Lecture 04. Fundamentals of Lidar Remote Sensing (2) – "Lidar Equation"

❑ Introduction

❑ Physical Picture of Lidar Equation

❑ Fundamental Lidar Equation

❑ Different Forms of Lidar Equation

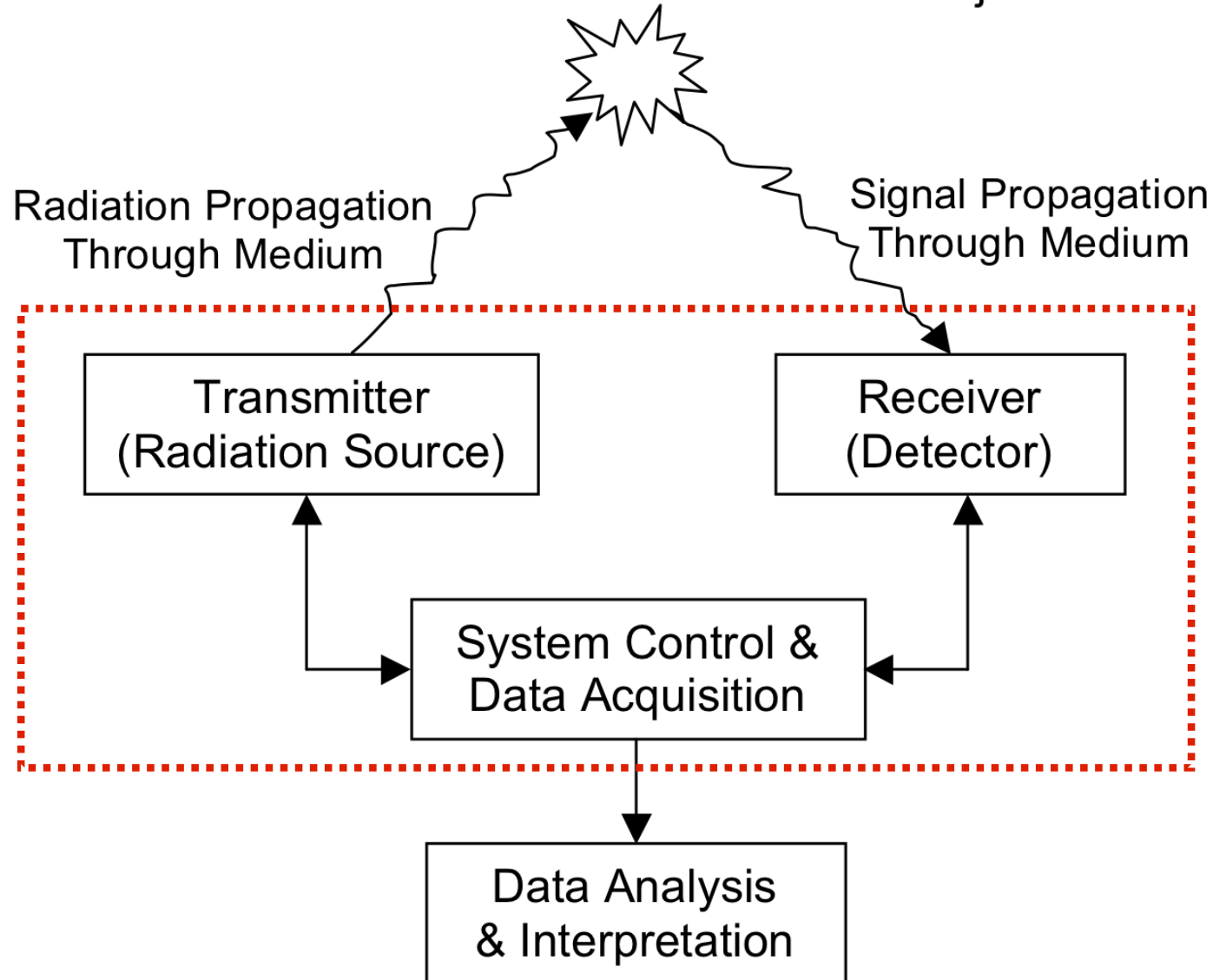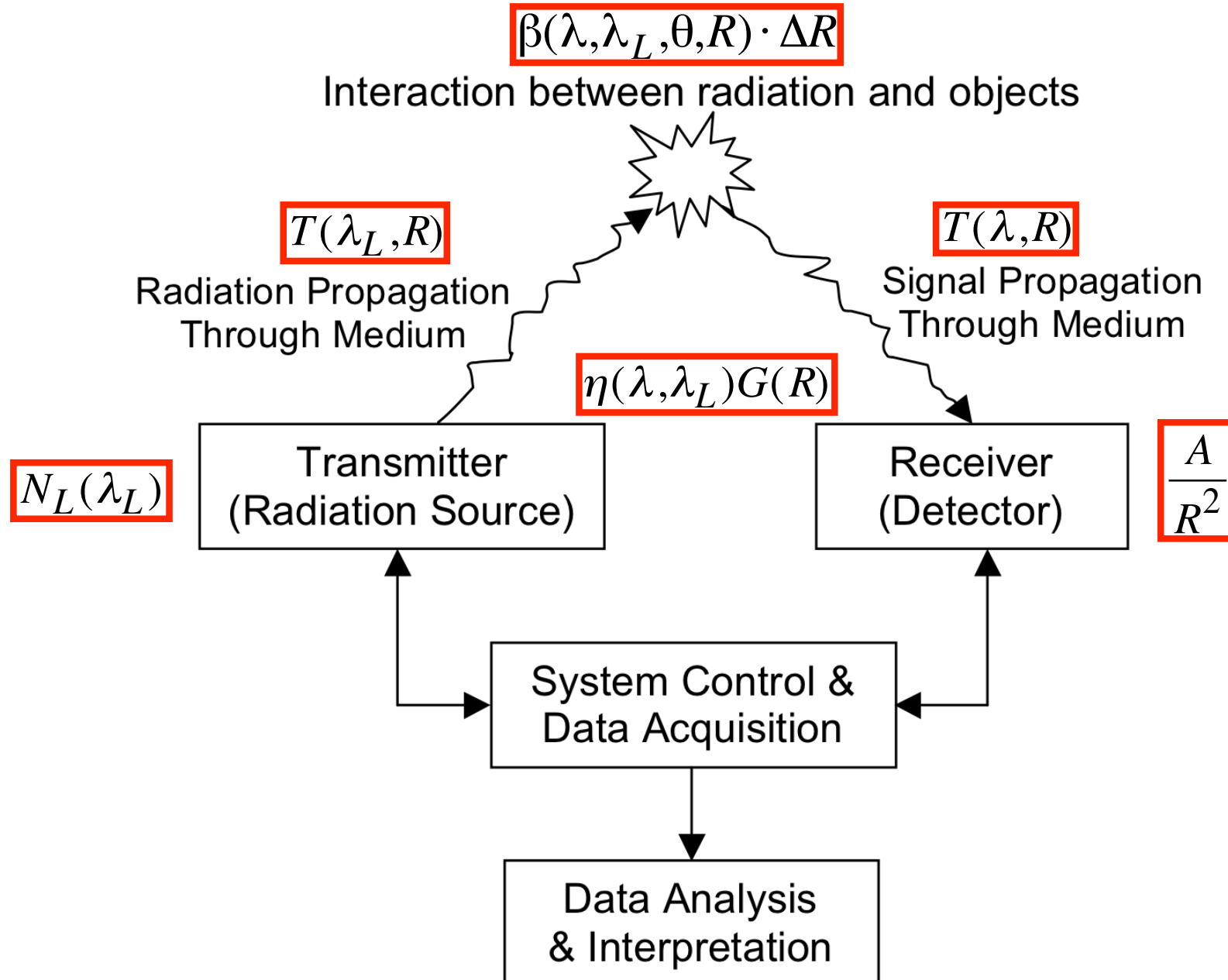❑ Illustration of Lidar Equation

