Lecture 08. Lidar Simulation - Application of Lidar Equation

- Review lidar fundamentals
- How to start lidar simulation?
- Parameters used in lidar simulation
- Lidar envelope estimate procedure
- Simulation of resonance fluorescence return
- Simulation of Rayleigh scattering return
- Summary

Review Lidar Equation

- Lidar equation is to link the expected lidar returns (N_S) to the lidar parameters (both transmitter and receiver), transmission through medium, physical interactions between light and objects, and background/noise conditions, etc.

- Keep in mind the big picture of a lidar system –
  - Radiation source
  - Radiation propagation in the medium
  - Interaction with the objects
  - Signal propagation in the medium
  - Photons are collected and detected

Can you derive a lidar equation by yourself?

Various Forms of Lidar Equations

- General lidar equation with angular scattering coefficient

$$N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

- General lidar equation with total scattering coefficient

$$N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta_T(\lambda,\lambda_L,R)\Delta R\right]\cdot\frac{A}{4\pi R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

- Lidar equation for Rayleigh lidar

$$N_S(\lambda,R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\beta(\lambda,R)\Delta R\right)\left(\frac{A}{R^2}\right)T^2(\lambda,R)\left(\eta(\lambda)G(R)\right)+N_B$$

- Lidar equation for resonance fluorescence lidar

$$N_S(\lambda,R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda,R)n_c(z)R_B(\lambda)\Delta R\right)\left(\frac{A}{4\pi R^2}\right)\left(T_a^2(\lambda,R)T_c^2(\lambda,R)\right)\left(\eta(\lambda)G(R)\right)+N_B$$

How to Start Lidar Simulation?

- Lidar simulation of return signals is a direct application of lidar equation involving physical processes.
- Let us start with the Arecibo K Doppler lidar as an example.
- 1st, write down all fundamental constants used in lidar.
- 2nd, gather lidar, atomic/molecular & atmosphere parameters.
- 3rd, start with the laser source of transmitter and follow the lidar picture from transmitted photons, through atmosphere transmission, backscatter probability, collection probability, and receiver efficiency, to detected photon numbers.
- 4th, understand the physical process of light interaction with objects to calculate the backscatter probability.
- 5th, background estimate considering many factors (both atmosphere conditions and lidar parameters like filter, FOV, ...)
- 6th, get the final results and verify them with reality.

Fundamental Constants in Lidar

- Fundamental constants for lidar simulation

1) Light speed in vacuum c
2) Planck constant h
3) Boltzmann constant k_B
4) Elementary charge e
5) Electron mass m_e
6) Proton mass m_p
7) Electric constant ε_0
8) Magnetic constant μ_0
9) Avogadro constant N_A
10) ……

Lidar Parameters

- Lidar parameters for lidar simulation

1) Laser pulse energy, repetition rate, pulse duration,
2) Laser central wavelength, linewidth, chirp
3) Laser divergence angle
4) Transmitter mirror reflectivity
5) Telescope primary mirror diameter and reflectivity
6) Telescope/receiver field-of-view (FOV)
7) Receiver mirrors' transmission,
8) Fiber coupling efficiency, transmission
9) Filter peak transmission, bandwidth, out-of-band rejection
10) Detector quantum efficiency and maximum count rate
11) ……

Atomic or Molecular Parameters

- Atomic and molecular parameters for lidar simulation

1) Atomic energy level structure, degeneracy
2) Spontaneous transition rate A_{ki}, oscillator strength f
3) Atomic mass or molecular weight
4) Resonance frequency or wavelength
5) Isotope shift, abundance, line strength
6) ……

Atmosphere Parameters

- Atmosphere parameters for lidar simulation

1) Lower atmosphere transmission
2) Atmosphere number density
3) Atmosphere pressure and temperature
4) Species number density or column abundance
5) Background sky radiance, solar angle, base altitude, etc.
6) ……

Levels of Lidar Simulation

- There are many different levels/steps of lidar simulation, depending on what we care about and how complicated the lidar detection procedure is. Three major levels are

1. Envelope estimation (non-range-resolved): integrated photon returns from an entire layer or region
2. Range-resolved simulation: range-dependent photon returns from different ranges
3. Range-resolved and spectral-resolved simulation: photon returns from different ranges and distribution in spectrum at each range.

