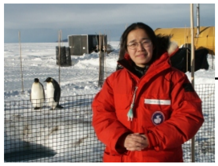
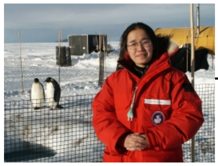


Lecture 10. Lidar Simulation and Error Analysis Overview (2)

- Introduction
- Accuracy versus Precision
- Classification of Measurement Errors
- Accuracy in lidar measurements
- Precision in lidar measurements
- General procedure of error analysis
- Monte Carlo Method
- Summary

Introduction

- For all physical experiments, errors or uncertainties exist that must be reduced by improved understanding of physical processes, improved experimental techniques, and repeated measurements. Those errors remaining must be estimated to establish the validity of our results.
- Error is defined as "the difference between an observed or calculated value and the true value".
- Usually we do not know the "true" value; otherwise there would be no reason for performing the experiment. We may know approximately what it should be, however, either from earlier experiments or from theoretical predictions. Such approximations can serve as a guide but we must always determine in a systematic way from the data and the experimental conditions themselves how much confidence we can have in our experimental results.



Introduction

- Before going further, let us rule out one kind of errors - illegitimate errors that originate from mistakes in measurement or computation.

- Have you heard about the measurement of "faster-than-light neutrinos", announced in September 2011?

-- It is totally due to a bad error in the experiments: wrong measurement of time of flight! The experiments were also badly designed – using GPS instead of atomic clock to count the time!

http://news.sciencemag.org/scienceinsider/2012/02/breaking-news-error-undoes-faster.html

http://www.bbc.co.uk/news/science-environment-17560379

- A good reference book for general error analysis is "Data Reduction and Error Analysis for the Physical Sciences" by Philip R. Bevington and D. Keith Robinson (3rd edition, 2003).

Accuracy versus Precision

- It is important to distinguish between the terms accuracy and precision, because in error analysis, accuracy and precision are two different concepts, describing different aspects of a measurement.
- The accuracy of an experiment is a measure of how close the result of the experiment is to the true value.
- The precision is a measure of how well the result has been determined, without reference to its agreement with the true value. The precision is also a measure of the reproducibility of the result in a given experiment.
- Accuracy concerns about bias, i.e., how far away is the measurement result from the true value? Precision concerns about uncertainty, i.e., how certain or how sure are we about the measurement result?
- For any measurement, the results are commonly expected to be a mean value with a confidence range: $x_i \pm \Delta x_i$

