$

$$N_R(\lambda,z_R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_R(\pi,\lambda)n_R(z_R)\Delta z\right)\left(\frac{A}{z_R^{\,2}}\right)\left(\eta(\lambda)T_a^{\,2}(\lambda,z_R)G(z_R)\right) \tag{12.35}$$

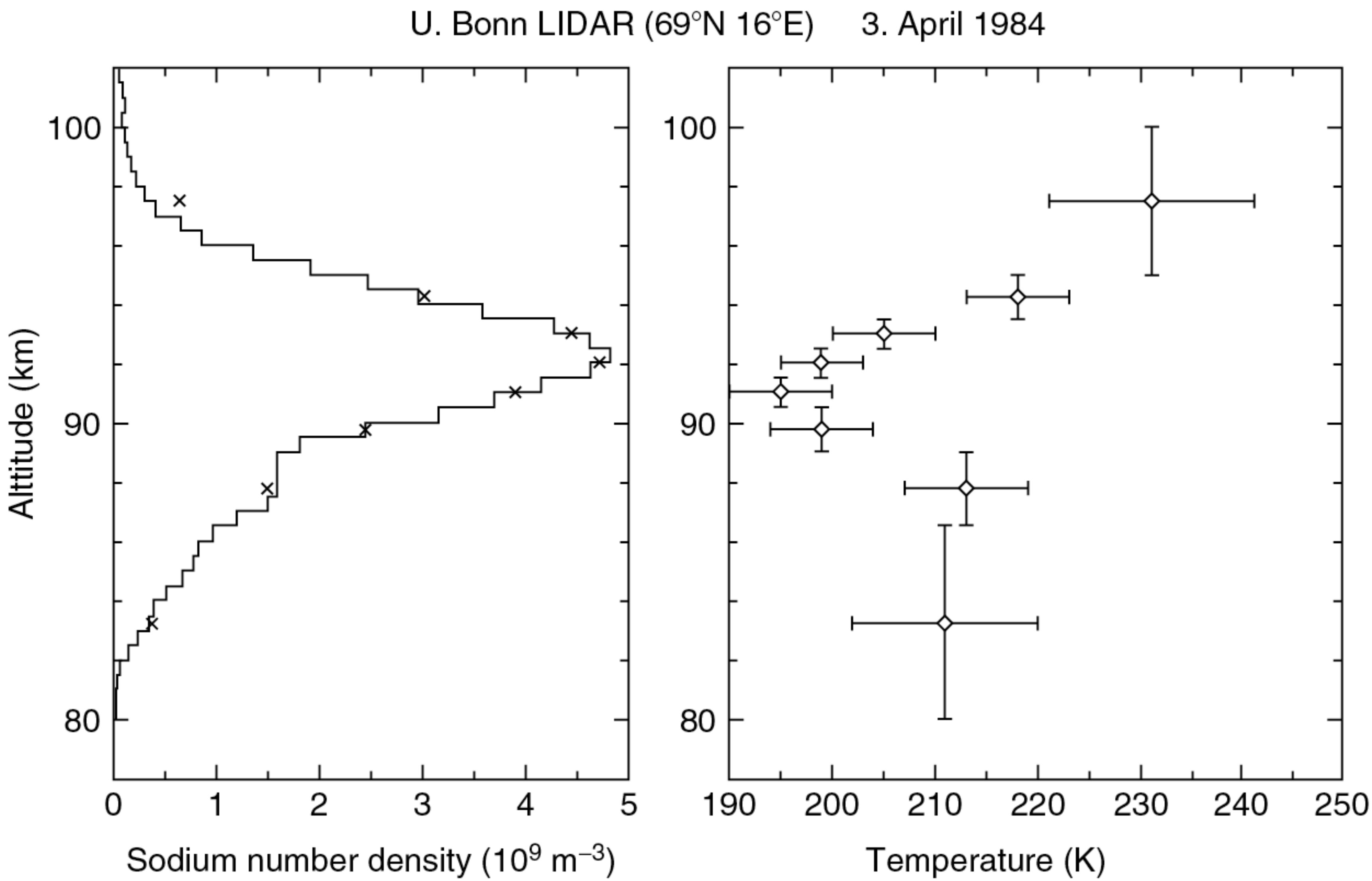
$$\sigma_{eff}(\lambda,z) = \frac{C(z)}{T_c^{\,2}(\lambda,z)}\frac{N_{Na}(\lambda,z)}{N_R(\lambda,z_R)} \tag{12.36}$$

where $C(z) = \dfrac{\sigma_R(\pi,\lambda)n_R(z_R)}{n_{Na}(z)}\dfrac{4\pi z^2}{z_R^{\,2}}$ (12.37)

