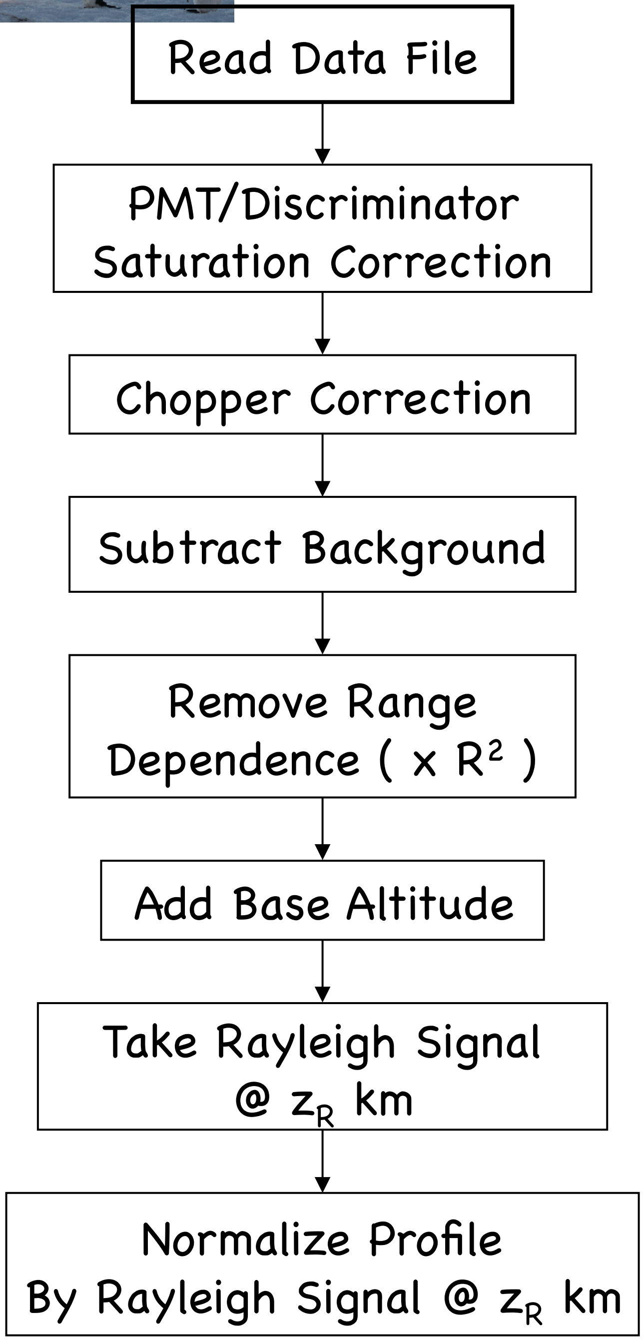
Lecture 18. Lidar Data Inversion (2)

- Review of Pre-process and Profile-process
- Main Process Procedure to Derive T and V_R Using Ratio Doppler Technique
- Derivations of n_c from narrowband resonance Doppler lidar
- Derivation of β
- Derivation of n_c from broadband resonance lidar
- Summary

Review of Preprocess & Profile Process Procedure



- Read data: for each set, and calculate T, W, and n for each set
- PMT/Discriminator saturation correction
- Chopper/Filter correction
- Background estimate and subtraction
- Range-dependence removal (not altitude)
- Base altitude adjustment
- Take Rayleigh signal @ z_R (Rayleigh fit or Rayleigh sum)
- Rayleigh normalization
- Subtract Rayleigh signals from Na/Fe/K region

$$N_N(\lambda,z) = \frac{N_S(\lambda,z) - N_B}{N_S(\lambda,z_R) - N_B} \frac{z^2}{z_R^2}$$

Solutions to Lidar Equation

- Lidar equation for pure Rayleigh backscattering

$$N_S(\lambda,z_R)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_R(\pi,\lambda)n_R(z_R)\right]\Delta z\left(\frac{A}{z_R^{\ 2}}\right)T_a^{\ 2}(\lambda,z_R)\left(\eta(\lambda)G(z_R)\right)+N_B$$

- Lidar equation for resonance fluorescence

$$N_S(\lambda,z)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_{eff}(\lambda,z)n_c(z)R_B(\lambda)+4\pi\sigma_R(\pi,\lambda)n_R(z)\right]\Delta z\left(\frac{A}{4\pi z^2}\right)$$
$$\times\left(T_a^2(\lambda)T_c^{\ 2}(\lambda,z)\right)\left(\eta(\lambda)G(z)\right)+N_B$$

$$n_c(z)=\left[\frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^{\ 2}}\frac{1}{T_c^{\ 2}(\lambda,z)}-\frac{n_R(z)}{n_R(z_R)}\right]\cdot\frac{4\pi\sigma_R(\pi,\lambda)n_R(z_R)}{\sigma_{eff}(\lambda)R_B(\lambda)}$$