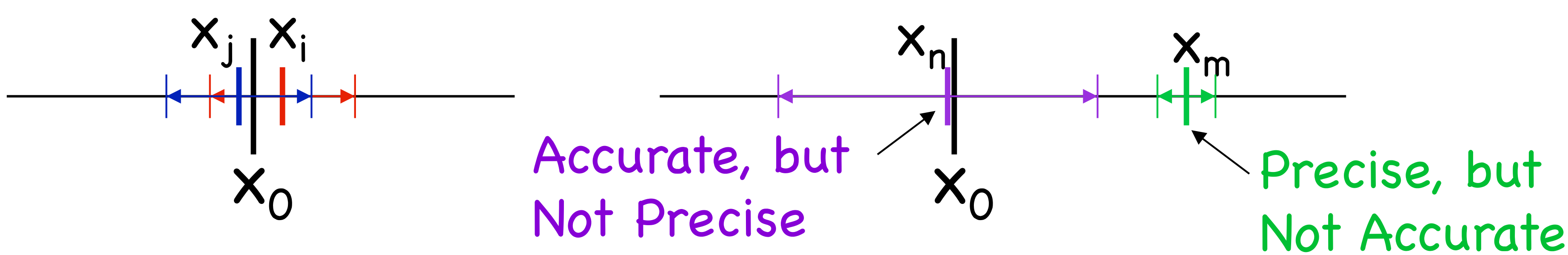


Illustration of Accuracy and Precision

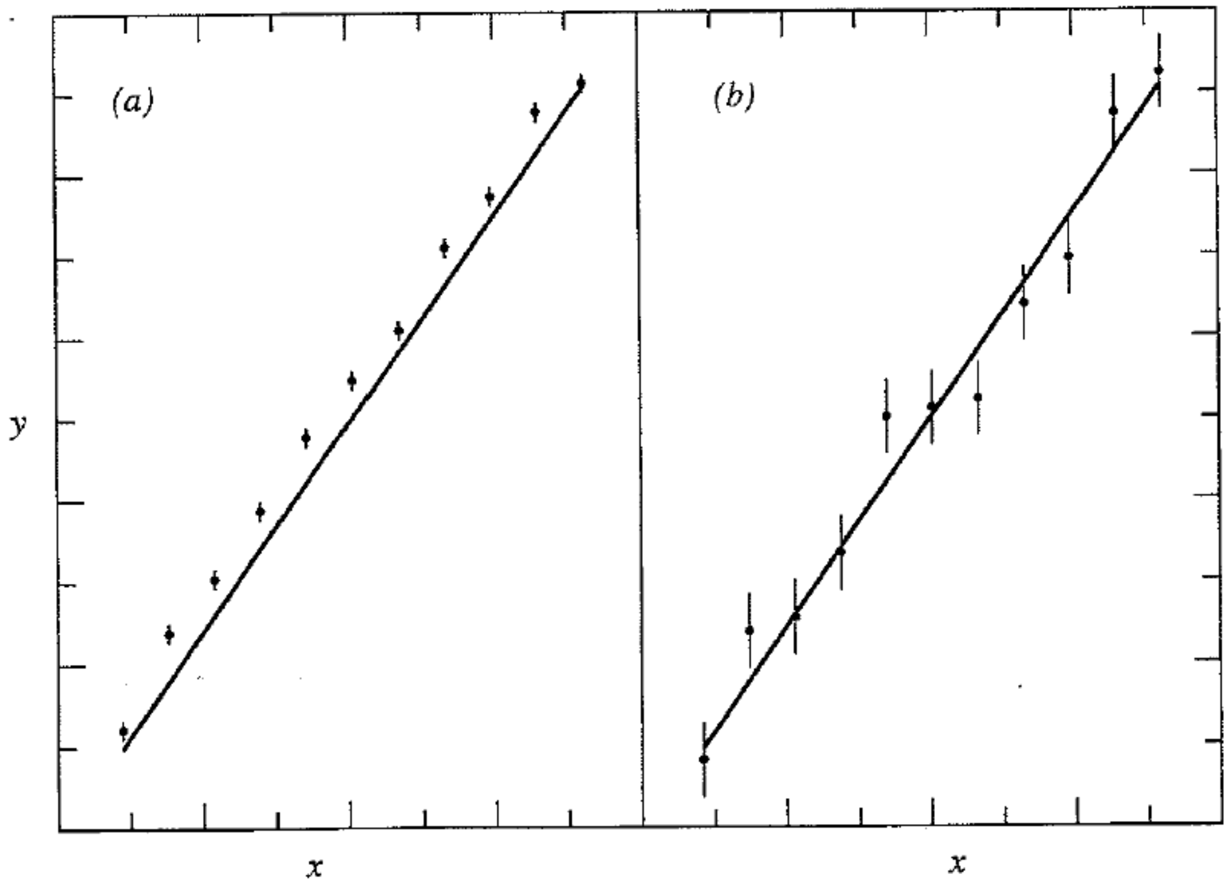


FIGURE 1.1
Illustration of the difference between precision and accuracy. (*a*) Precise but inaccurate data. (*b*) Accurate but imprecise data. True values are represented by the straight lines.

[Data Reduction and Error Analysis, Bevington and Robinson, 2003]

Classification of Measurement Errors

- Measurement errors are classified into two major categories: Systematic errors and random errors.
- Systematic errors are errors that will make our results different from the "true" values with reproducible discrepancies. Errors of this type are not easy to detect and not easily studied by statistical analysis. They must be estimated from an analysis of the experimental conditions, techniques, and our understanding of physical interactions. A major part of the planning of an experiment should be devoted to understanding and reducing sources of systematic errors.
- Random errors are fluctuations in observations that yield different results each time the experiment is repeated, and thus require repeated experimentation to yield precise results.
- Another way to describe systematic and random errors are: Experimental uncertainties that can be revealed by repeating the measurements are called random errors; those that cannot be revealed in this way are called systematic errors.