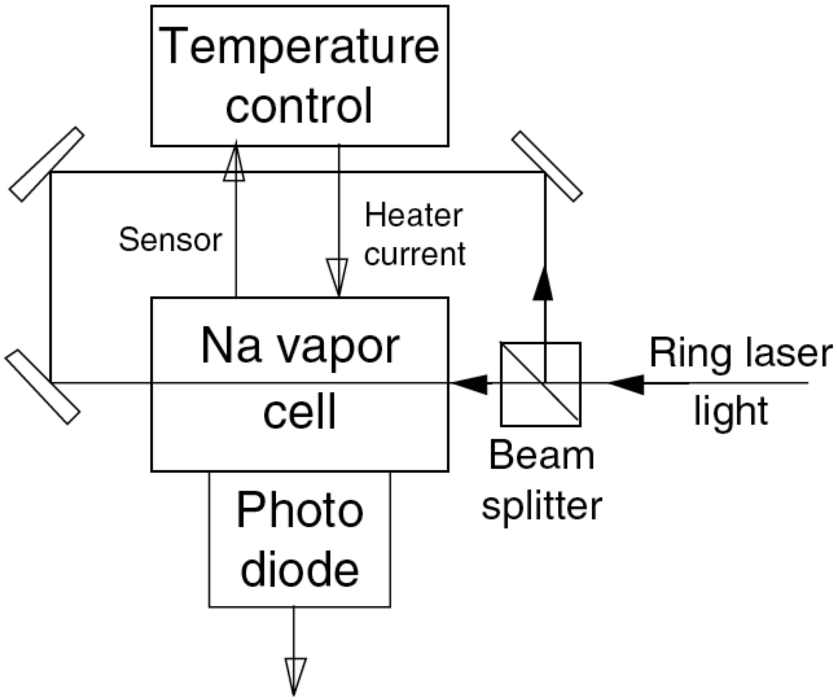
[Fricke and von Zahn, JATP, 1985]