❑ Summary

Introduction

❑ Lidar equation is the fundamental equation in laser remote sensing field to relate the received photon counts (or light power) with the transmitted laser photon numbers (or laser power), the light transmission in atmosphere or medium, the physical interaction between light and objects, the photon receiving probability, and the lidar system efficiency and geometry, etc.

❑ The lidar equation is based on the physical picture of lidar remote sensing, and derived under two assumptions: independent and single scattering.

❑ Different lidars may use different forms of the lidar equation, but all come from the same picture.

Picture of Lidar Remote Sensing

Interaction between radiation and objects

Radiation Propagation
Through Medium

Signal Propagation
Through Medium

Transmitter
(Radiation Source)

Receiver
(Detector)

System Control &
Data Acquisition

Data Analysis
& Interpretation

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Physical Picture of Lidar Equation

$$\beta(\lambda,\lambda_L,\theta,R)\cdot\Delta R$$

Interaction between radiation and objects

$T(\lambda_L,R)$

$T(\lambda,R)$

Radiation Propagation
Through Medium

Signal Propagation
Through Medium

$\eta(\lambda,\lambda_L)G(R)$

$N_L(\lambda_L)$

Transmitter (Radiation Source)	Receiver (Detector)

$\dfrac{A}{R^2}$

System Control & Data Acquisition

Data Analysis & Interpretation

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Factors in Lidar Equation

The received photon counts N_S are related to the factors

$$N_S(\lambda, R) \propto$$

$$N_L(\lambda_L)$$ Transmitted laser photon number

$$T(\lambda_L, R)$$ Laser photon transmission through medium

$$\beta(\lambda, \lambda_L, \theta, R) \cdot \Delta R$$ Probability of a transmitted photon to be scattered

$$T(\lambda, R)$$ Signal photon transmission through medium

$$\frac{A}{R^2}$$ Probability of a scattered photon to be collected

$$\eta(\lambda, \lambda_L) G(R)$$ Lidar system efficiency and geometry factor

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Considerations for Lidar Equation

❑ In general, the interaction between the light photons and the particles is a scattering process.