Illustration of Accuracy and Precision

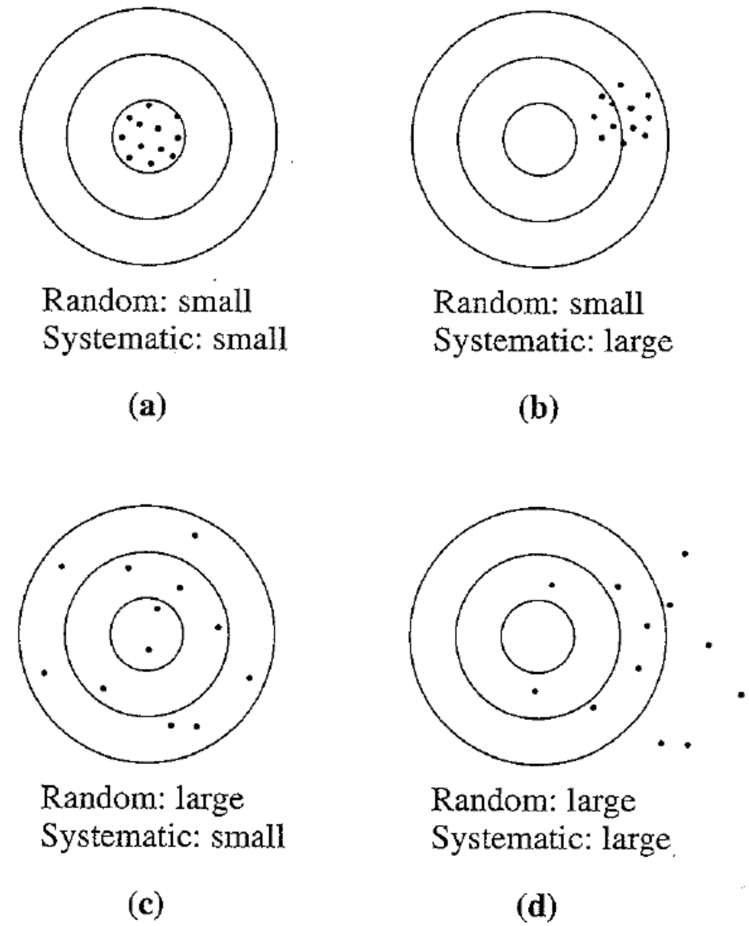


Figure 4.1. Random and systematic errors in target practice. (a) Because all shots arrived close to one another, we can tell the random errors are small. Because the distribution of shots is centered on the center of the target, the systematic errors are also small. (b) The random errors are still small, but the systematic ones are much larger—the shots are "systematically" off-center toward the right. (c) Here, the random errors are large, but the systematic ones are small—the shots are widely scattered but not systematically off-center. (d) Here, both random and systematic errors are large.

Illustration of Accuracy and Precision

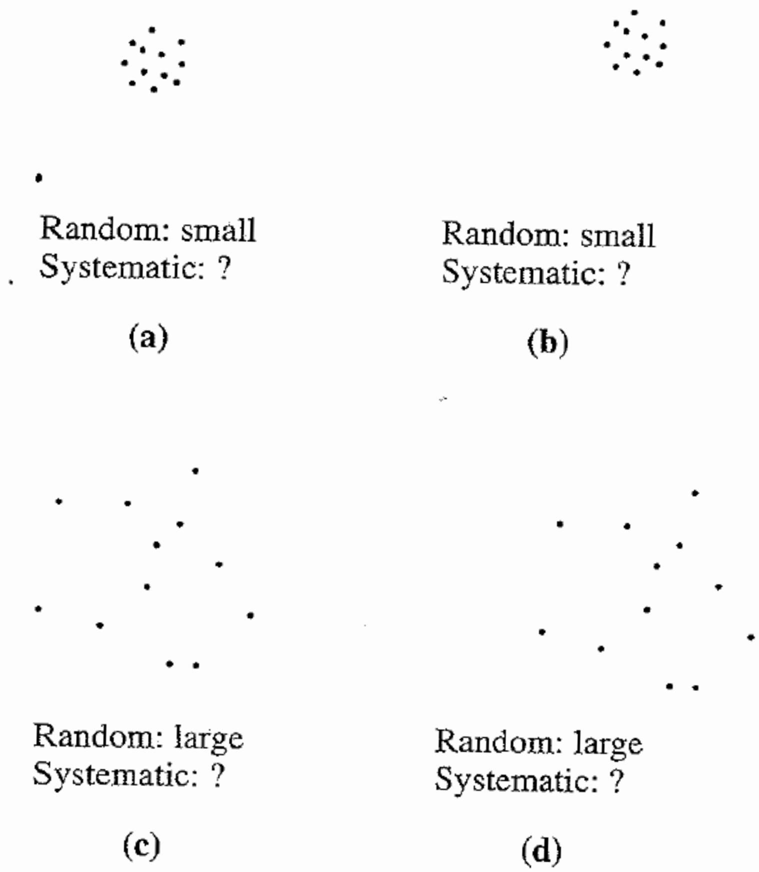
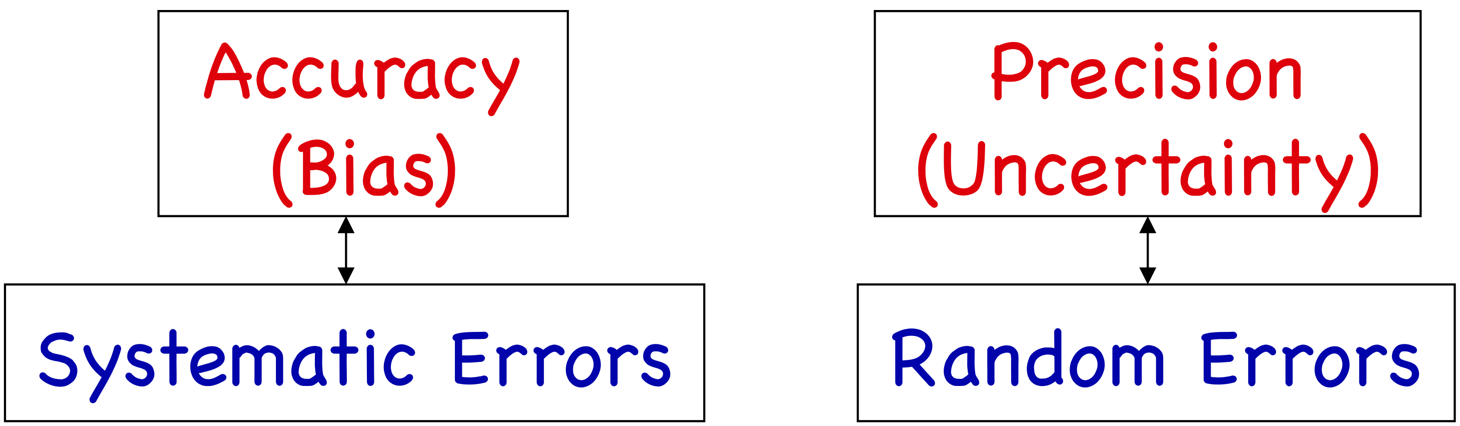