- Envelope estimate is discussed in this lecture and range-resolved simulation will be studied in next lecture.

Arecibo K Lidar Estimate

- We want to use the fundamental lidar equation to estimate the detected photon counts of K signals returned from the entire K layers using the Arecibo K lidar parameters.
- This is the first step (envelope estimate) of lidar simulations to assess a lidar potential and system performance. It provides an idea of what performance can be expected.
- Let us start with the general lidar equation

$$\boxed{N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B}$$

- Resonance fluorescence lidar uses the lidar equation

$$\boxed{N_S(\lambda,R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda,R)n_c(z)R_B(\lambda)\Delta R\right)\left(\frac{A}{4\pi R^2}\right)\left(T_a^2(\lambda,R)T_c^2(\lambda,R)\right)\left(\eta(\lambda)G(R)\right)+N_B}$$

Lidar Estimate Procedure (1)

- First, estimate the transmitted laser photon numbers for single lidar pulse

$$N_L(\lambda_L) = \frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L} = \frac{E_{pulse}}{hc/\lambda_L}$$

- Arecibo K Doppler lidar parameters:
  Laser pulse energy: E_{pulse} = 100 mJ
  Laser central wavelength: λ_L = 770.1088 nm
- h is Planck constant and c is light speed

- Therefore, a single lidar pulse sends out photons of

$N_L = 3.88 \times 10^{17}$

Lidar Estimate Procedure (2)

- Second, consider the transmitter steering mirror reflectivity and atmosphere transmission, and estimate the number of laser photons that reach K layers

$$\boxed{N_{Trans} = N_L \cdot R_{Tmirror} \cdot T_{atmos}}$$

- Arecibo K Doppler lidar parameters:

  Transmitter mirrors: 3 mirrors @ R = 99.8%

  ⇨ $R_{tmirror} = (0.998)^3 = 0.994$

- Lower atmosphere transmission at 770 nm:

  $T_{atmos} = 80\%$

- Therefore, the number of photons reaching K layers

  $N_{Trans} = 3.08 \times 10^{17}$

Lidar Estimate Procedure (3)

- Third, consider the absorption and spontaneous emission procedure, estimate scattering probability and estimate the number of resonance fluorescence photons produced by entire K layers (ignoring extinction in K)

$$N_{Fluorescence} = N_{Trans} \cdot P_{scattering} = N_{Trans} \cdot \sigma_{eff} \cdot KAbdn$$

- Peak effective cross-section of K D_{1a} line:

$$\sigma_{eff} = 10 \times 10^{-16}\ m^2$$

- K layer column abundance:

$$KAbdn = 6 \times 10^{7}\ cm^{-2} = 6 \times 10^{11}\ m^{-2}$$

- The scattering probability is given by:

$$P_{scattering} = \sigma_{eff} \times Kabdn = 6 \times 10^{-4}$$

- Therefore, the number of fluorescence photons

$$N_{Fluorescene} = 1.85 \times 10^{14}$$

Lidar Estimate Procedure (4)

- Fourth, consider the atmosphere transmission for return signals and estimate the number of fluorescence photons that reach the sphere surface at receiver range

$$\boxed{N_{Sphere} = N_{Fluorescence} \cdot T_{atmos}}$$

- Lower atmosphere transmission at 770 nm:

  $T_{atmos} = 80\%$
- Note: we ignore the extinction caused by K layers

- Thus, the number of photons reaching the sphere

  $N_{Sphere} = 1.48 \times 10^{14}$

Lidar Estimate Procedure (5)

- Fifth, consider the telescope primary mirror area, estimate the collection probability, and estimate the number of photons reaching the primary mirror

$$\boxed{N_{\mathrm{Pr}imary} = N_{Sphere} \cdot P_{collection} = N_{Sphere} \cdot \frac{A}{4\pi R^2}}$$