❑ The expected photon counts are proportional to the product of the

(1) transmitted laser photon number,

(2) probability that a transmitted photon is scattered,

(3) probability that a scattered photon is collected,

(4) light transmission through medium, and

(5) overall system efficiency.

❑ Background photon counts and detector noise also contribute to the expected photon counts.

Lidar Equation Development

❑ With the picture of lidar remote sensing in mind, one can follow the photon path to develop a lidar equation to quantify how the received photon number or light power is related to the transmitted photons, light transmission, scattering probability, receiver probability, system efficiency and geometry factors as well as background light and detector noise --

(4.1)

$$N_S(\lambda, R) = N_L(\lambda_L) \cdot \eta(\lambda_L) \cdot T(\lambda_L, R) \cdot \left[\beta(\lambda, \lambda_L, \theta, R) \Delta R \right] \cdot T(\lambda, R) \cdot \frac{A}{R^2} \cdot \eta(\lambda) \cdot G(R) + N_B$$

❑ In this development, photon or light power is regarded as scalar quantities, so the calculation sequence does not matter. However, if vector or matrix is involved, e.g., for polarization study, then it is necessary to consider the matrix computation sequence for the lidar equation.

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Fundamental Lidar Equation

$$N_S(\lambda,R) = N_L(\lambda_L) \cdot \left[\beta(\lambda,\lambda_L,\theta,R)\Delta R \right] \cdot \frac{A}{R^2} \cdot \left[T(\lambda_L,R)T(\lambda,R) \right] \cdot \left[\eta(\lambda,\lambda_L)G(R) \right] + N_B$$

(4.2)

- ❑ N_S -- expected photon counts detected at λ and R
- ❑ 1st term -- the transmitted laser photon number;
- ❑ 2nd term -- the probability of a transmitted photon to be scattered by the objects into a unit solid angle;
- ❑ 3rd term -- the probability of a scatter photon to be collected by the receiving telescope;
- ❑ 4th term -- the light transmission through medium for the transmitted laser and return signal photons;
- ❑ 5th term -- the overall system efficiency;
- ❑ 6th term N_B -- background and detector noise counts.

Basic Assumptions in Lidar Equation

❑ The lidar equation is developed under two assumptions: the scattering processes are independent, and only single scattering occurs.

❑ Independent scattering means that particles are separated adequately and undergo random motion so that the contribution to the total scattered energy by many particles have no phase relation. Thus, the total intensity is simply a sum of the intensity scattered from each particle.

❑ Single scattering implies that a photon is scattered only once. Multiple scatter is excluded in our considerations.

1st Term: Transmitted Photon Number

$$N_S(\lambda,R) = N_L(\lambda_L) \cdot \left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right] \cdot \frac{A}{R^2} \cdot \left[T(\lambda_L,R)T(\lambda,R)\right] \cdot \left[\eta(\lambda,\lambda_L)G(R)\right] + N_B$$

$$N_L(\lambda_L) = \left(\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right) \qquad (4.3)$$

$$= \frac{\text{Laser Power} \times \text{time bin length}}{\text{Planck constant} \times \text{Laser frequency}}$$

$$= \frac{\text{Transmitted laser energy within time bin}}{\text{Single laser photon energy}}$$

$$= \text{Transmitted laser photon number within time bin length}$$

Let's estimate how many photons are sent out with a 20 mJ laser pulse at 589 nm

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2nd Term: Probability to be Scattered

$$N_S(\lambda, R) = N_L(\lambda_L) \cdot \left[\beta(\lambda, \lambda_L, \theta, R)\Delta R\right] \cdot \frac{A}{R^2} \cdot \left[T(\lambda_L, R)T(\lambda, R)\right] \cdot \left[\eta(\lambda, \lambda_L)G(R)\right] + N_B$$

Angular scattering probability – the probability that a transmitted photon is scattered by scatters into a unit solid angle.

Angular scattering probability =
   volume scatter coefficient β
      × scattering layer thickness ΔR

(4.4)

Volume Scatter Coefficient β

Volume scatter coefficient β is equal to

$$\beta(\lambda,\lambda_L,R) = \sum_i \left[\frac{d\sigma_i(\lambda_L,\theta)}{d\Omega} n_i(R) p_i(\lambda) \right] \quad \text{(m}^{-1}\text{sr}^{-1}) \quad (4.5)$$

$\dfrac{d\sigma_i(\lambda_L)}{d\Omega}$ is the differential scatter cross–section of single particle in species i at scattering angle θ (m²sr⁻¹)

$n_i(R)$ is the number density of scatter species i (m⁻³)

$p_i(\lambda)$ is the probability of the scattered photons falling into the wavelength λ.