Figure 4.2. The same experiment as in Figure 4.1 redrawn without showing the position of the target. This situation corresponds closely to the one in most real experiments, in which we do not know the true value of the quantity being measured. Here, we can still assess the random errors easily but cannot tell anything about the systematic ones.

Errors vs. Accuracy & Precision

- The accuracy of an experiment is generally dependent on how well we can control or compensate for systematic errors.
- The precision of an experiment depends upon how well we can overcome random errors.
- A given accuracy implies an equivalent precision and, therefore, also depends on random errors to some extent.

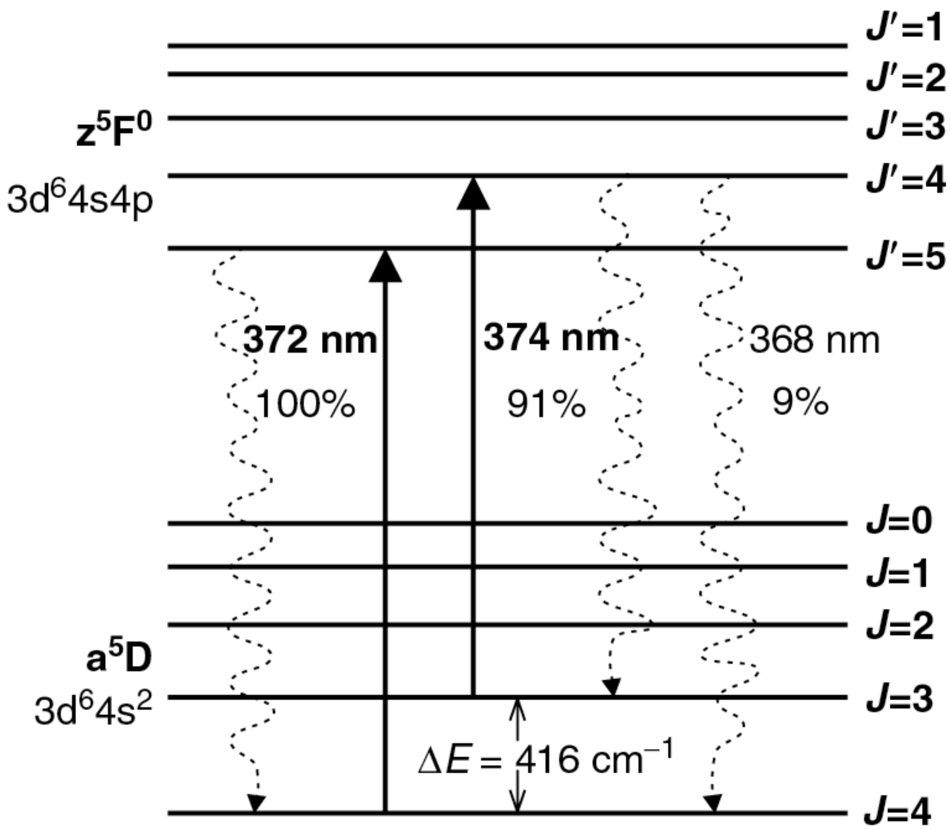


Accuracy in Lidar Measurements

- Systematic errors determine the measurement accuracy.
- Accuracy is mainly determined by: (1) How much we understand the physical interactions and processes involved in the measurements or observations, e.g., atomic parameters and absorption cross-section, isotopes, branching ratio, Hanle effect, atomic layer saturation effect, transmission/extinction, interference absorption, etc. (2) How well we know the lidar system parameters, e.g., laser central frequency, laser linewidth and lineshape, photo detector/discriminator calibration, receiver filter function, overlapping function, chopper function, etc.
- It happened in the history of physical experiments (e.g., quantum frequency standard) that when people understood more about the physical processes or interactions, the claimed experimental accuracy decreased. This is because some systematic errors (bias) caused by certain interactions were not included in earlier error analysis, as people were not aware of them. This could also happen to lidar measurements.
- In the following lectures on different types of lidars, keep in mind such a question: What affects the lidar measurement accuracy?