Constituent Density

- Normalized Photon Count to the density estimation

$$n_c(z) = \left[\frac{N_S(\lambda,z) - N_B}{N_R(\lambda,z_R) - N_B} \cdot \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)} \right] \cdot \frac{4\pi\sigma_R(\pi,\lambda) n_R(z_R)}{\sigma_{eff}(\lambda) R_B(\lambda)}$$

Normalized Photon Count
From the preprocess

Temperature and wind dependent

→ we need to estimate the temperature and wind first to estimate the density

Basic Clue: Ratio Computation

- From physics, we calculate the ratios of R_T and R_W as

$$\boxed{R_T = \frac{\sigma_{eff}(f_+,z)+\sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}} \qquad \boxed{R_W = \frac{\sigma_{eff}(f_+,z)-\sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}}$$

- From actual photon counts, we calculate the ratios as

$$R_T = \frac{N_{Norm}(f_+,z)+N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}$$

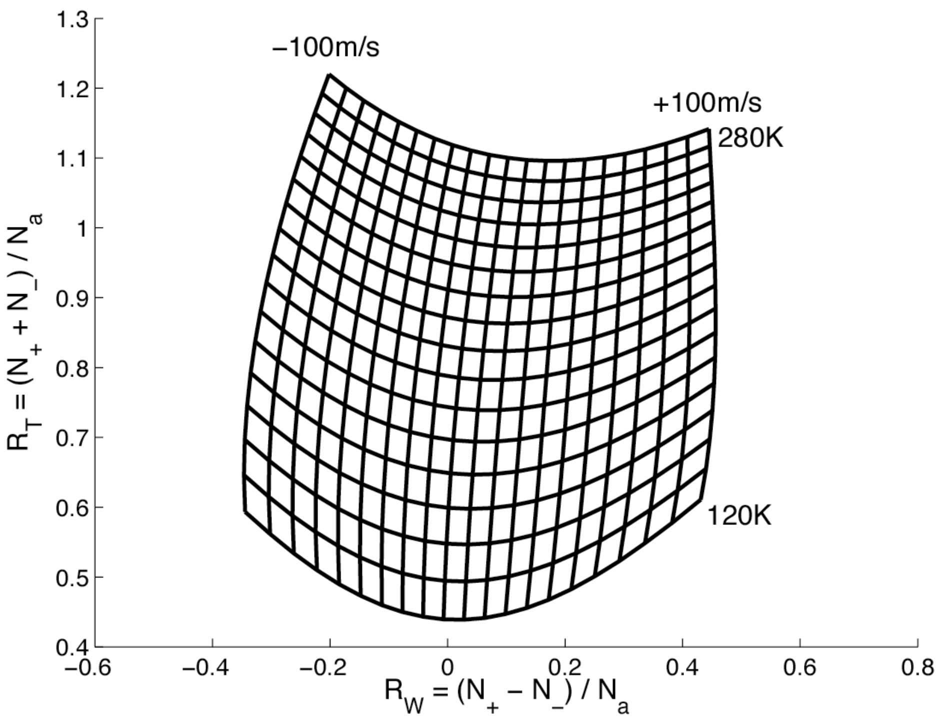
$$= \frac{\left(\frac{N_S(f_+,z)-N_B}{N_S(f_+,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_+,z)}-\frac{n_R(z)}{n_R(z_R)}\right)+\left(\frac{N_S(f_-,z)-N_B}{N_S(f_-,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_-,z)}-\frac{n_R(z)}{n_R(z_R)}\right)}{\frac{N_S(f_a,z)-N_B}{N_S(f_a,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_a,z)}-\frac{n_R(z)}{n_R(z_R)}}$$

$$R_W = \frac{N_{Norm}(f_+,z)-N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}$$

$$= \frac{\left(\frac{N_S(f_+,z)-N_B}{N_S(f_+,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_+,z)}-\frac{n_R(z)}{n_R(z_R)}\right)-\left(\frac{N_S(f_-,z)-N_B}{N_S(f_-,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_-,z)}-\frac{n_R(z)}{n_R(z_R)}\right)}{\frac{N_S(f_a,z)-N_B}{N_S(f_a,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_a,z)}-\frac{n_R(z)}{n_R(z_R)}}$$

