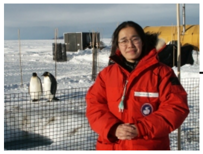
Lecture 09. Lidar Simulation and Error Analysis Overview

- Overview of Lidar Simulation and Error Analysis
- Range-resolved lidar simulation procedure
- Error analysis and photon noise
- Summary



Lidar Simulation and Error Analysis

- Why are lidar simulation and error analysis necessary?

- Analogy to atmospheric modeling which is a complex code to integrate everything together to see what happens in a complicated system like the atmosphere. The basis is the atmospheric science theory.

- Lidar remote sensing is a complicated procedure with many factors involved (both human-controllable and non-controllable). It is difficult to imagine what happens just from lidar theory (lidar equation). So we want to write a computer code to integrate all factors together in a proper way based on the lidar theory. By doing so we can investigate what the lidar outcome is supposed to be and how the outcome changes with the factors.

- What are the lidar simulation and error analysis?

- The lidar simulation and error analysis are lidar modeling to integrate complicated lidar remote sensing processes together.

- The basis for lidar simulation and error analysis is the lidar theory, spectroscopy, and measurement principles.

Lidar Simulation and Error Analysis

- Practice of lidar simulation and error analysis help deepen our understanding of every process, especially physical processes, in the entire lidar detection procedure.
- It also helps us understand how each factor in the procedure contributes to the signal strength and the measurement error (accuracy, precision, and resolution).
- Four Major Goals of Lidar Simulation and Error Analysis

1. Estimate of expected lidar returns (signal level and shape)
2. Analysis of expected measurement precision & resolution, i.e., errors (uncertainty) introduced by photon noise and trade-off among lidar parameters (e.g., off-zenith angle determination)
3. Analysis of expected measurement accuracy & precision (caused by uncertainty in system parameters) and lidar measurement sensitivity to atomic, lidar, and atmospheric parameters
4. Forward model to test data retrieval code and metrics

Lidar Simulation and Error Analysis

- Merits, Functions, or Applications of each aspect

1. Estimate of expected lidar returns (signal level and shape)

- Show what signals you can expect on real systems;
- Assess the potential of a lidar system;
- Comparison to actual signals to help diagnose lidar efficiency and other system problems, including reality check

2. Analysis of expected measurement precision & resolution, i.e., errors (uncertainty) introduced by photon noise

- Help determine needed laser power, receiver aperture, system efficiency, filter width, FOV, etc
- Help determine whether daytime measurement is doable
- Trade-off between resolution and precision
- Determination of optimum lidar parameters

Lidar Simulation and Error Analysis

- Merits, Functions, or Applications of each aspect

3. Analysis of expected measurement accuracy & precision (caused by uncertainty in system parameters) and lidar measurement sensitivity to atomic, lidar, and atmospheric parameters

- Help define requirements on lidar system parameters, like frequency accuracy, linewidth, stability, etc.
- Provide a guideline to system development
- Help determine measurement accuracy (bias)

4. Forward model to test data retrieval code and metrics

- Test data retrieval code and its sensitivity to noise
- Help compare different metrics to minimize cross-talk between different measurement errors

Lidar Simulation Build-up

- Analogy to atmospheric modeling, it is not practical to make a lidar simulation code complete for the first try, because so many things are involved. Therefore, we will build up a lidar simulation code step by step.

- First, we set up a platform using MatLab: gather needed information, set up necessary variables, and put these parameters into right places. For some parameters, you may use a place-holder before getting into the details.

- Then, we begin the simulation based on lidar equations and particular lidar procedure of each application. For commonly used or complicated parts, you may write 'functions' for each individual function, and then call them from the main code.

- The more we understand the lidar detection procedure and the more factors we consider, the more sophisticated will the lidar simulation code become.