Volume scatter coefficient β is the probability per unit distance travel that a photon is scattered into wavelength λ in unit solid angle at angle θ.

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3rd Term: Probability to be Collected

$$N_S(\lambda,R) = N_L(\lambda_L) \cdot \left[\beta(\lambda,\lambda_L,\theta,R)\Delta R \right] \cdot \boxed{\frac{A}{R^2}} \cdot \left[T(\lambda_L,R)T(\lambda,R) \right] \cdot \left[\eta(\lambda,\lambda_L)G(R) \right] + N_B$$

The probability that a scatter photon is collected by the receiving telescope, i.e., the solid angle subtended by the receiver aperture to the scatterer.

Δz

z

A

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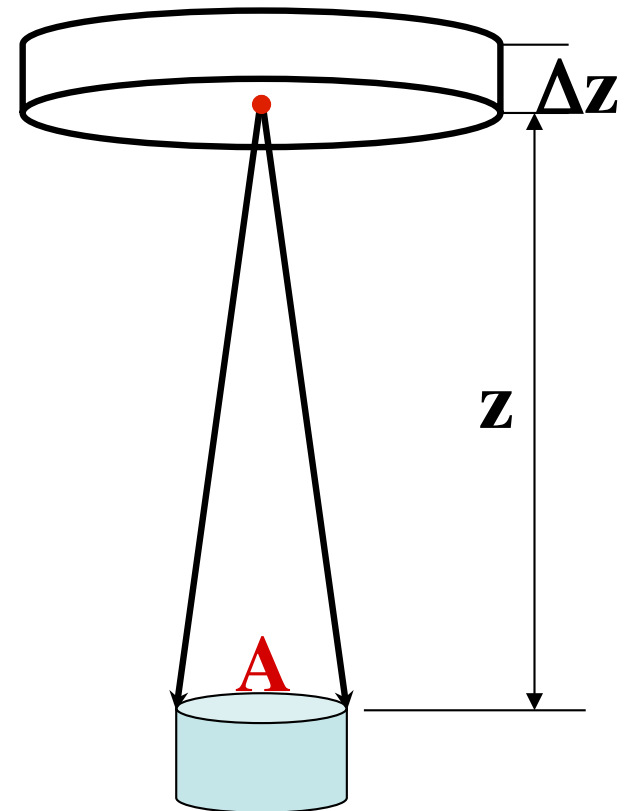
4th Term: Light Transmission

$$N_S(\lambda,R) = N_L(\lambda_L) \cdot \left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right] \cdot \frac{A}{R^2} \cdot \left[T(\lambda_L,R)T(\lambda,R)\right] \cdot \left[\eta(\lambda,\lambda_L)G(R)\right] + N_B$$

The atmospheric transmission of laser light at outgoing wavelength λ_L and return signal at wavelength λ

Transmission for laser light $\longrightarrow$

Transmission for return signal $\longrightarrow$

$$T(\lambda_L,R) = \exp\left[-\int_0^R \alpha(\lambda_L,r)dr\right]$$

$$T(\lambda,R) = \exp\left[-\int_0^R \alpha(\lambda,r)dr\right]$$

(4.6)

Where $\alpha(\lambda_L, R)$ and $\alpha(\lambda, R)$ are extinction coefficients (m^{-1})

Extinction Coefficient α

$$\alpha(\lambda,R) = \sum_i \left[\sigma_{i,ext}(\lambda) n_i(R) \right] \qquad (4.7)$$

$\sigma_{i,ext}(\lambda)$ is the extinction cross-section of species i

$n_i(R)$ is the number density of species i

Extinction = Absorption + Scattering (Integrated)

$$\sigma_{i,ext}(\lambda) = \sigma_{i,abs}(\lambda) + \sigma_{i,sca}(\lambda) \qquad (4.8)$$

Total Extinction = Aerosol Extinction + Molecule Extinction

$$\alpha(\lambda,R) = \alpha_{aer,abs}(\lambda,R) + \alpha_{aer,sca}(\lambda,R) + \alpha_{mol,abs}(\lambda,R) + \alpha_{mol,sca}(\lambda,R)$$

(4.9)

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5th Term: Overall Efficiency

$$N_S(\lambda,R) = N_L(\lambda_L) \cdot \left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right] \cdot \frac{A}{R^2} \cdot \left[T(\lambda_L,R)T(\lambda,R)\right] \cdot \left[\eta(\lambda,\lambda_L)G(R)\right] + N_B$$

$\eta(\lambda,\lambda_L) = \eta_T(\lambda_L) \cdot \eta_R(\lambda)$ is the lidar hardware optical efficiency

(4.10)   e.g., mirrors, lens, filters, detectors, etc

$G(R)$ is the geometrical form factor, mainly concerning the overlap of the area of laser irradiation with the field of view of the receiver optics

6th Term: Background Noise

$$N_S(\lambda,R) = N_L(\lambda_L) \cdot \left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right] \cdot \frac{A}{R^2} \cdot \left[T(\lambda_L,R)T(\lambda,R)\right] \cdot \left[\eta(\lambda,\lambda_L)G(R)\right] + N_B$$

N_B is the expected photon counts due to background noise, detector and circuit shot noise, etc.