Main Process Procedure

- Compute Doppler calibration curves from physics



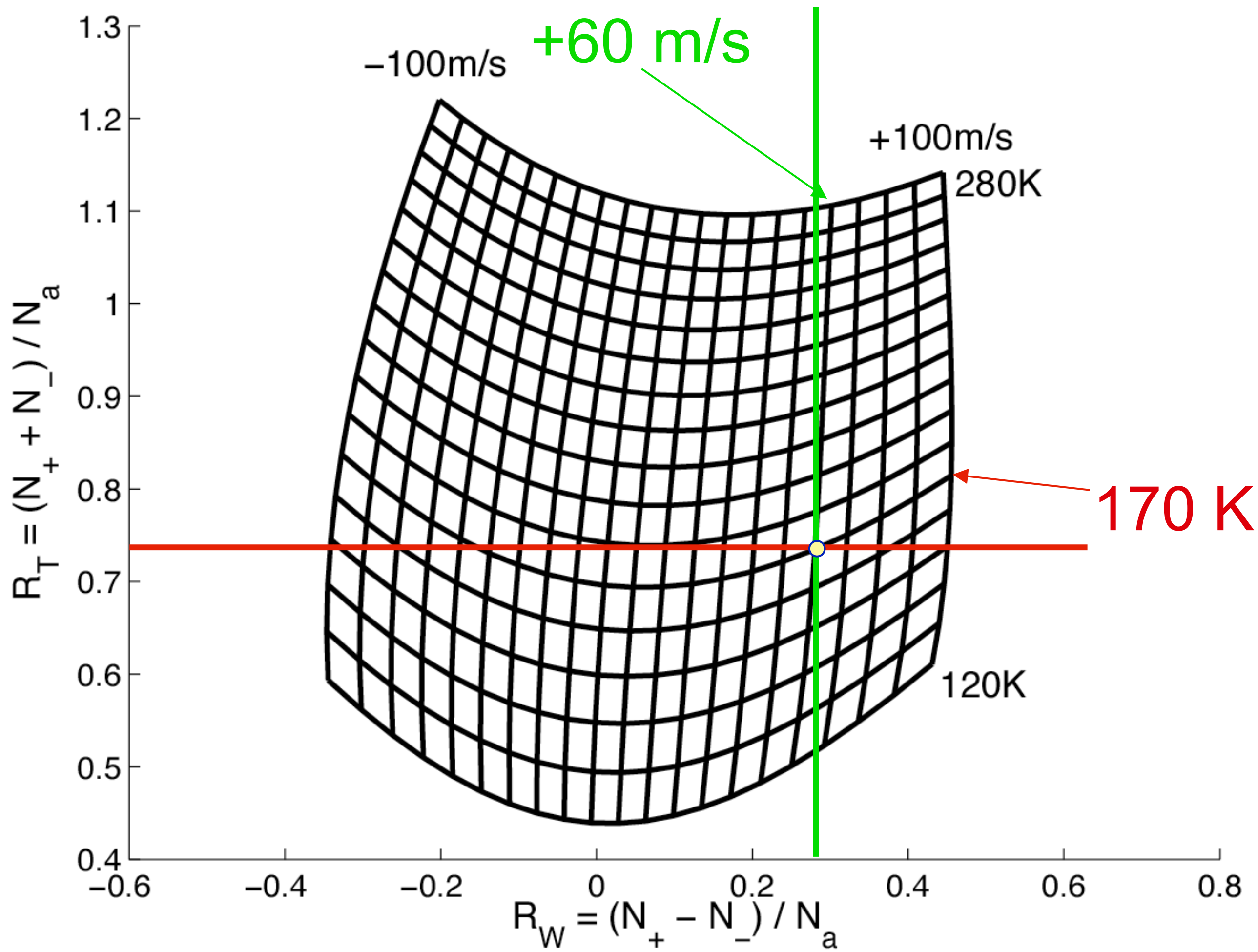
$$R_W = \frac{\sigma_{eff}(f_+, z) - \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

$$R_T = \frac{\sigma_{eff}(f_+, z) + \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

$$\sigma_{\text{eff}}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_{\text{e}}} \frac{e^2 f}{4\epsilon_0 m_{\text{e}} c} \sum_{n=1}^{6} A_n \exp\left(-\frac{\left[\nu_n - \nu\left(1 - \frac{v_{\text{R}}}{c}\right)\right]^2}{2\sigma_{\text{e}}^2}\right)$$

Main Process Procedure

- Compute actual ratios R_T and R_W from photon counts
- Look up these two ratios on the calibration curves to infer the corresponding Temperature and Wind from isoline/isogram.



Main Ideas to Derive Na T and W

- In the ratio technique, Na number density is cancelled out. So we have two ratios R_T and R_W that are independent of Na density but both dependent on T and W.
- The idea is to derive temperature and radial wind from these two ratios first, and then derive Na number density using computed temperature and wind at each altitude bin.
- To derive T and W from R_T and R_W, the basic idea is to use look-up table or iteration methods to derive them: (1) compute R_T and R_W from physics point-of-view to generate the table or calibration curves, (2) compute R_T and R_W from actual photon counts, (3) check the table or calibration curves to find the corresponding T and W. (4) If R_T and R_W are out of range, then set to nominal T and W.
- However, because the Na extinction coefficient is involved, the upper bins are related to lower bins, and extinction coefficient is related to Na density and effective cross-section. The solution is to start from the bottom of the Na layer.