- Arecibo K lidar telescope: primary mirror diameter

  D = 80 cm ⇨ A = 0.50 m^2

- K layer centroid altitude:

  R = 90 km = 9 x 10^4 m

- The collection probability is given by:

  $P_{collection} = A/(4\pi R^2) = 4.94 \times 10^{-12}$

- Therefore, the number of photons reaching the primary mirror: $N_{Sphere} = 730.8$

Lidar Estimate Procedure (6)

- Sixth, estimate the receiver efficiency considering primary mirror reflectivity, collimating optics transmission, filter transmission, and PMT QE

$$\boxed{\eta_{receiver} = R_{primary} \cdot \eta_{fiber} \cdot T_{Rmirror} \cdot T_{IF} \cdot QE}$$

- Arecibo K lidar receiver parameters:
  - primary mirror reflectivity $R_{primary} = 91\%$
  - Fiber coupling efficiency $\eta_{fiber} = 75\%$
  - receiver mirror transmittance $T_{Rmirror} = 74\%$
  - Interference filter peak transmission $T_{IF} = 80\%$
  - PMT quantum efficiency $QE = 15\%$
- Therefore, the receiver efficiency is

$$\eta_{receiver} = 6.06\%$$

Lidar Estimate Procedure (7)

- Seventh, consider the receiver efficiency and estimate the number of photons detected by PMT

$$\boxed{N_{S(K)} = N_{primary} \cdot \eta_{receiver}}$$

- Using the results from steps 5th and 6th,

$$N_{S(K)} = 730.8 \times 6.06\% = 44.3$$

- Therefore, the number of photons detected by PMT, (i.e., the K lidar return signal counts), for each single lidar pulse from the entire K layers are

$$N_{S(K)} = 44.3$$

- Note: these photon counts originate from 3.88×10^{17} laser photons!!!

Comparison to Actual Lidar Return

- Typical lidar return signals of the Arecibo K Doppler lidar are about 10-50 counts per shot from the entire K layers, depending on seasons and atmosphere conditions.

- Our estimate is surprisingly close to the actual situation – K lidar people have tried their best to measure the system efficiencies precisely.

- From this estimate, how do you feel about the upper atmosphere lidar: What is the major killer of the signal strength?

Long range - weak signal !

Simulation of Resonance Fluorescence

- Besides common issues in lidar simulation, the main point in simulation of resonance fluorescence return is to correctly estimate the effective cross section and column abundance / density of these atomic species, e.g., K.
- Effective scattering cross section can be affected by laser central frequency, linewidth, saturation, optical pumping, branching ratio, isotopes, Hanle effect, etc.
- Correct estimate of this involves comprehensive understanding of the physical process and spectroscopy knowledge - This is why spectroscopy class is important!
- Column abundance and density vary with season, latitude, and are also affected by waves etc. Usually we use a mean column abundance as a representative for envelope estimate.

Effective Cross-Section for K Atoms

- Absorption cross section of K atom's D1 line is given by

$$\sigma_{abs}(\nu) = \sum_{A=39}^{41} \left\{ Abdn(A) \frac{1}{\sqrt{2\pi}\sigma_D} \frac{e^2 f}{4\varepsilon_0 m_e c} \sum_{n=1}^{4} A_n \exp\left(-\frac{[\nu_n - \nu(1 - V_R/c)]^2}{2\sigma_D^2} \right) \right\}$$

Isotope abundance: 93.2581% (^{39}K), 0.0117% (^{40}K), 6.7302% (^{41}K)
Line strength: An = 5/16, 1/16, 5/16, 5/16
Oscillator strength: f, Doppler broadening: σ_D