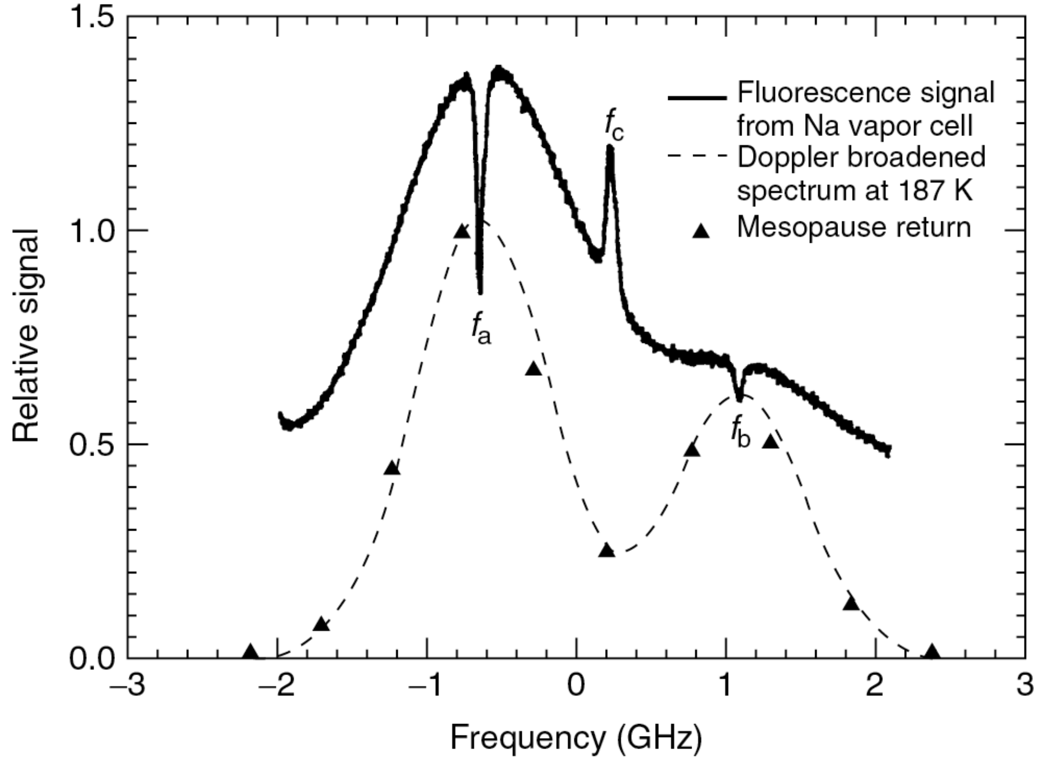


Scanning Na Lidar Results

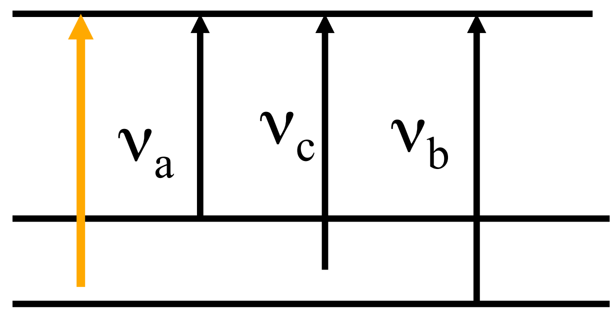


Doppler-Free Na Spectroscopy





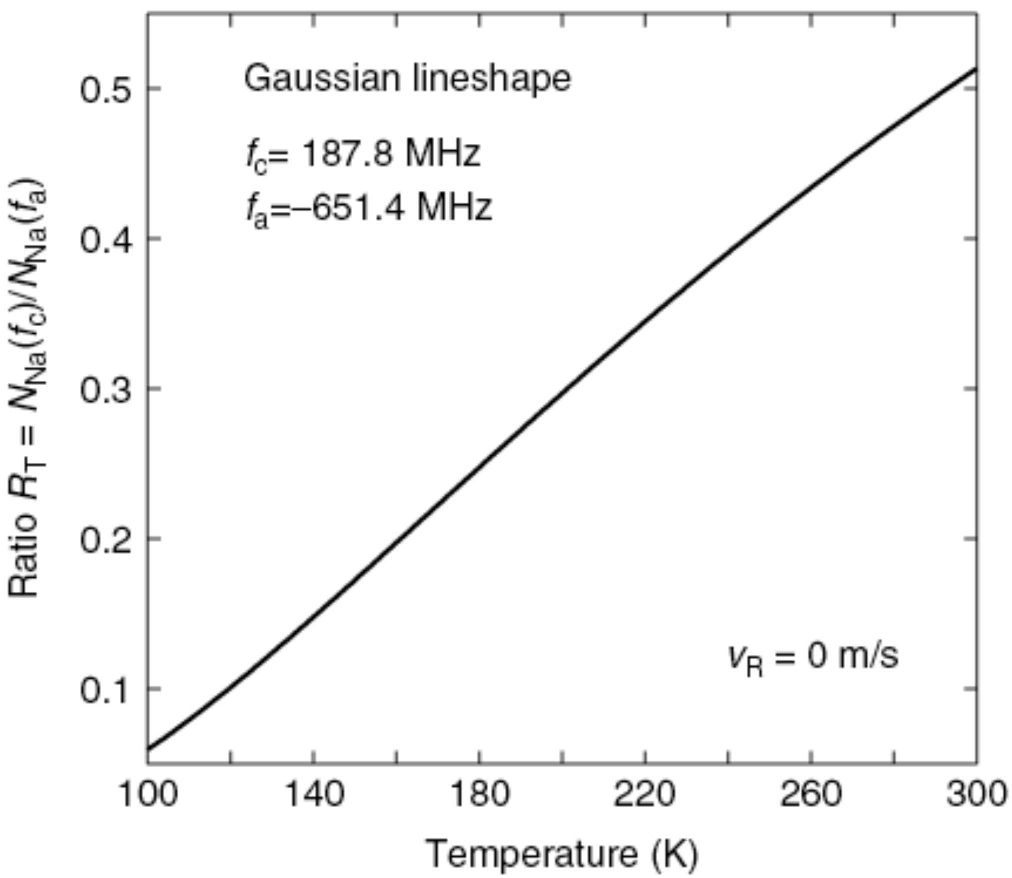
See detailed explanation on Na Doppler-free saturation-fluorescence spectroscopy in Textbook Chapter 5.2.2.3.2



$$\nu_c = (\nu_a + \nu_b)/2 \quad (12.38)$$

Doppler Ratio Technique: 2-Frequency

$$R_T(z) = \frac{N_{norm}(f_c, z, t_1)}{N_{norm}(f_a, z, t_2)} = \frac{\sigma_{eff}(f_c, z) n_{Na}(z, t_1)}{\sigma_{eff}(f_a, z) n_{Na}(z, t_2)} \approx \frac{\sigma_{eff}(f_c, z)}{\sigma_{eff}(f_a, z)} \qquad (12.39)$$