Main Process

Load Atmosphere n_R, T_R, P_R
Profiles from MSIS00

↓

Start from Na layer bottom
$E\ (z=z_b) = 1$
Calculate $N_{norm}\ (z=z_b)$ from
photon counts and MSIS
number density for each freq

$$N_{Norm}(\lambda,z) = \frac{N_S(\lambda,z) - N_B}{N_S(\lambda,z_R) - N_B}\,\frac{z^2}{{z_R}^2}\,\frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}$$

↓

Calculate R_T and R_W from N_{Norm}

↓

Are ratios reasonable?

No → Set to nominal values
$T = 200$ K, $W = 0$ m/s
→ Calculate Na density $n_c(z)$

Yes → Find T and W
from the Table
→ Look-up Table Calibration
→ Calculate Na density $n_c(z)$

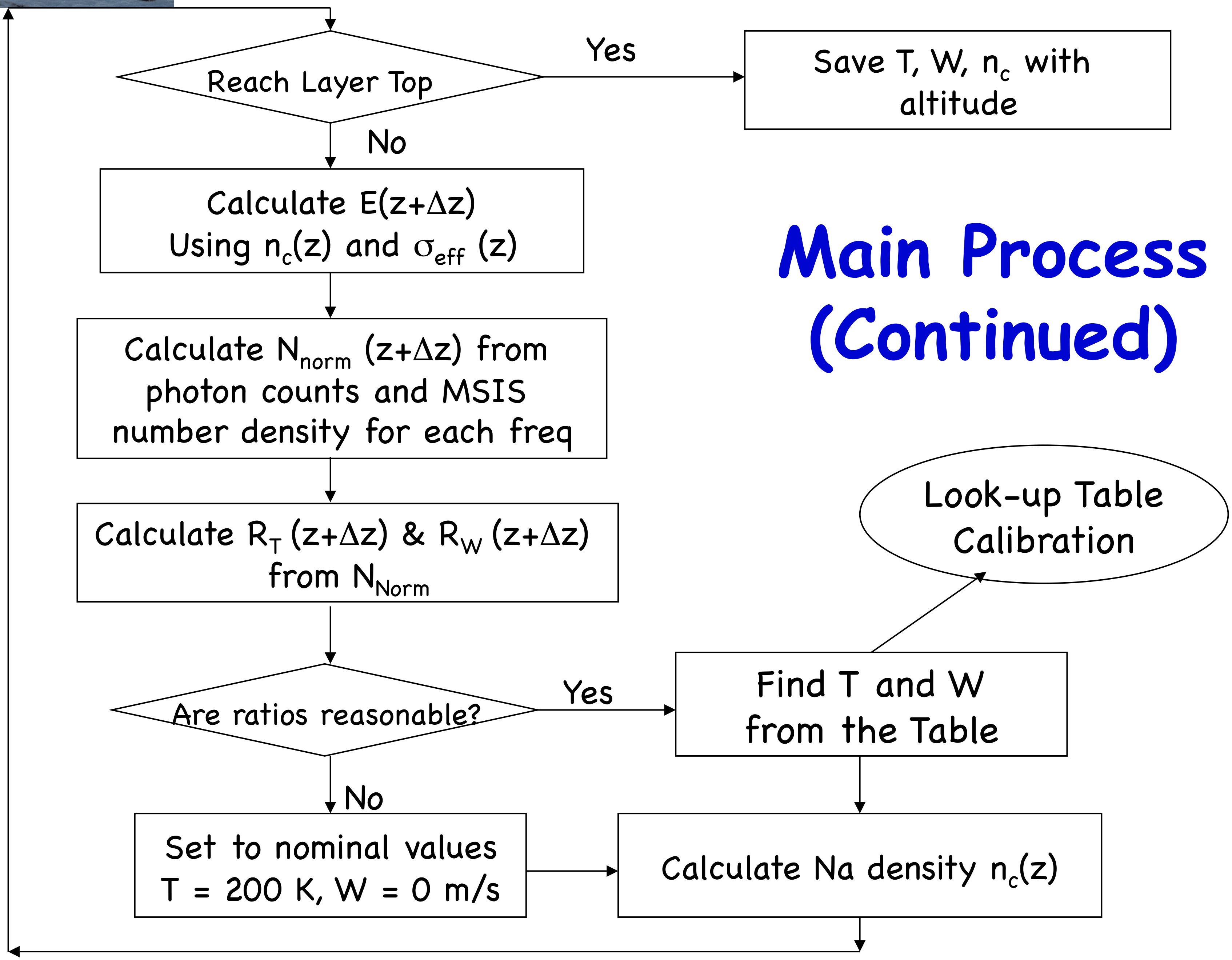
Create look-up table or calibration curves
From physics

$$R_T = \frac{\sigma_{eff}(f_+,z) + \sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}$$

$$R_W = \frac{\sigma_{eff}(f_+,z) - \sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}$$