Lidar Simulation Levels or Layers

- There are many different levels or layers of lidar simulation, depending on what we care about and how complicated the lidar detection procedure is. Three major levels are

1. Envelope estimation (non-range-resolved): integrated photon returns from an entire layer or region
2. Range-resolved simulation: photon returns from different ranges
3. Range-resolved and spectral-resolved simulation: photon returns from different ranges and distribution in spectrum at each range

- Other factors to consider:

1) Background (MODTRAN code is a good resource),

2) Noise (Poisson distribution),

3) Geometry,

4) Polarization,

5) Precise computation of backscatter cross section

6) ……

Review Envelope Estimate

- What we did on Friday was to estimate the return photon counts from the entire K layers using lidar equation and lidar/atomic/atmospheric parameters.
- 1st, write down all fundamental constants used in lidar.
- 2nd, gather lidar, atomic/molecular & atmosphere parameters.
- 3rd, start with the laser source of transmitter and follow the lidar picture from transmitted photons, through atmosphere transmission, backscatter probability, collection probability, and receiver efficiency, to detected photon numbers.
- 4th, understand the physical process of light interaction with objects to calculate the backscatter probability.
- 5th, background estimate considering many factors (both atmosphere conditions and lidar parameters like filter, FOV, …)
- 6th, get the final results and verify them with reality.

Fundamental Constants, Lidar Parameters, Atomic or Molecular Parameters, Atmospheric Parameters

- Always use NIST latest fundamental constants
- Try to use NIST atomic and molecular parameters
- Gather all possible lidar parameters
- Gather possible atmospheric parameters
- Another possible way is to scale from existing lidar measurements

Envelope Estimate Procedure

$$N_L(\lambda_L) = \frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L} = \frac{E_{pulse}}{hc/\lambda_L}$$

$N_L = 3.88 \times 10^{17}$

$$N_{Trans} = N_L \cdot R_{Tmirror} \cdot T_{atmos}$$

$N_{Trans} = 3.08 \times 10^{17}$

$$N_{Fluorescence} = N_{Trans} \cdot P_{scattering} = N_{Trans} \cdot \sigma_{eff} \cdot KAbdn$$

$N_{Fluorescene} = 1.85 \times 10^{14}$

$$N_{Sphere} = N_{Fluorescence} \cdot T_{atmos}$$

$N_{Sphere} = 1.48 \times 10^{14}$

$$N_{Primary} = N_{Sphere} \cdot P_{collection} = N_{Sphere} \cdot \frac{A}{4\pi R^2}$$

$N_{Sphere} = 730.8$

$$\eta_{receiver} = R_{primary} \cdot \eta_{fiber} \cdot T_{Rmirror} \cdot T_{IF} \cdot QE$$

$\eta_{receiver} = 6.06\%$

$$N_{S(K)} = N_{primary} \cdot \eta_{receiver}$$

$N_{S(K)} = 44.3$

Comparison to reality: 10-50 count/shot

Range-Resolved Lidar Simulation

- Three main steps:

(1) Initialization

- Define constants and parameters

(2) Simulation of photon counts vs. altitude (range)

- Rayleigh, resonance fluorescence, background, aerosol, noise

(3) Computation of signal-to-noise ratio (SNR)

- SNR will be useful in the error analysis

Initialization

- Initialization: to define constants and parameters

1) Fundamental constants: c, h, Q_e, M_e, …
2) Atomic parameters: resonance wavelengths, frequencies, strength, oscillator strength, A_{ki}, degeneracy factors, isotopes, abundance, …

or Molecular parameters: CO_2 structure, etc.

3) Lidar transmitter and receiver parameters: pulse energy, linewidth, frequency, repetition rate, telescope diameter, R, T, η_{QE}, T_{IF}, …
4) User-controlled parameters: integration time (shots number), bin width ΔR, pointing up or down, base altitude, pointing angle, model choice, …
5) Atmospheric parameters (taken from models, e.g., MSIS00): number density, pressure, temperature, transmission, …
6) Na/K/Fe layer parameters: distribution, Z_0, σ_{rms}, Abdn, …

Simulation of N(R) or N(z)