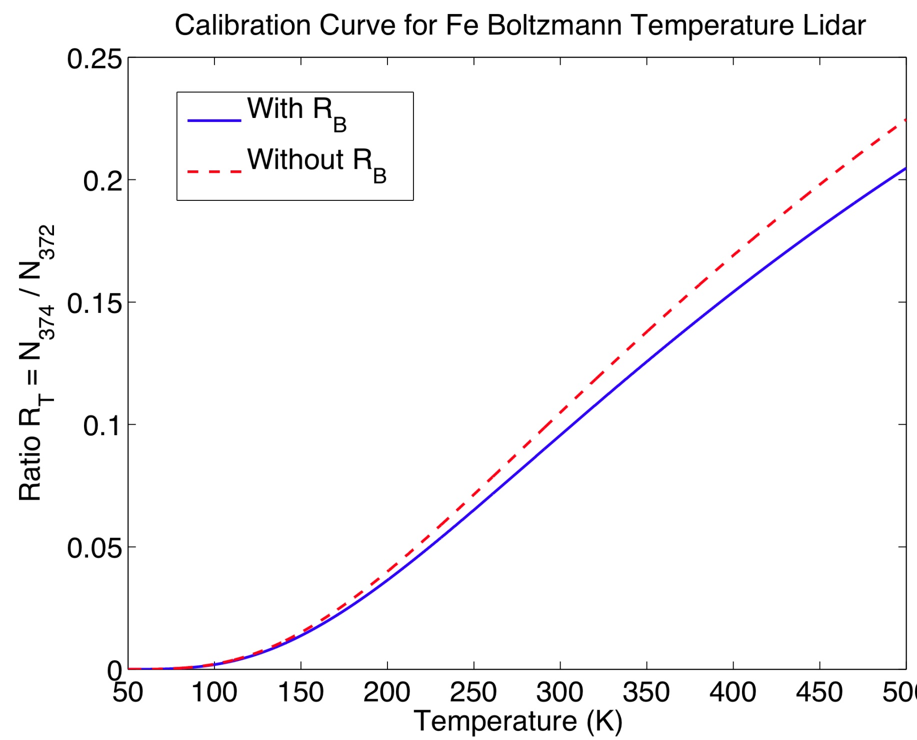
Example: Fe Boltzmann Lidar

- Systematic bias of temperature measurements can be caused by ignoring the branching ratio in the Fe Boltzmann lidar.



Atomic Fe Energy Level

[Gelbwachs, 1994; Chu et al., 2002]



Calibration curve for Fe Boltzmann lidar

Fe Boltzmann Temperature Ratio

- Fe resonance fluorescence lidar equation

$$N_{Fe}(\lambda,z)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_{eff}(\lambda,\sigma_L,T,v_R)R_{B\lambda}n_{Fe}(z)\right]\Delta z\left(\frac{A}{4\pi z^2}\right)\left(T_a^2(\lambda)T_c^2(\lambda,z)\right)\left(\eta(\lambda)G(z)\right) \quad (10.1)$$

Rayleigh scattering lidar equation at Rayleigh-normalization altitude z_R

$$N_R(\lambda,z_R)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_R(\pi,\lambda)n_R(z_R)\right]\Delta z\left(\frac{A}{z_R{}^2}\right)T_a{}^2(\lambda,z_R)\left(\eta(\lambda)G(z_R)\right) \quad (10.2)$$

Rayleigh normalization leads to normalized photon counts

$$N_{Norm}(\lambda,z)=\frac{N_{Fe}(\lambda,z)}{N_R(\lambda,z_R)}\frac{1}{T_c^2(\lambda,z)}\frac{z^2}{z_R^2}=\frac{\sigma_{eff}(\lambda,\sigma_L,T,v_R)R_{B\lambda}n_{Fe}(\lambda,z)}{\sigma_R(\lambda,)n_R(\lambda,z_R)} \quad (10.3)$$

- Take the Boltzmann temperature ratio as

$$R_T(z)=\frac{N_{norm}(\lambda_{374},z)}{N_{norm}(\lambda_{372},z)}=\frac{g_2}{g_1}\frac{R_{B374}}{R_{B372}}\left(\frac{\lambda_{374}}{\lambda_{372}}\right)^{4.0117}\frac{\sigma_{eff}(\lambda_{374},\sigma_{L374},T,v_R)}{\sigma_{eff}(\lambda_{372},\sigma_{L372},T,v_R)}\exp\left(-\Delta E/k_BT\right) \quad (10.4)$$

- Therefore, temperature can be derived as

$$T(z)=\frac{\Delta E/k_B}{\ln\left[\frac{g_2}{g_1}\frac{R_{B374}}{R_{B372}}\left(\frac{\lambda_{374}}{\lambda_{372}}\right)^{4.0117}\frac{\sigma_{eff}(\lambda_{374},\sigma_{L374},T,v_R)}{\sigma_{eff}(\lambda_{372},\sigma_{L372},T,v_R)}\frac{1}{R_T(z)}\right]} \quad (10.5)$$

Accuracy in Lidar Measurements

- For lidar researchers, one of our major tasks is to understand the physical processes as good as possible (e.g., measuring atomic parameters accurately from lab experiments, seeking and understanding all possible physical interactions involved in the scattering or absorption and fluorescence processes like saturation effects, understanding the details of laser and detection process) and improve our experimental conditions to either avoid or compensate for the systematic errors.
- These usually demand experimenters to be highly knowledgeable of atomic, molecular, and laser physics and spectroscopy, measurement procedure, etc. That's why we emphasize the spectroscopy knowledge is fundamental to lidar technology, rather than optical/laser engineering.
- Achieving high accuracy also requires experimenters to control and measure the lidar parameters very accurately and precisely.
- On the lidar design aspect, it would be good to develop lidar systems that are stable and less subject to systematic errors, e.g., freq chirp.
- Also, sometimes it is necessary to take the trade-off between accuracy and precision, depending on the experimental purposes/goals.