Look-up Table
Calibration

Main Process (Continued)



Derivation of T_c (Extinction)

- The T_c (caused by constituent extinction) can be derived from

$$T_c(\lambda,z) = \exp\left(-\int_{z_{bottom}}^{z} \sigma_{eff}(\lambda,z) n_c(z) \, dz\right) = \exp\left(-\sum_{z_{bottom}}^{z} \sigma_{eff}(\lambda,z) n_c(z) \Delta z\right)$$

- The effective cross-section

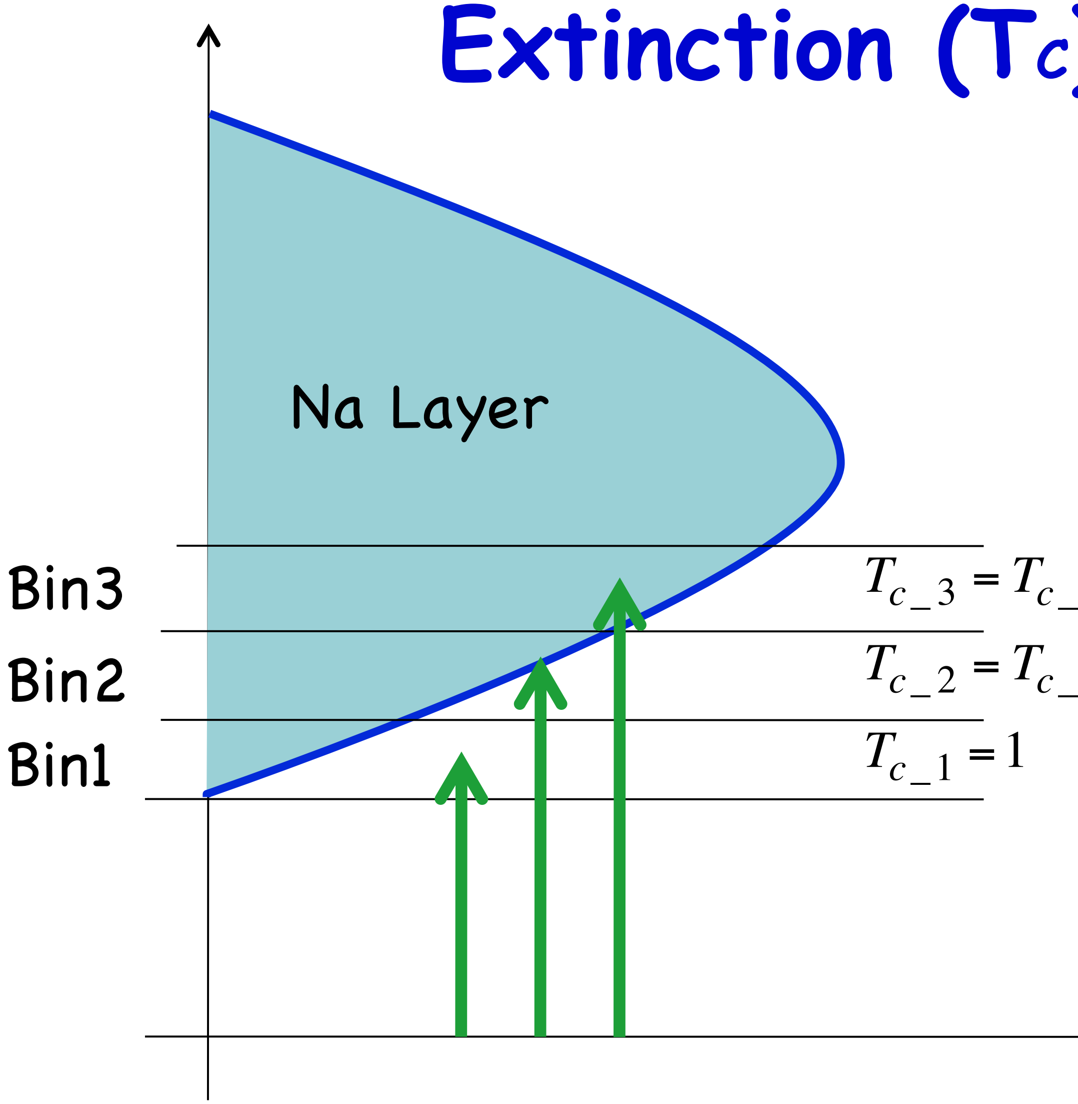
$$\sigma_{\mathrm{eff}}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{e}}} \frac{e^2 f}{4\epsilon_0 m_{\mathrm{e}} c} \sum_{n=1}^{6} A_n \exp\left(-\frac{\left[\nu_n - \nu\left(1 - \frac{v_{\mathrm{R}}}{c}\right)\right]^2}{2\sigma_{\mathrm{e}}^2}\right)$$