- Photon counts vs. altitude/range: Sum of the following terms

1) Rayleigh scattering signals: take atmosphere number density distribution profile n(z) from a model, e.g., MSIS00, set bin width Δz (e.g., 0.5 km), compute $\sigma_R \cdot n(z) \cdot \Delta z$ or further considering the pointing angle, then follow the normal simulation procedure for each bin
2) Resonance fluorescence signals: compute the Na/K/Fe density distribution profile from Z_0, σ_{rms}, Abdn, compute effective cross-section σ_{eff} from atomic and laser spectroscopy, compute $\sigma_{eff} \cdot n_{Na}(z) \cdot \Delta z$ or further considering the pointing angle, consider the transmission (extinction) caused by atomic absorption, follow the normal simulation procedure for each bin
3) Aerosol signals: usually in specific regions. It may involve complicated procedure and information, seeking models or existing data
4) Background counts: scale from real measurements or use MODTRAN code to compute background (needing lidar parameters, like FOV, filter function, etc., also solar and atmosphere information)
5) Noise: Poisson distribution from photon counts $\Delta N(z) = \sqrt{N(z)}$

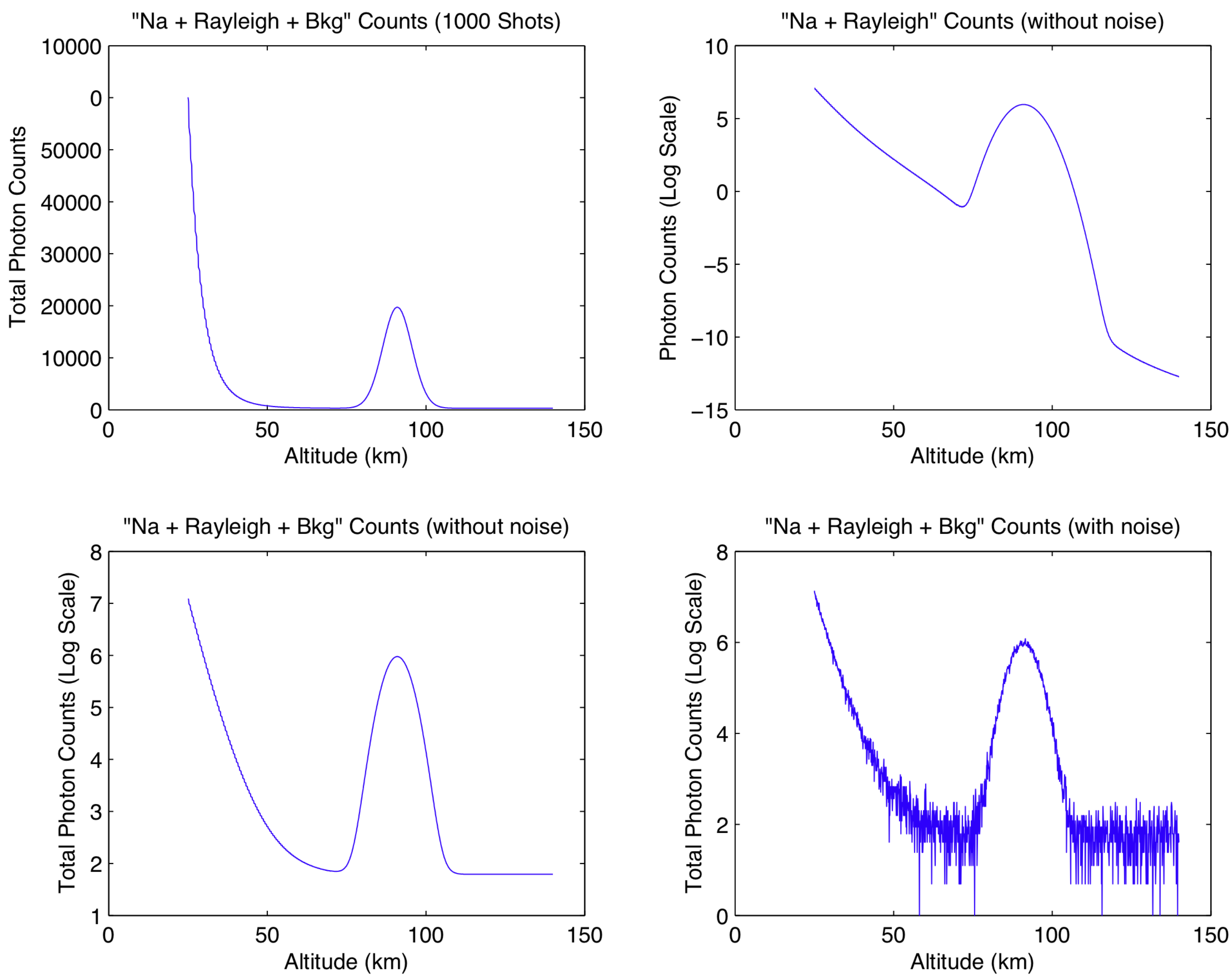
Simulation of N(R) in non-resonance

- What to do if a lidar is to study non-resonance interactions?

1) Rayleigh scattering signals: there will always be Rayleigh signals no matter what lidar is considered
2) Non-resonance fluorescence: Raman, differential absorption, fluorescence, aerosol, or reflection signals - need to get some rough distribution, cross-section, reflectivity, etc. information
3) Aerosol signals: aerosols play a role in both backscatter coefficient and atmospheric extinction, also need some distribution information - could be what to be detected or just as background or noise
4) Background counts: there will always be some background
5) Noise: It is always there, so needs to be counted in.

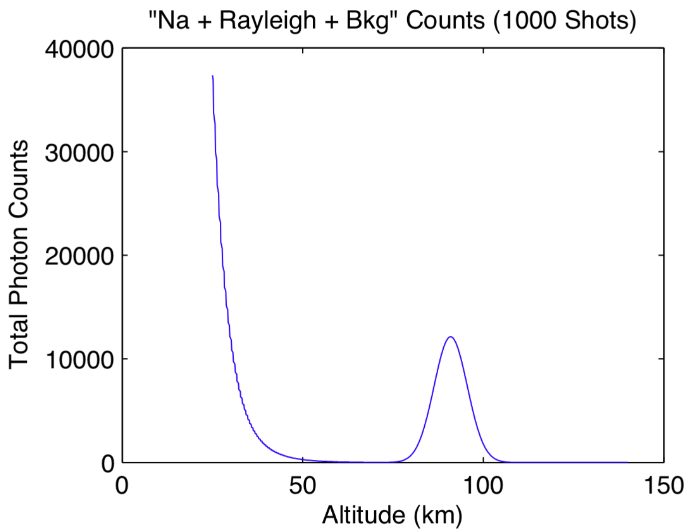
Example 1: Low Background – Nighttime

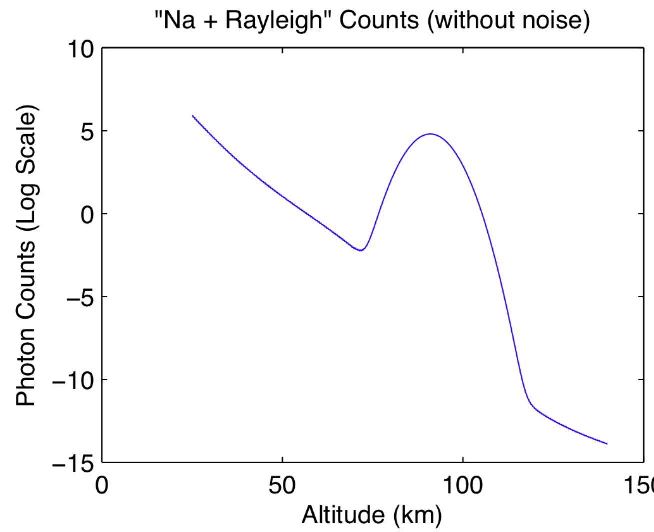
Resonance + Rayleigh + Background + Photon Noise