Precision in Lidar Measurements

- Random errors determine the measurement precision.
- Possible sources: (1) shot noise associated with photon-counting system, (2) random uncertainty associated with laser jitter and electronic jitter. The former ultimately limits the precision because of the statistic nature of photon-detection processes.
- In normal lidar photon counting, photon counts obey Poisson distribution. Therefore, for a given photon count N, the corresponding uncertainty is

$$\Delta N = \sqrt{N} \qquad (10.6)$$

- Precision is usually concerned with the random errors - errors that can be reduced by more repeated measurements or errors that can be reduced by sacrificing temporal or spatial resolutions.
- By making many times of the same measurements and then taking the mean of all measurements, the random errors of the measurements can be reduced. The accumulation of more lidar shots is equivalent to repeating the same measurements to reduce uncertainties caused by photon noise, laser frequency jitter, and linewidth fluctuation.

Precision in Lidar Measurements

- Photon noise is the major limitation to measurement precision. From the error equation, we know the larger the signal photon counts, the smaller the error caused by photon noise. Why so?

- A single shot results in a photon count of N with fluctuation of ΔN, leading to an error of ΔN/N. When many (m) shots are integrated together, we have the photon counts roughly mN with fluctuation of Δ(mN), leading to the error of Δ(mN)/mN. This error should have been reduced if we regard this integration procedure as taking a mean of repeated measurements.

$$\frac{\Delta(mN)}{mN} = \frac{\sqrt{mN}}{mN} = \frac{1}{\sqrt{m}} \cdot \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{m}} \cdot \frac{\Delta N}{N} \qquad (10.7)$$

- Therefore, the precision error caused by photon noise can be improved by two ways: (1) sacrifice of temporal resolution by integrating more shots together; (2) sacrifice of spatial resolution by integrating more range bins together； or the combination of both.

Error Propagation

- Propagation of Errors is an important aspect in lidar error analysis. This is because the temperature, wind, backscatter coefficient, etc. that we want to determine are dependent variables that are a function of one or more different measured variables (e.g., photon counts, laser frequency and linewidth). We must know how to propagate or carry over the uncertainties in the measured variables to determine the uncertainty in the dependent variables.
- For example, photon noise causes the uncertainty in the measured photon counts, then the photon count uncertainty leads to the uncertainty in the temperature and wind ratios R_T and R_W, which will result in errors in the inferred temperature T and wind W. -- Error propagation procedure
- Basic rules for propagation of error can be found in many textbooks, e.g., addition, subtraction, multiplication, division, product of power, and mixture of them, along with many other complicated functions.
- We will introduce a universal procedure through the use of differentials of the corresponding ratios R_T and R_W as illustrated below. This method is mathematically based on the Taylor expansion.

General Procedure of Error Analysis

- We introduce a differentiation method as a general procedure of error analysis. It is applicable to all lidars as well as to normal measurement error analysis. Let us use the Fe Boltzmann lidar as an example to explain the error analysis procedure.

- The temperature ratio for Fe Boltzmann temperature lidar is

$$R_T = \frac{N_{norm}(\lambda_{374})}{N_{norm}(\lambda_{372})} = \frac{g_2}{g_1}\frac{R_{B374}}{R_{B372}}\left(\frac{\lambda_{374}}{\lambda_{372}}\right)^{4.0117}\frac{\sigma_{eff}(f_{374},\sigma_{L374},T,v_R)}{\sigma_{eff}(f_{372},\sigma_{L372},T,v_R)}\exp\left(-\Delta E/k_B T\right) \quad (10.8)$$

- Through the ratio R_T, the temperature T is an implicit function of photon count ratio R_T, laser frequencies f_{L374} and f_{L372}, laser linewidths σ_{L374} and σ_{L372}, radial wind v_R, branching ratios R_{B374} and R_{B372}, etc. Each parameter could have some uncertainty or error, leading to errors in the measured temperature.