Up-looking lidar: main background noise comes from solar scattering, solar straight radiation, star/city light.

Down-looking lidar: besides above noise, it could have three extra backgrounds: (1) Specular reflection from water/ice surface; (2) Laser reflectance from ground; (3) Solar reflectance from ground.

Different Forms of Lidar Equation

❑ The main difference between upper and lower atmosphere lidars lies in the treatment of backscatter coefficient and atmosphere transmission (extinction).

❑ Upper atmosphere lidar cares about the backscatter coefficient more than anything else, because (1) the lower atmosphere transmission is cancelled out during Rayleigh normalization, and (2) the extinction caused by atomic absorption can be precisely calculated, thus, extinction is not an issue to upper atmosphere lidar.

❑ Lower atmosphere lidar relies on both backscatter coefficient and atmospheric extinction, as these are what they care about or something that cannot be cancelled out.

General Form of Lidar Equation

$$N_S(\lambda,R) = N_L(\lambda_L) \cdot \left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right] \cdot \frac{A}{R^2} \cdot \left[T(\lambda_L,R)T(\lambda,R)\right] \cdot \left[\eta(\lambda,\lambda_L)G(R)\right] + N_B$$

(4.2)   — Lidar Equation in Photon Count and $\beta(\theta)$

$$P_S(\lambda,R) = P_L(\lambda_L) \cdot \left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right] \cdot \frac{A}{R^2} \cdot \left[T(\lambda_L,R)T(\lambda,R)\right] \cdot \left[\eta(\lambda,\lambda_L)G(R)\right] + P_B$$

(4.11) — Lidar Equation in Light Power and β (θ)

$$N_S(\lambda,R) = N_L(\lambda_L) \cdot \left[\beta_T(\lambda,\lambda_L,R)\Delta R\right] \cdot \frac{A}{4\pi R^2} \cdot \left[T(\lambda_L,R)T(\lambda,R)\right] \cdot \left[\eta(\lambda,\lambda_L)G(R)\right] + N_B$$

(4.12)   — Lidar Equation in Photon Count and β_T

General Form of LIDAR Equation

Angular volume scattering coefficient

$$\beta(\lambda,\lambda_L,\theta,R) = \sum_i \left[\frac{d\sigma_i(\lambda_L)}{d\Omega} n_i(R) p_i(\lambda) \right]$$

(4.5)

Total scattering coefficient

$$\beta_T(\lambda,\lambda_L,R) = \int_0^{4\pi} \beta(\lambda,\lambda_L,\theta,R) d\Omega$$

$$= \int_0^{4\pi} \sum_i \left[\frac{d\sigma_i(\lambda_L)}{d\Omega} n_i(R) p_i(\lambda) \right] d\Omega$$

$$= \sum_i \left[\sigma_i(\lambda_L) n_i(R) p_i(\lambda) \right]$$

(4.13)

Fluorescence Form of Lidar Equation

$$N_S(\lambda,R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) \left(\sigma_{eff}(\lambda,R) n_c(z) R_B(\lambda) \Delta R \right) \left(\frac{A}{4\pi R^2} \right) \left(T_a^2(\lambda,R) T_c^2(\lambda,R) \right) \left(\eta(\lambda) G(R) \right) + N_B$$

(4.14)

❑ Here, $T_c(R)$ is the transmission resulted from the constituent extinction

$$T_c(R) = \exp\left(-\int_{R_{bottom}}^{R} \sigma_{eff}(\lambda,r') n_c(r') \mathrm{d}r' \right) = \exp\left(-\int_{R_{bottom}}^{R} \alpha_c(\lambda,r') \mathrm{d}r' \right)$$

(4.15)

❑ Here, $\alpha(\lambda,R)$ is the extinction coefficient caused by the constituent absorption.

$$\alpha_c(\lambda,R) = \sigma_{eff}(\lambda,R) n_c(R) \qquad (4.16)$$

❑ Resonance fluorescence and laser-induced-fluorescence are NOT instantaneous processes, but have delays due to the radiative lifetime of the excited states.