Ready to estimate the constituent density

$$\sigma_e = \sqrt{\sigma_D^2 + \sigma_L^2}$$

$$n_c(z) = \left[\frac{N_S(\lambda,z) - N_B}{N_R(\lambda,z_R) - N_B} \cdot \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}\right] \cdot \frac{4\pi\sigma_R(\pi,\lambda) n_R(z_R)}{\sigma_{eff}(\lambda) R_B(\lambda)}$$

A Key Step to Main Process Extinction (T_c)



$$T_{c\_3} = T_{c\_2} \times \exp(-\sigma_{eff\_2} \cdot n_{c\_2} \cdot \Delta z)$$

$$T_{c\_2} = T_{c\_1} \times \exp(-\sigma_{eff\_1} \cdot n_{c\_1} \cdot \Delta z)$$

$$T_{c\_1} = 1$$

Main Process Step 1: Starting Point

1. Extinction (Tc) at the bottom of Na layer is 1
2. Calculate the normalized photon count for each frequency

$$\boxed{N_{Norm}(\lambda,z) = \frac{N_S(\lambda,z) - N_B}{N_S(\lambda,z_R) - N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}}$$

3. Based on the normalized photon counts, you get R_T and R_W

$$\boxed{R_T = \frac{N_{Norm}(f_+,z) + N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}} \qquad \boxed{R_W = \frac{N_{Norm}(f_+,z) - N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}}$$

4. Estimate the temperature and wind using the calibration curves computed from physics

Main Process Step 2: Bin-by-Bin Procedure

5. Calculate the effective cross section using temperature and wind derived

6. Using the effective cross-section and Tc = 1 (at the bottom), calculate the Na density.

$$n_c(z) = \left[\frac{N_S(\lambda,z) - N_B}{N_R(\lambda,z_R) - N_B} \cdot \frac{z^2}{z_R^{\ 2}} \frac{1}{T_c^{\ 2}(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)} \right] \cdot \frac{4\pi\sigma_R(\pi,\lambda) n_R(z_R)}{\sigma_{eff}(\lambda) R_B(\lambda)}$$

7. From effective cross-section and Na density, calculate the extinction for the next bin.

$$T_c(\lambda,z) = \exp\left(-\int_{z_{bottom}}^{z} \sigma_{eff}(\lambda,z) n_c(z) \mathrm{d}z\right) = \exp\left(-\sum_{z_{bottom}}^{z} \sigma_{eff}(\lambda,z) n_c(z) \Delta z\right)$$

Na Density Derivation

- The Na density can be inferred from the peak freq signal

$$n_{Na}(z)=\frac{N_{norm}(f_a,z)}{\sigma_a}4\pi n_R(z_R)\sigma_R=\frac{N_{norm}(f_a,z)}{\sigma_a}4\pi\times 2.938\times 10^{-32}\frac{P(z_R)}{T(z_R)}\cdot\frac{1}{\lambda^{4.0117}}$$

- The Na density can be inferred from a weighted average of all three frequency signals.
- The weighted effective cross-section is

$$\sigma_{eff\_wgt}=\sigma_a+\alpha\sigma_+ +\beta\sigma_-$$

where α and β are chosen so that

$$\frac{\partial\sigma_{eff\_wgt}}{\partial T}=0;\qquad \frac{\partial\sigma_{eff\_wgt}}{\partial v_R}=0$$

- The Na density is then calculated by

$$n_{Na}(z)=4\pi n_R(z_R)\sigma_R\frac{N_{norm}(f_a,z)+\alpha N_{norm}(f_+,z)+\beta N_{norm}(f_-,z)}{\sigma_a+\alpha\sigma_+ +\beta\sigma_-}$$

Process Procedure for Deriving β

Read Data File

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PMT Saturation Correction

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Chopper Correction

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Subtract Background

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Remove Range Dependence (x R^2)

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Add Base Altitude

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Take Rayleigh Signal @ z_R km

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Normalize Profile By Rayleigh Signal @ z_R km

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Load Atmosphere n_R, T, P Profiles from MSIS00

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Calculate $R(z)$ and $\beta(z)$

↓

Smooth $R(z)$ and $\beta(z)$ By Hamming Window FWHM = 250 m

↓

Calculate z_c, σ_{rms}, β_{total}

Process Procedure for β of PMC

Load Atmosphere n_R, T, P Profiles from MSIS00

↓

Calculate $R(z)$ and $\beta(z)$

↓

Smooth $R(z)$ and $\beta(z)$ By Hamming Window FWHM = 250 m

↓

Calculate z_c, σ_{rms}, β_{total}

$$R = \frac{[N_S(z) - N_B]\cdot z^2}{[N_S(z_{RN}) - N_B]\cdot z_{RN}^2}\cdot\frac{n_R(z_{RN})}{n_R(z)}$$

$$\beta_{PMC}(z) = \left[\frac{[N_S(z) - N_B]\cdot z^2}{[N_S(z_{RN}) - N_B]\cdot z_{RN}^2} - \frac{n_R(z)}{n_R(z_{RN})}\right]\cdot\beta_R(z_{RN})$$