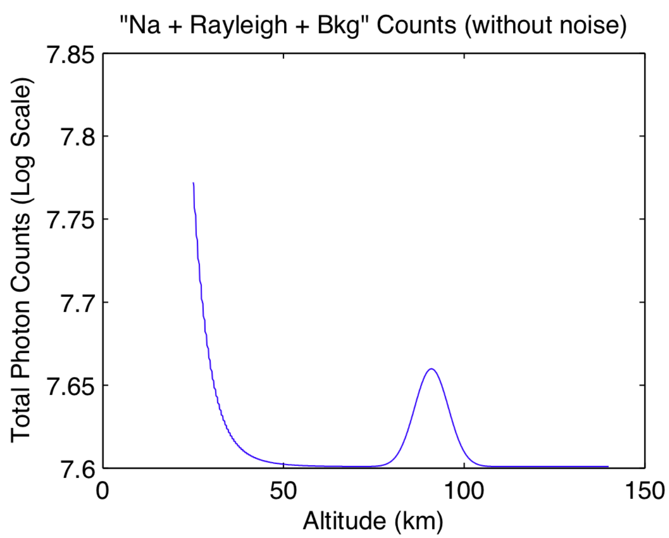


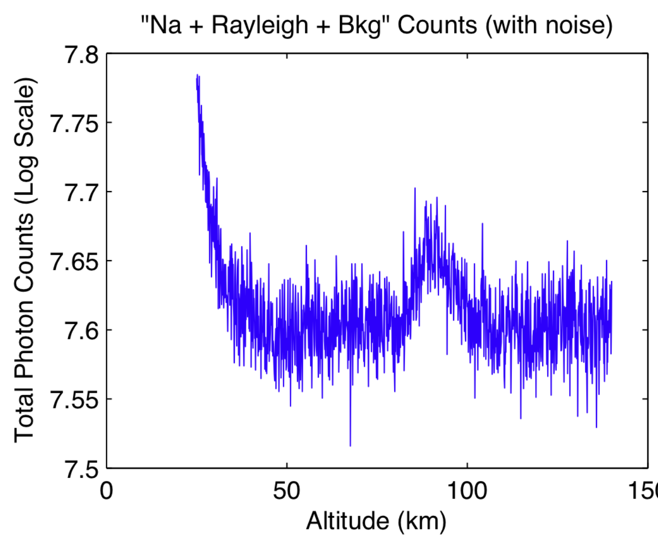
Example 2: High Background – Daytime

Resonance + Rayleigh + Background + Photon Noise

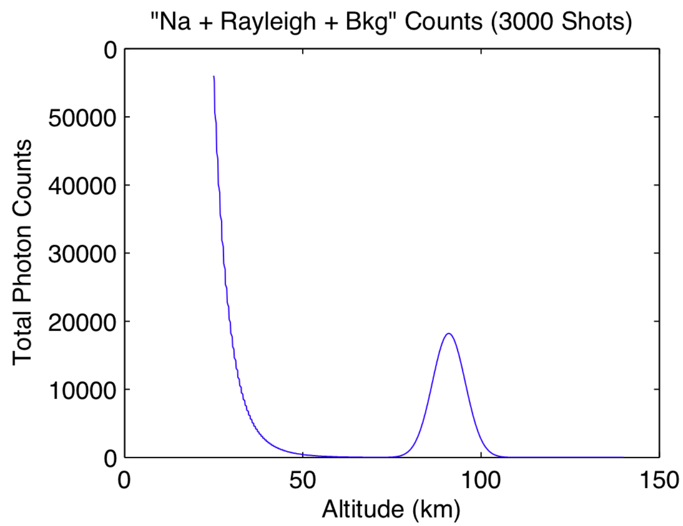


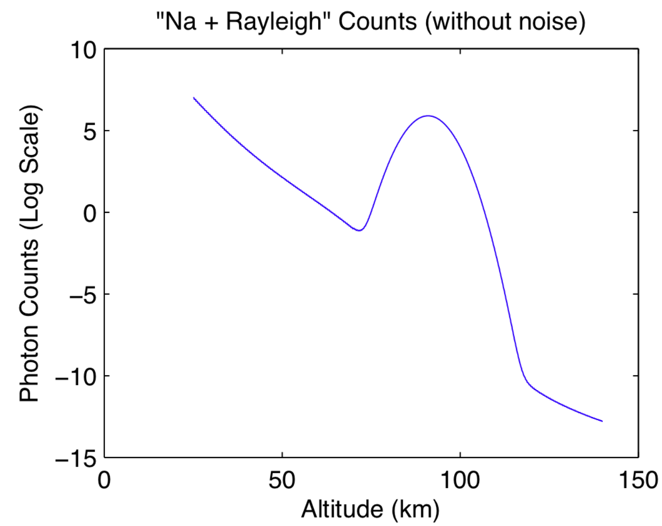


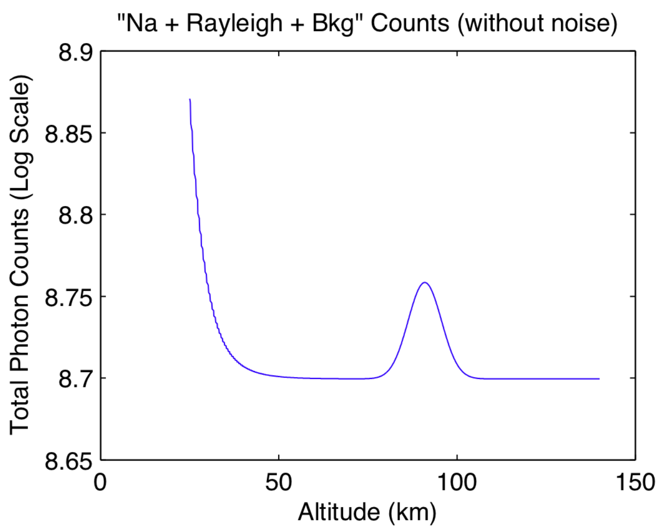


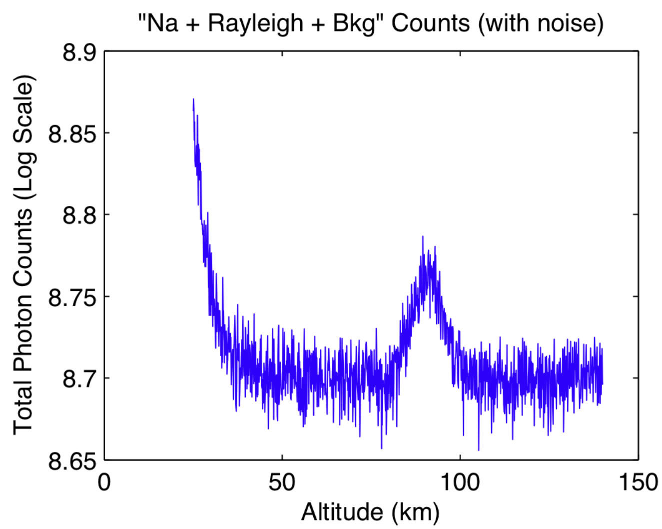


Example 3: High Background – Daytime
with Longer Integration Time









Signal-to-Noise Ratio (SNR)

- Computation of signal-to-noise ratio (SNR) from simulation results

$$SNR(z) = \frac{N_{Signal}(z)}{\Delta N_{Signal}(z)}$$

- However, we must consider how the N(z) is obtained

$$N_{Signal}(z) = N_{Total}(z) - N_B$$

$$\Delta N_{Signal}(z) = \Delta N_{Total}(z) - \Delta N_B$$

$$\left(\Delta N_{Signal}(z)\right)_{rms} = \sqrt{\left(\Delta N_{Total}(z)\right)^2 + \left(\Delta N_B\right)^2}$$

$$\left(\Delta N_{Signal}(z)\right)_{rms} = \sqrt{N_{Total}(z) + \left(\Delta N_B\right)^2} \approx \sqrt{N_{Total}(z)}$$