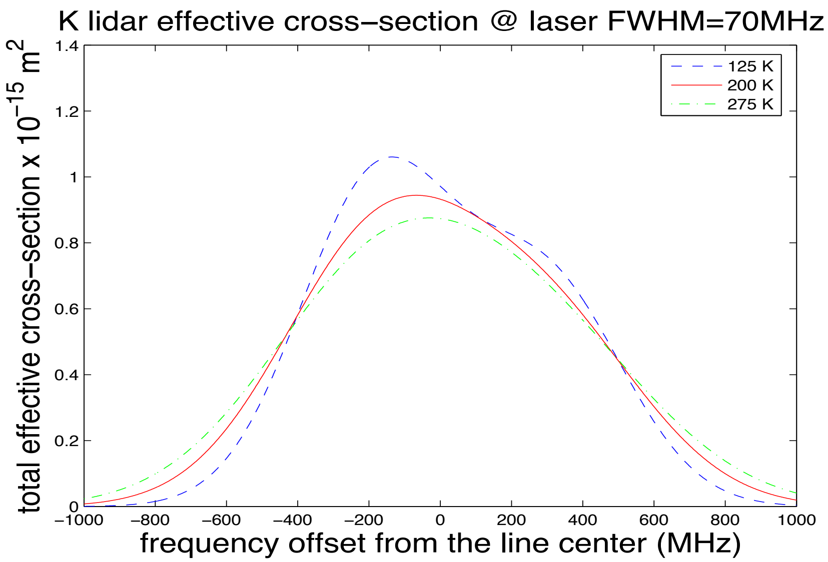
- The effective total scattering cross section of K atom's D1 line is the convolution of the absorption cross section and the laser lineshape. Under the assumption of Gaussian lineshape of the laser, it is given by

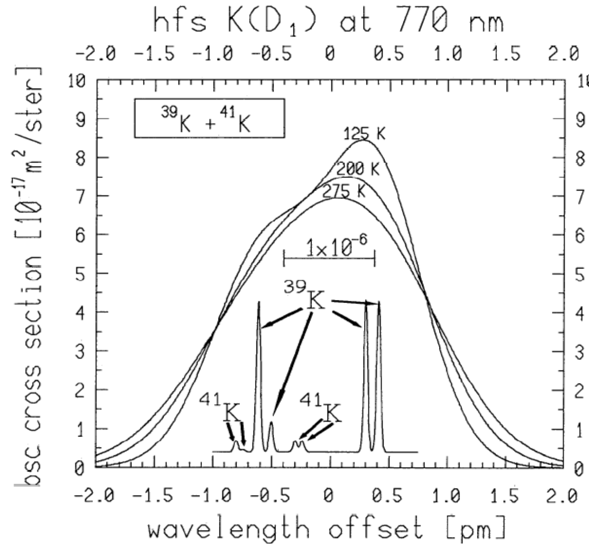
$$\sigma_{eff}(\nu) = \sum_{A=39}^{41} \left\{ Abdn(A) \frac{1}{\sqrt{2\pi}\sigma_e} \frac{e^2 f}{4\varepsilon_0 m_e c} \sum_{n=1}^{4} A_n \exp\left(-\frac{[\nu_n - \nu(1 - V_R/c)]^2}{2\sigma_e^2} \right) \right\}$$

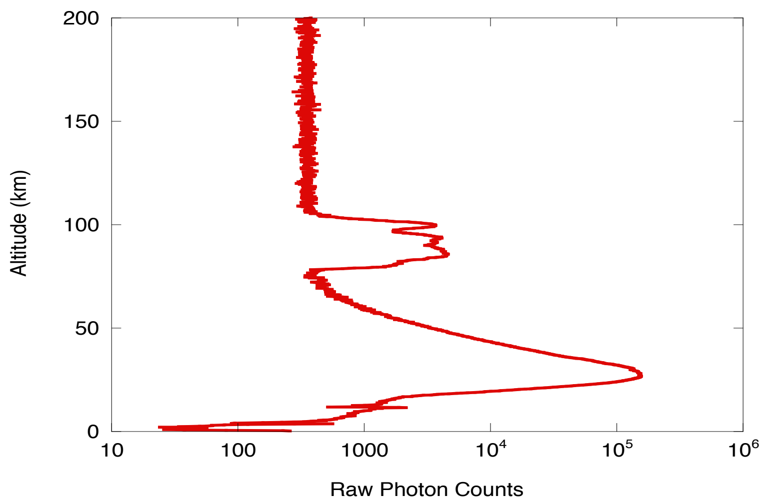
where $\sigma_e = \sqrt{\sigma_D^2 + \sigma_L^2}$ and $\sigma_D = \sqrt{\dfrac{k_B T}{M\lambda_0^2}}$