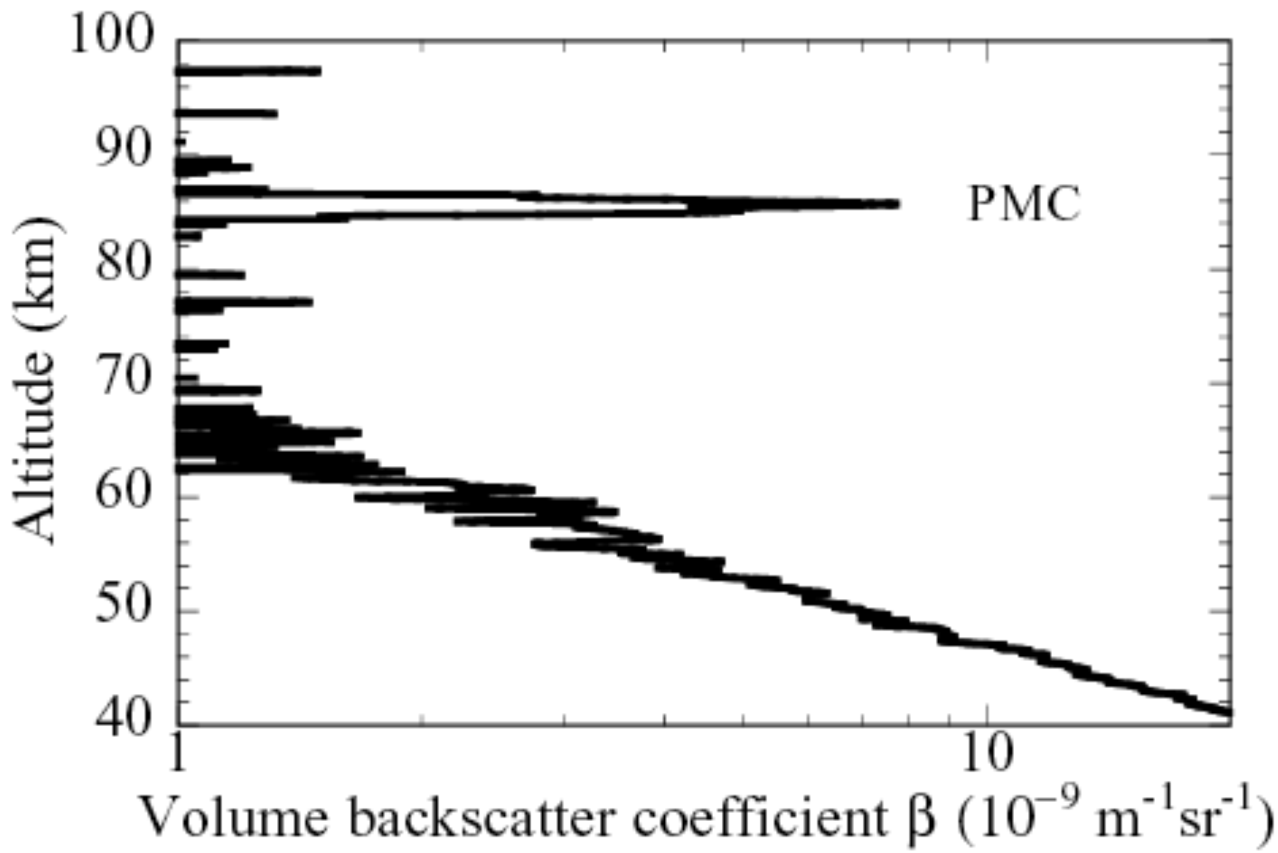
$$\beta_R(z_{RN},\pi) = \frac{\beta}{4\pi}P(\pi) = 2.938\times 10^{-32}\frac{P(z_{RN})}{T(z_{RN})}\cdot\frac{1}{\lambda^{4.0117}}$$