SNR Continued

$$\left(\Delta N_{Signal}(z)\right)_{rms} = \sqrt{N_{Total}(z) + \left(\Delta N_B\right)^2} \approx \sqrt{N_{Total}(z)}$$

- The last approximate equal is obtained, considering the fact that the error associated with the estimate of N_B is negligible, because N_B is estimated from many bins in the background range where no Rayleigh, aerosol or resonance fluorescence signals are presented.

$$N_B = \frac{1}{m_B}\sum_{i=1}^{m_B} N_{Bi} \Rightarrow \left(\Delta N_B\right)_{rms} = \sqrt{\frac{\left(\Delta N_{Bi}\right)^2}{m_B}} = \frac{\left(\Delta N_{Bi}\right)_{rms}}{\sqrt{m_B}} << \left(\Delta N_{Bi}\right)_{rms}$$

- Thus, the SNR is given by

$$SNR(z) = \frac{N_{Signal}(z)}{\Delta N_{Signal}(z)} = \frac{N_T(z) - N_B}{\sqrt{N_T(z)}} = \begin{cases} \sqrt{N_T}, when \quad N_B \approx 0 \\ \sqrt{N_T} - \dfrac{N_B}{\sqrt{N_T}}, when \quad N_B >> 0 \end{cases}$$

Photon Noise vs. SNR and Error

- Photon counting undergoes a statistic of Poisson distribution.
- Any photon count has an error associated with it – the photon noise that is given by

$$\Delta N = \sqrt{N}$$

- If photon noise didn't exist, the high background count N_B wouldn't affect the SNR, as shown in the plots of "Na + Rayleigh + Bkg" Counts (without noise) on slides 15-17. A constant can be subtracted easily.
- Unfortunately, photon noise is unavoidable. The high background N_B introduces large photon noise, which exhibits as the "wider" width of the line in the raw photon count profiles (as shown in the plots of "Na + Rayleigh + Bkg" Counts (with noise) on slides 15-17.
- Thus, the SNR decreases with higher background N_B. The relationship between SNR and N_B is quantified in the equations derived on slides 18-19.
- Comparing the plot in slide 15 with that in slide 16, when the background is changed from nighttime 0.06 cnt/shot/km to daytime 20 cnts/shot/km, the SNR significantly decreases.

Errors Caused by Photon Noise

- Further comparing the plot in slide 17 with that in slide 16, it is clear that the longer integration time (3000 shots vs. 1000 shots) improves the SNR by increasing the photon count N_T while keeping the same N_B.
- The SNR can also be improved by sacrificing the range resolution.
- Photon noise causes the uncertainty (photon noise) in the measured photon counts, then the photon count uncertainty leads to the uncertainty (error) in the physical quantities measured by lidar.

-- Error propagation procedure

$$\frac{\beta(z)}{\beta_R(z_R)} = \frac{N_S(z) - N_B}{N_R(z_R) - N_B} \cdot \frac{z^2}{z_R^2} \cdot \frac{T_a^2(z_R)G(z_R)}{T_a^2(z)G(z)} = \frac{N_{T(S)}(z) - N_B}{N_{T(R)}(z_R) - N_B} \cdot \frac{z^2}{z_R^2}$$

- Considering the errors associated with the estimates of N_B and N_R are negligible, the relative error of the estimate of backscatter coefficient is given by

$$\frac{\Delta\beta(z)}{\beta(z)} = \frac{\Delta N_T(z)}{N_T(z) - N_B} \approx \frac{1}{SNR}$$

- Thus, higher SNR leads to smaller measurement error.

Summary

- Lidar simulation and error analysis are an integral of our understanding of lidar principle, technology and actual application procedure.

- It provides a model to investigate how the lidar returns depend on different parameters and how measurement accuracy, precision, and resolution depend on different parameters.

- It can be used to verify lidar return shapes, assess lidar potentials, guide lidar design and instrumentation, test data retrieval code and metrics, etc.

- A complete lidar simulation and error analysis code is very complicated. We will go step by step to achieve it.