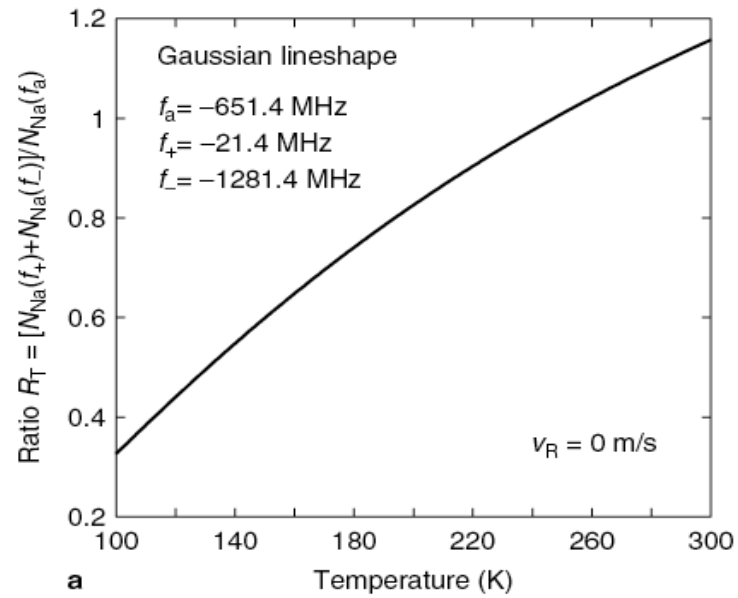


$$N_{norm}(f, z, t) = \frac{N_{Na}(f, z, t)}{N_R(f, z, t) T_c^2(f, z)} \qquad (12.40)$$

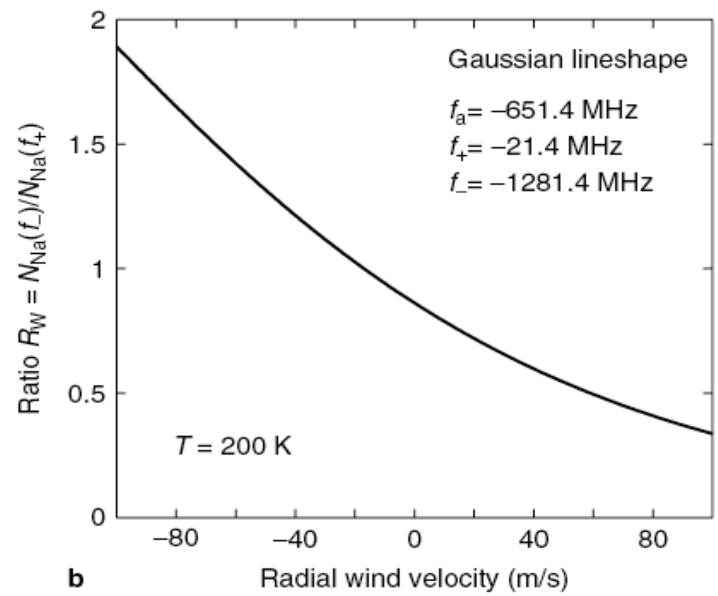
$$N_{norm}(f, z, t) = \frac{\sigma_{eff}(f) n_{Na}(z)}{\sigma_R(\pi, f) n_R(z_R)} \frac{z_R^2}{4\pi z^2} \qquad (12.41)$$