$$z_c = \frac{\sum_i \beta_{PMC}(z_i)\cdot z_i}{\sum_i \beta_{PMC}(z_i)}$$

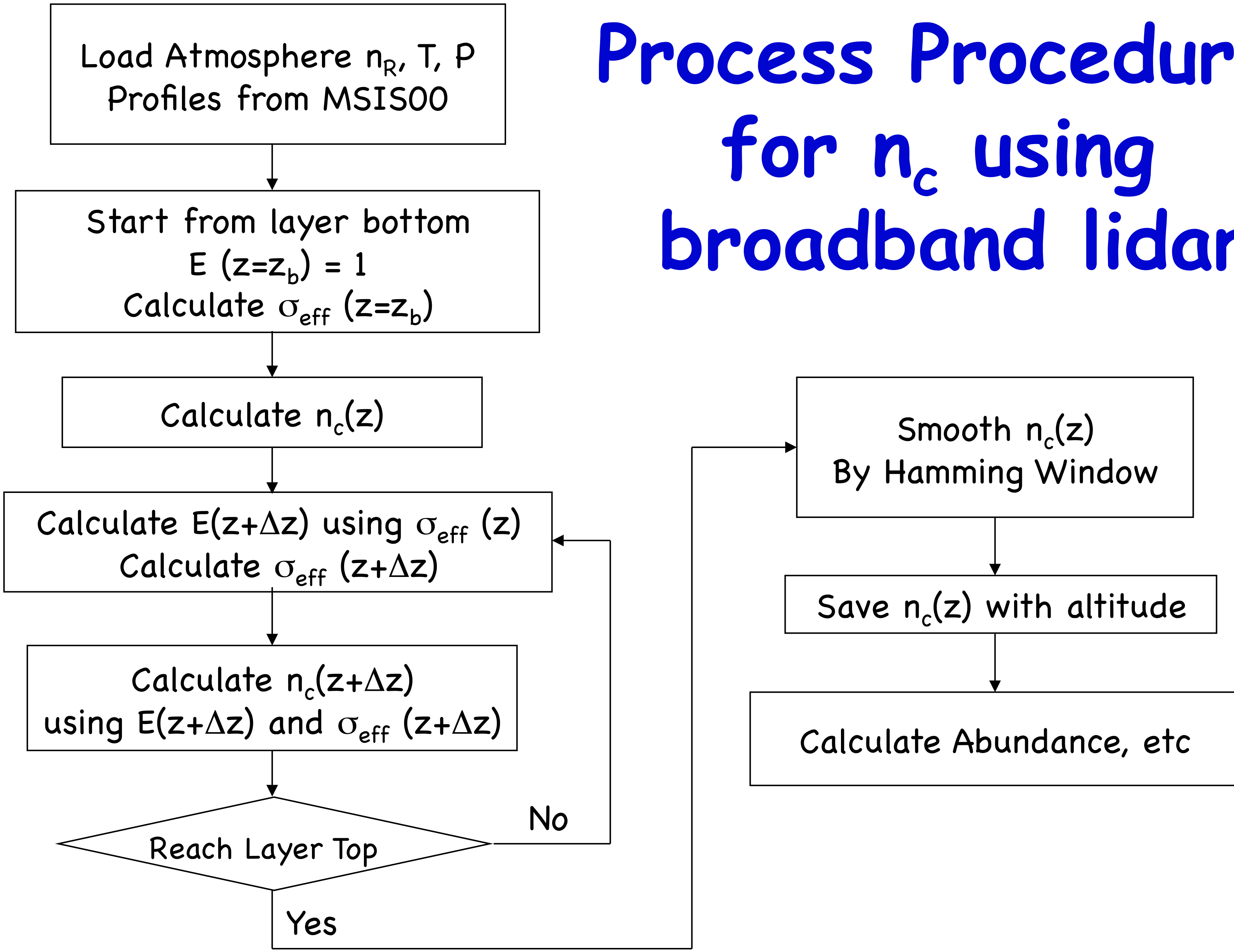
$$\sigma_{rms} = \sqrt{\frac{\sum_i (z_i - z_c)^2\beta_{PMC}(z_i)}{\sum_i \beta_{PMC}(z_i)}}$$

$$\beta_{total} = \int \beta_{PMC}(z)dz$$

Example Result: South Pole PMC



Process Procedure for n_c using broadband lidar



Process Procedure for n_c

- Computation of effective cross-section (concerning laser shape, assuming nominal T and W)
- Spatial resolution - binning or smoothing
- temporal resolution - integration

-- in order to improve SNR

- Extinction coefficient
- Calculate density
- Calculate abundance, peak altitude, etc.
- Show Na lidar data as examples in class

To Improve SNR

- In order to improve signal-to-noise ratio (SNR), we have to sacrifice spatial and/or temporal resolutions.
- Spatial resolution
  - binning
  - smoothing
- temporal resolution
  - integration

Summary

- The pre-process and profile-process are to convert the raw photon counts to corrected and normalized photon counts in consideration of hardware properties and limitations.
- The main process of T and V_R is to convert the normalized photon counts to T and V_R through iteration or looking-up table methods.
- The main process of n_c or β is to convert the normalized photon counts to number density or volume backscatter coefficient, in combination with prior acquired knowledge or model knowledge of certain atmosphere information or atomic/molecular spectroscopy.
- Data inversion procedure consists of the following processes:

  (1) pre- and profile-process,

  (2) process of T and V_R,

  (3) process of n_c and β, etc.