General Lidar Equation in β and α
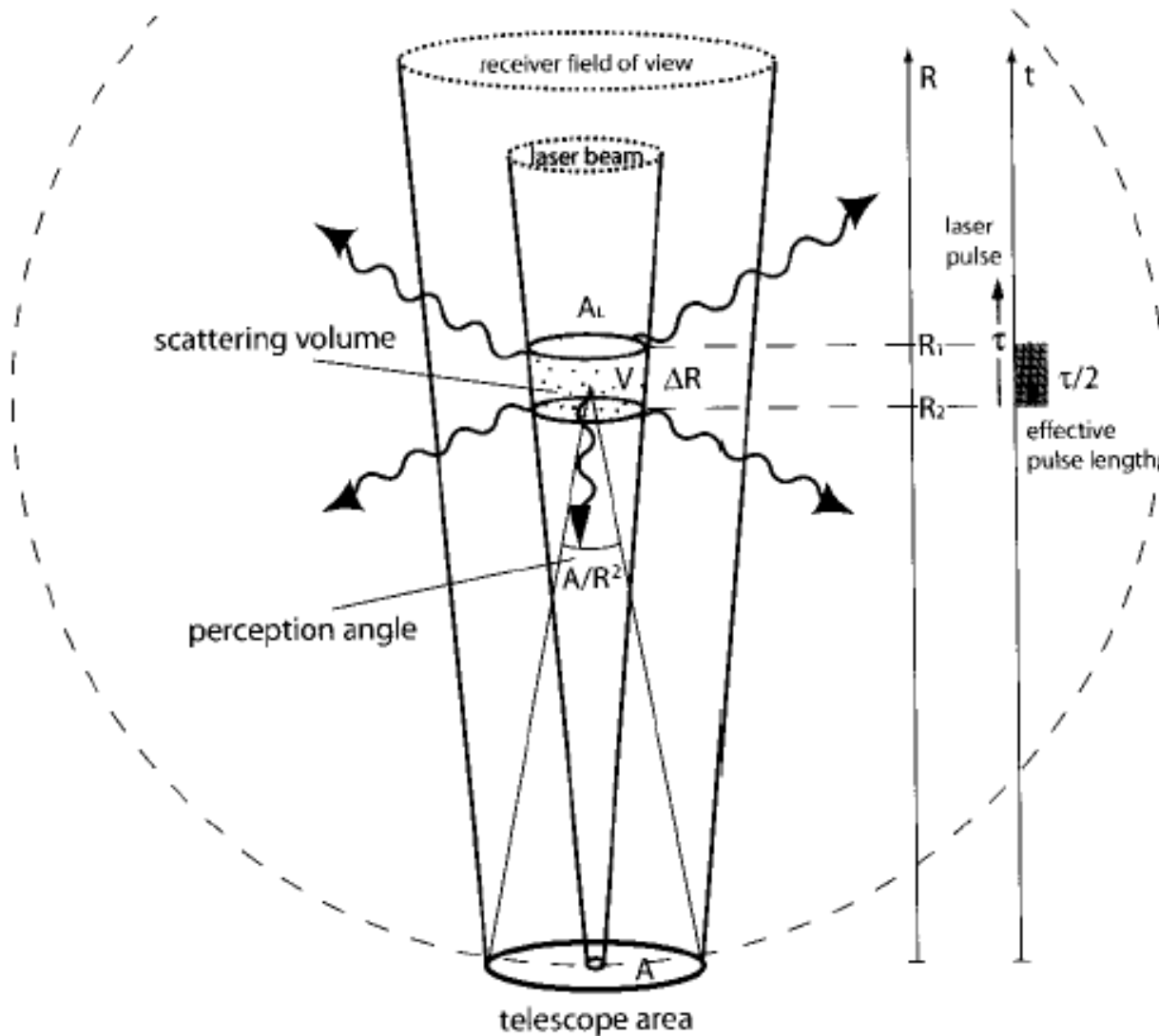
$$N_S(\lambda, R) = \left[\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right]\left[\beta(\lambda, \lambda_L, \theta, R)\Delta R\right]\left(\frac{A}{R^2}\right)$$

$$\cdot \exp\left[-\int_0^R \alpha(\lambda_L, r')dr'\right]\exp\left[-\int_0^R \alpha(\lambda, r')dr'\right]\left[\eta(\lambda, \lambda_L)G(R)\right] + N_B$$

β is the volume scatter coefficient   (4.17)
α is the extinction coefficient

Volume scatter coefficient $\beta(\lambda, \lambda_L, R) = \sum_i\left[\frac{d\sigma_i(\lambda_L)}{d\Omega}n_i(R)p_i(\lambda)\right]$ (4.5)

Transmission $T(\lambda_L, R)T(\lambda, R) = \exp\left[-\left(\int_0^R \alpha(\lambda_L, r)dr + \int_0^R \alpha(\lambda, r)dr\right)\right]$ (4.18)

Illustration of LIDAR Equation

-- Courtesy of Ulla Wandinger [Introduction to Lidar]

Summary

❑ Lidar equation is the fundamental equation governing the lidar remote sensing field.