- Therefore, the temperature error is given by the derivatives

$$\Delta T = \frac{\partial T}{\partial R_T}\Delta R_T + \frac{\partial T}{\partial f_L}\Delta f_L + \frac{\partial T}{\partial \sigma_L}\Delta \sigma_L + \frac{\partial T}{\partial R_B}\Delta R_B + \frac{\partial T}{\partial \mathrm{v_R}}\Delta \mathrm{v_R} \quad (10.9)$$

Differentiation Method

- The root-mean-square (rms) temperature error is given by

$$(\Delta T)_{rms} = \sqrt{\left(\frac{\partial T}{\partial R_T}\Delta R_T + \frac{\partial T}{\partial f_L}\Delta f_L + \frac{\partial T}{\partial \sigma_L}\Delta \sigma_L + \frac{\partial T}{\partial R_B}\Delta R_B + \frac{\partial T}{\partial v_R}\Delta v_R\right)^2} \qquad (10.10)$$

- If the error sources are independent from each other, then the means of cross terms are zero. Then we have

$$(\Delta T)_{rms} = \sqrt{\left(\frac{\partial T}{\partial R_T}\Delta R_T\right)^2 + \left(\frac{\partial T}{\partial f_L}\Delta f_L\right)^2 + \left(\frac{\partial T}{\partial \sigma_L}\Delta \sigma_L\right)^2 + \left(\frac{\partial T}{\partial R_B}\Delta R_B\right)^2 + \left(\frac{\partial T}{\partial v_R}\Delta v_R\right)^2} \qquad (10.11)$$

- The above error equation indicates that many laser parameters and radial wind errors could affect the inferred temperature because they all influence the effective cross sections or the temperature ratio. In the meantime, photon noise can cause uncertainty in the ratio R_T, resulting in temperature error.

Error Derivation: Implicit Differentiation

- How to derive the error coefficients, like $\frac{\partial T}{\partial R_T}, \frac{\partial T}{\partial f_L}, etc.$?
- We may use the implicit differentiation through R_T as below:

$$\Delta T = \Delta R_T \left(\frac{\partial R_T}{\partial R_T} \Big/ \frac{\partial R_T}{\partial T} \right) + \Delta f_L \left(\frac{\partial R_T}{\partial f_L} \Big/ \frac{\partial R_T}{\partial T} \right) + \Delta \sigma_L \left(\frac{\partial R_T}{\partial \sigma_L} \Big/ \frac{\partial R_T}{\partial T} \right) + \Delta R_B \left(\frac{\partial R_T}{\partial R_B} \Big/ \frac{\partial R_T}{\partial T} \right) + \Delta \mathrm{v_R} \left(\frac{\partial R_T}{\partial \mathrm{v_R}} \Big/ \frac{\partial R_T}{\partial T} \right) \quad (10.12)$$

- For the photon-noise induced temperature error,

$$\Delta T = \frac{1}{\partial R_T / \partial T} \cdot \Delta R_T = \frac{R_T}{\partial R_T / \partial T} \cdot \frac{\Delta R_T}{R_T} \quad (10.13)$$

Reciprocal of Sensitivity

Relative error of R_T

- The relative error of R_T can be derived in terms of measured signal and background photon counts (see later slides).

Derivation of Error Coefficients

- The temperature error coefficient can be derived numerically

$$\frac{R_T}{\partial R_T / \partial T} = \frac{R_T}{[R_T(T+\delta T) - R_T(T)]/\delta T} \qquad (10.14)$$

- Two approaches to derive the above numerical solution:

(1) One way is to use the equation of R_T in terms of physical processes. You don't have to go through the entire simulation process each time when you change the temperature, but just calculate the R_T from the effective cross section and Boltzmann equation.

$$R_T = \frac{g_2}{g_1}\frac{R_{B374}}{R_{B372}}\left(\frac{\lambda_{374}}{\lambda_{372}}\right)^{4.0117}\frac{\sigma_{eff}(f_{374},\sigma_{L374},T,v_R)}{\sigma_{eff}(f_{372},\sigma_{L372},T,v_R)}\exp\left(-\Delta E / k_B T\right) \qquad (10.15)$$

(2) Another way is to use the equation of R_T in terms of photon counts, and then go through the entire simulation procedure to re-compute R_T for each new temperature. This method is more universal than the first approach, because not all cases could have a R_T written in terms of pure physical cross sections.

$$R_T = \frac{N_{norm}(\lambda_{374})}{N_{norm}(\lambda_{372})} \qquad (10.16)$$

Derivation of $\Delta R_T/R_T$

- Temperature ratio R_T will be derived from the actual photon counts of N_{374} and N_{372}

$$R_T = \frac{N_{374}}{N_{372}} \qquad (10.17)$$

Using differentiation method, we have

$$\Delta R_T = \frac{\partial R_T}{\partial N_{374}} \Delta N_{374} + \frac{\partial R_T}{\partial N_{372}} \Delta N_{372} = \frac{1}{N_{372}} \Delta N_{374} - \frac{N_{374}}{N_{372}^{\ 2}} \Delta N_{372} \qquad (10.18)$$

Combining Eq. (10.17) with Eq. (10.18), we have

$$\frac{\Delta R_T}{R_T} = \frac{\Delta N_{374}}{N_{374}} - \frac{\Delta N_{372}}{N_{372}} \qquad (10.19)$$

Regarding the errors from two channels are uncorrelated, we have

$$\left(\frac{\Delta R_T}{R_T}\right)_{rms} = \sqrt{\overline{\left(\frac{\Delta N_{374}}{N_{374}} - \frac{\Delta N_{372}}{N_{372}}\right)^2}} = \sqrt{\overline{\left(\frac{\Delta N_{374}}{N_{374}}\right)^2} + \overline{\left(\frac{\Delta N_{372}}{N_{372}}\right)^2}} \qquad (10.20)$$