Doppler Ratio Technique: 3-Frequency

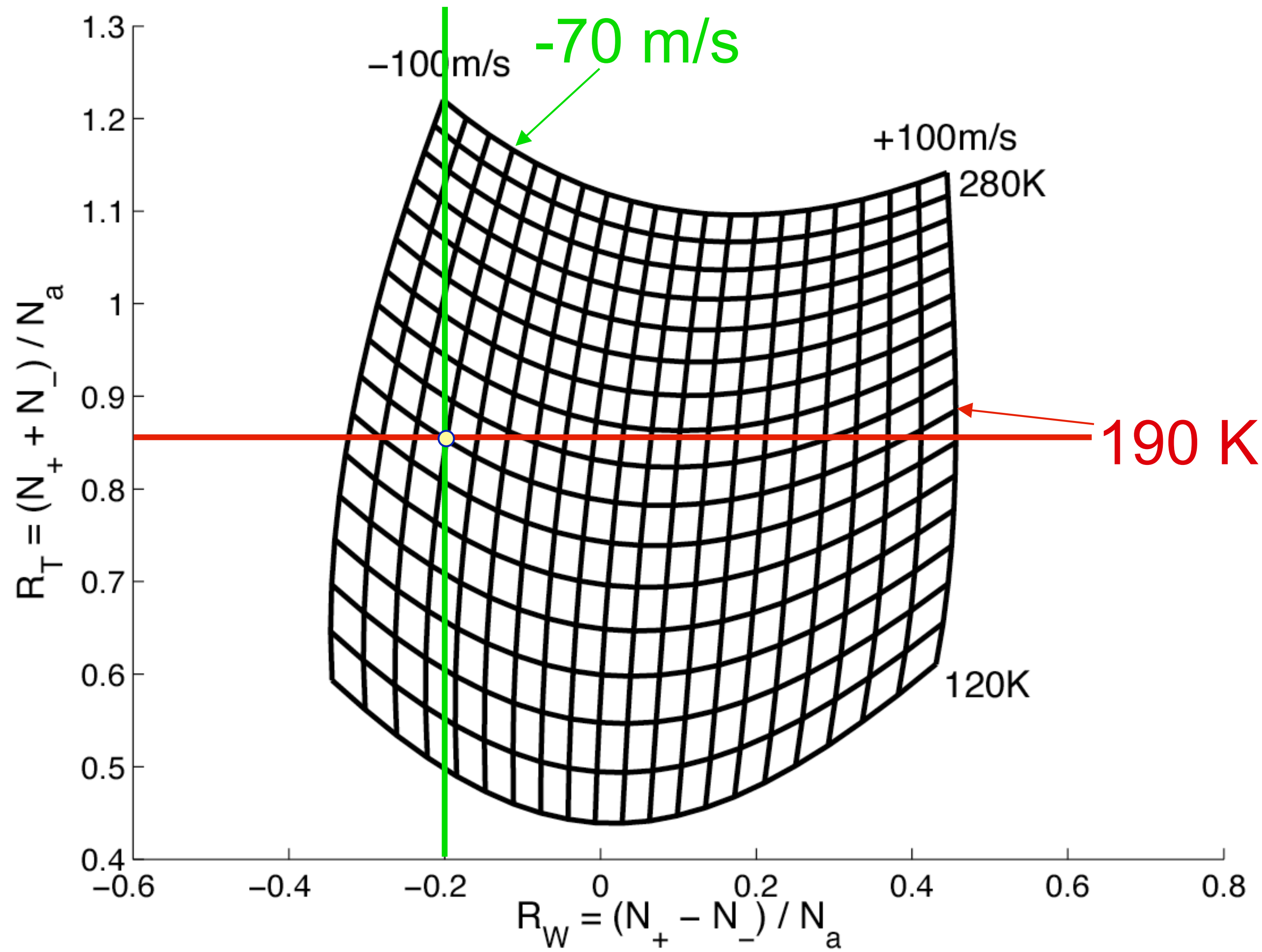
$$R_T(z) = \frac{N_{norm}(f_+, z, t_1) + N_{norm}(f_-, z, t_2)}{N_{norm}(f_a, z, t_3)} \approx \frac{\sigma_{eff}(f_+, z) + \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)} \quad (12.42)$$



$$R_W(z) = \frac{N_{norm}(f_-, z, t_2)}{N_{norm}(f_+, z, t_1)} \approx \frac{\sigma_{eff}(f_-, z)}{\sigma_{eff}(f_+, z)} \quad (12.43)$$



Na Doppler Lidar Calibration Curves



Summary

- The key point to measure temperature is to find and use temperature-dependent effects and phenomena to make measurements.

- Doppler technique utilizes the Doppler effect (frequency shift and linewidth broadening) by moving particles to infer wind and temperature information. It is widely applied in lidar, radar and sodar technique as well as passive optical remote sensing.

- Resonance fluorescence Doppler lidar technique applies scanning or ratio technique to infer the temperature and wind from the Doppler spectroscopy, while the Doppler spectroscopy is inferred from intensity ratio at different frequencies.