❑ Lidar equation relates the received photon counts to the transmitted laser photon numbers, light transmission through medium, probability of a transmitted photon to be scattered, properties of scatters, probability of a scattered photon to be collected, and lidar system efficiency and geometry factors.

❑ Different lidars may use different forms of the lidar equation, depending on the needs and emphasis.

Chapter 1 of "Laser Remote Sensing" textbook
IntroLidar.pdf
Chapter 5. Sections 5.1 and 5.2.1 textbook
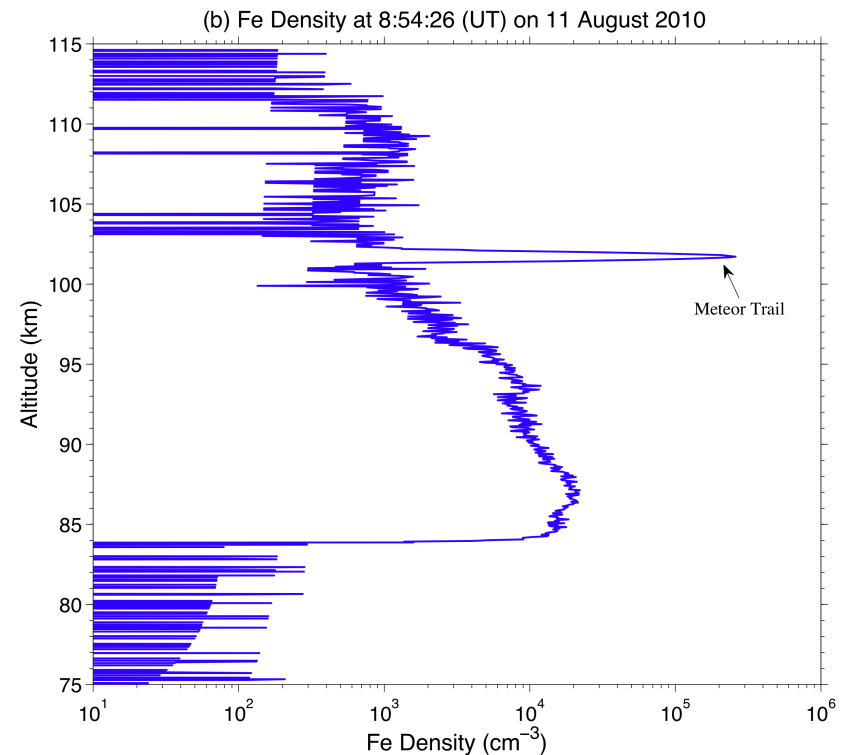
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More on TOF Range Determination

From Lecture 03, we have stated the followings:

❑ When scatters are continuously distributed, the ultimate resolution of range determination is limited by the pulse duration time τ: $\Delta R = c \cdot \tau / 2$.