Considering the signal photon counts are derived by subtracting the background counts from the total photon counts, the photon count uncertainty is given by

$$\overline{(\Delta N_{374})^2} = N_{374} + B, \quad \overline{(\Delta N_{372})^2} = N_{372} + B \qquad (10.21)$$

Derivation of $\Delta R_T/R_T$ Cont'd

- Substituting Eq. (10.21) into Eq. (10.20) and considering Eq. (10.17), we obtain

$$\left(\frac{\Delta R_T}{R_T}\right)_{rms} = \sqrt{\frac{N_{374}+B}{N_{374}^{\ 2}} + \frac{N_{372}+B}{N_{372}^{\ 2}}} = \sqrt{\frac{R_T N_{372}+B}{\left(R_T N_{372}\right)^2} + \frac{N_{372}+B}{N_{372}^{\ 2}}} \qquad (10.22)$$

- Some algebra derivation leads us to the final result

$$\left(\frac{\Delta R_T}{R_T}\right)_{rms} = \frac{\left(1+\frac{1}{R_T}\right)^{1/2}}{\left(N_{372}\right)^{1/2}}\left[1+\frac{B}{N_{372}}\frac{\left(1+\frac{1}{R_T^{\ 2}}\right)}{\left(1+\frac{1}{R_T}\right)}\right]^{1/2} \qquad (10.23)$$

- If we change the expression to SNR of the 372-nm channel, then we have an approximate expression as below:

$$\left(\frac{\Delta R_T}{R_T}\right)_{rms} \approx \frac{1}{SNR_{372}}\sqrt{1+\frac{1}{R_T}} \qquad (10.24)$$

where SNR is defined as

$$SNR_{f_a} \equiv \frac{N_{372}}{\Delta N_{372}} = \frac{N_{372}}{\sqrt{N_{372}+B}} \qquad (10.25)$$

Temperature Error Due to Photon Noise

- Integrating above equations together, we obtain the equation for the temperature error caused by photon noise as below:

$$\Delta T = \frac{R_T}{\partial R_T / \partial T} \cdot \frac{\Delta R_T}{R_T}$$

$$= \frac{R_T}{[R_T(T+\delta T) - R_T(T)]/\delta T} \cdot \frac{\left(1+\frac{1}{R_T}\right)^{1/2}}{\left(N_{372}\right)^{1/2}} \left[1 + \frac{B}{N_{372}} \frac{\left(1+\frac{2}{{R_T}^2}\right)}{\left(1+\frac{1}{R_T}\right)}\right]^{1/2} \qquad (10.26)$$

- The photon counts in the above equation can be written in terms of signal to noise ratio (SNR), if it is more convenient or desirable for some analyses.
- Temperature error is inversely proportional to the sensitivity (S_T) and the signal to noise ratio (SNR).

$$S_T = \frac{\partial R_T / \partial T}{R_T} \qquad (10.27)$$

Summary

- Accuracy and precision are two different concepts for lidar error analysis. Accuracy concerns about bias, usually determined by systematic errors. Precision concerns about uncertainty, mainly determined by random errors, and in lidar photon counting case, mainly by photon noise.

- Error analysis is an important part for lidar research. Many confusing ideas are the in field, especially on the accuracy versus precision issues.

- The differentiation of metric ratio method can apply to both systematic and random errors, depending on the nature of the errors. Error sources could be systematic bias or random jitter, and measurement errors could also be systematic or random errors.

- One approach is to use the "differentiation method", and another one is the Monte Carlo approach.

Monte Carlo Method

- To reveal how random error sources affect the measurement precision and accuracy, an approach different than the above analytical "differentiation method" is the "Monte Carlo Method".
- It is not easy to repeat lidar observations in reality, but it is definitely achievable in lidar simulation and error analysis. The Monte Carlo method is to repeat the simulation many times with random sampling of the interested lidar or atmospheric or atomic parameters within their random error ranges and then check how the measurement results are deviated from the true values.
- For example, the laser central frequency has random errors due to frequency jitter. To investigate how it affects the measurements, we may run the simulation of single shot many times and for each shot we let the laser central frequency randomly pick one value within its jitter range. By integrating many shots together, we then look at how the temperature or wind ratios are deviated from the expected ratios if all the shots have the accurate laser frequency.

Summary

- Lidar simulation and error analysis are the "lidar modeling". It is an integration of complicated lidar remote sensing procedure.
- The key is still our understanding of the lidar theory and the physical interactions between the laser light and the objects you want to study. Only when we clearly understand the interactions in the atmosphere and the entire lidar detection procedure could we do good lidar simulation and error analysis.
- Calculation of errors for ratio technique utilizes the differentiation of the metric ratios as described in the textbook and in this lecture. It works for both systematic and random errors. It also works for both error analysis and sensitivity analysis.
- Reference our textbook: section 5.2.2.5.2