Refer to our textbook Chapter 5 and references therein

Effective Cross-Section for K Atoms







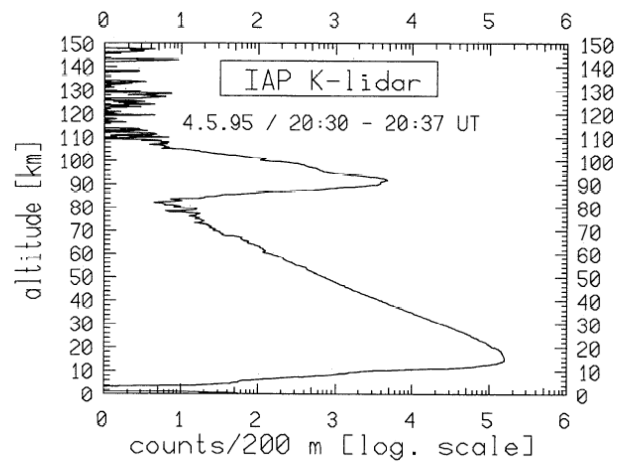


Figure 3: Count rate profile with 200 m altitude resolution (smoothed 1:2:4:2:1), background substracted, measured May 4, 1995. Integration time 7 min (= 10000 laser pulses).

[Friedman et al., JASTP, 2003] [von Zahn and Höffner, GRL, 1996]

Simulation of Rayleigh Return

- This is relatively simpler compared to resonance fluorescence, because Rayleigh scattering is straightforward.
- The key is to correctly estimate the Rayleigh backscatter cross section and atmosphere number density.
- Usually atmosphere number density can be taken from standard US atmosphere or MSIS model. MSIS number density varies with season and location (latitude).
- Estimate of atmosphere transmission or extinction is also very important. If dealing with lower atmosphere, it could be tricky as absorption and scattering by aerosols may interfere the lidar return signals.
- Rayleigh backscatter cross section can be calculated precisely using scattering theory. Refer to a book "Atmospheric Radiation" by Goody and Yung.

Rayleigh Backscatter Cross Section

- It is common in lidar field to calculate the Rayleigh backscatter cross section using the following equation

$$\frac{d\sigma_m(\lambda)}{d\Omega} = 5.45 \cdot \left(\frac{550}{\lambda}\right)^4 \times 10^{-32} \left(m^2 sr^{-1}\right)$$

where λ is the wavelength in nm.

- The Rayleigh backscatter cross section can also be estimated from the Rayleigh backscatter coefficient

$$\beta_{Rayleigh}(\lambda, z, \theta = \pi) = 2.938 \times 10^{-32} \frac{P(z)}{T(z)} \cdot \frac{1}{\lambda^{4.0117}} \left(m^{-1} sr^{-1}\right)$$

where λ is the wavelength in meter, P in mbar, T in Kelvin.

$$\therefore \frac{d\sigma_m(\lambda)}{d\Omega} = \frac{\beta_{Rayleigh}(\lambda, z, \pi)}{n_{atmos}(z)} \left(m^2 sr^{-1}\right)$$

Summary

- ❑ Lidar simulation is a direct application of lidar equation to practical problems.
- ❑ Our first step demonstrated in this lecture is an envelope estimate of the expected lidar returns from lidar parameters, transmittance through medium, physical interactions, etc. More functions of lidar simulation will be discussed in later lectures.
- ❑ The major difficulty in upper atmosphere lidar is the tiny collection efficiency (10^{-12}) caused by the long range (A/R^2) ⇨ weak signals.
- ❑ Receiver efficiency is another important factor that must be given careful considerations.

HWK Project #1 Assignment - Estimate of lidar returns
Due on Monday, February 14, 2011