❑ The distinct peak due to the strong reflection of light from surface or target, the range resolution can be significantly improved by digitizing the return pulse and compare shape.
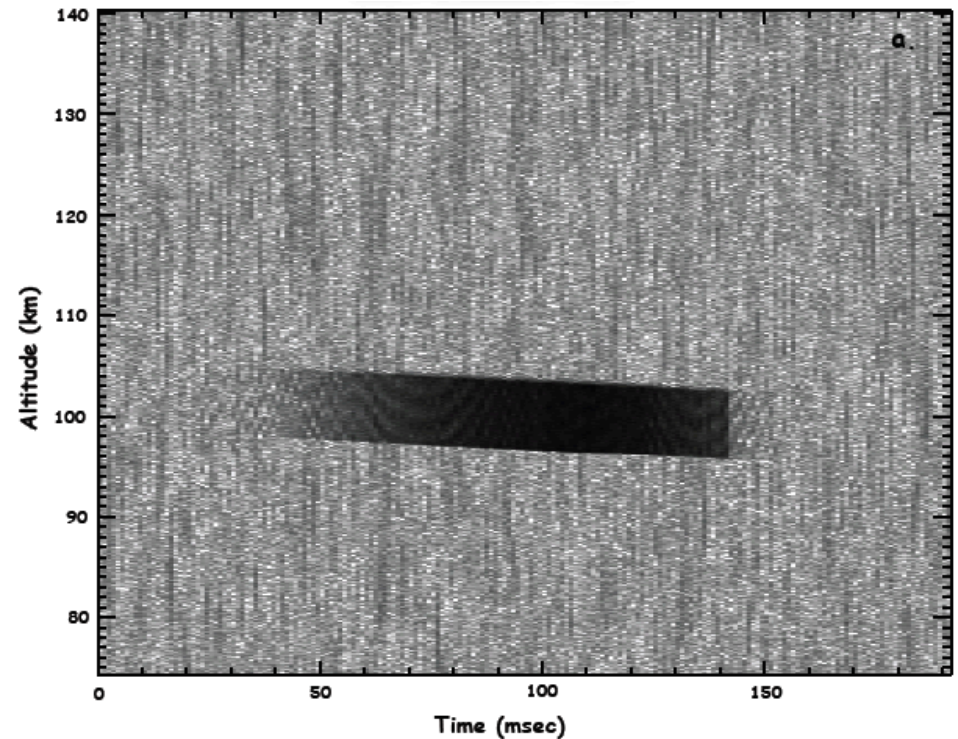
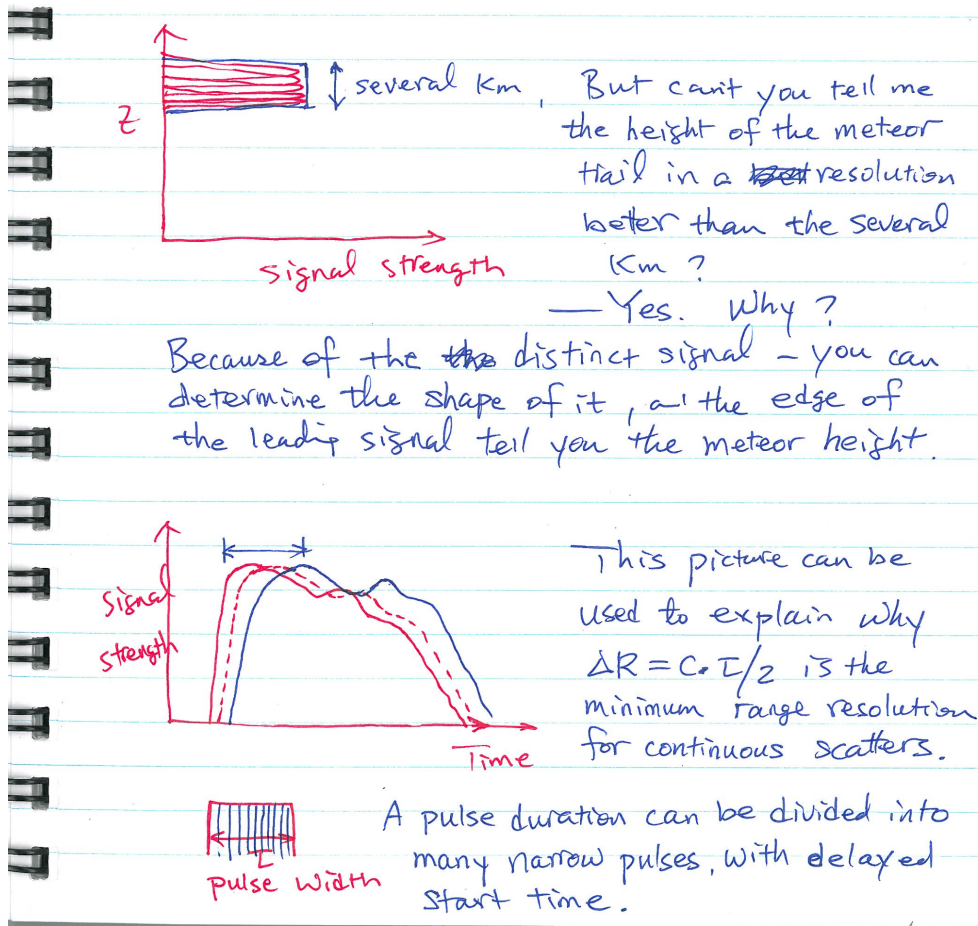How to understand these statements?

τ
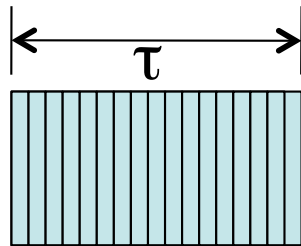
A pulse with long duration can be divided into many narrow pulses with sequentially delayed start times.

(b) Fe Density at 8:54:26 (UT) on 11 August 2010

Meteor Trail

Altitude (km)

Fe Density (cm^{-3})

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More on TOF Range Determination

↕ several km, But can't you tell me the height of the meteor tail in a ~~bet~~ resolution better than the several km?
— Yes. Why?
Because of the ~~the~~ distinct signal – you can determine the shape of it, a the edge of the leading signal tell you the meteor height.

This picture can be used to explain why $\Delta R = c \cdot \tau / 2$ is the minimum range resolution for continuous scatters.

A pulse duration can be divided into many narrow pulses, with delayed start time.

A meteor head echo recorded by Arecibo 430-MHz radar with a 45 µs pulse
(min ΔR = 6.75 km)