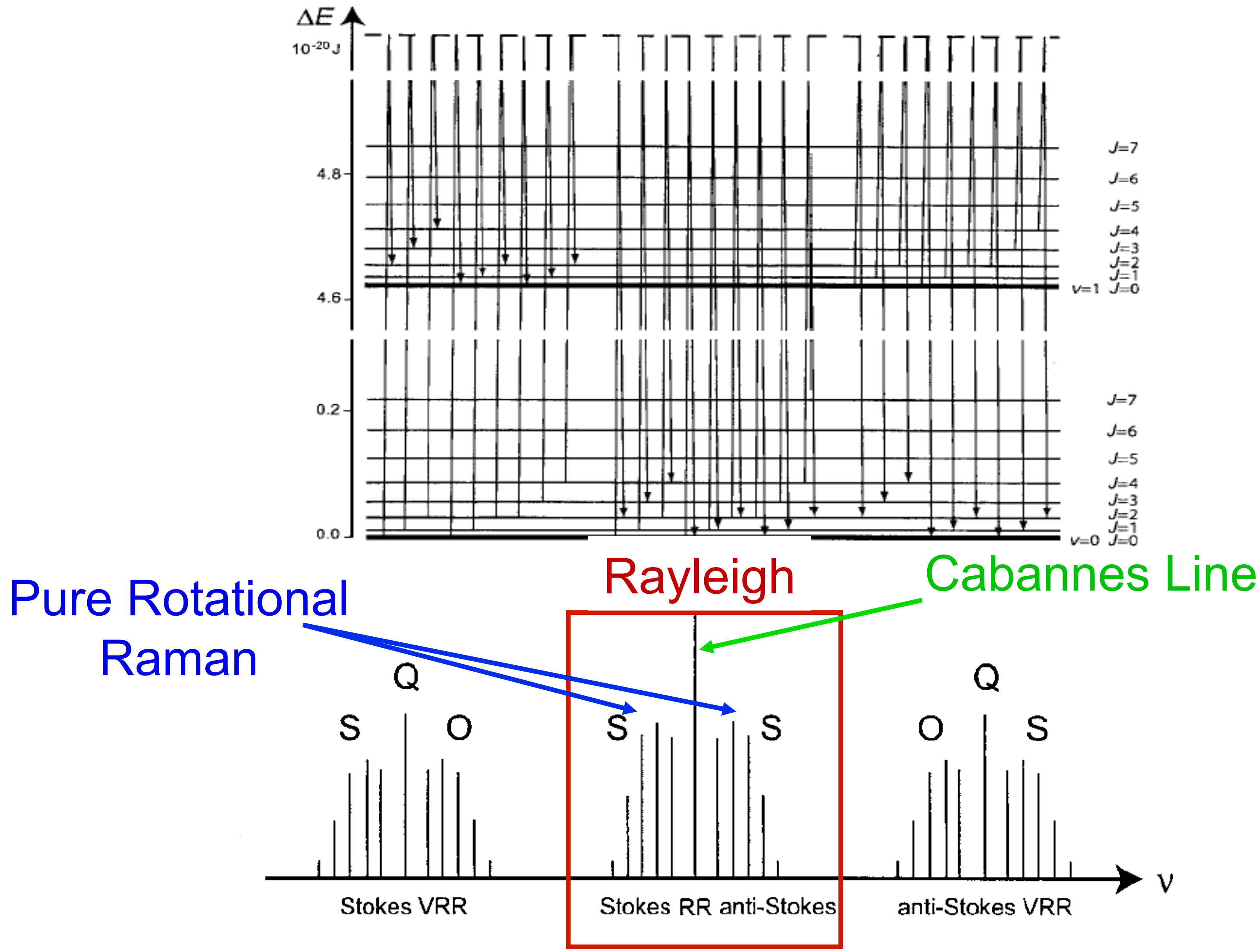
Lecture 06. Fundamentals of Lidar Remote Sensing (4)

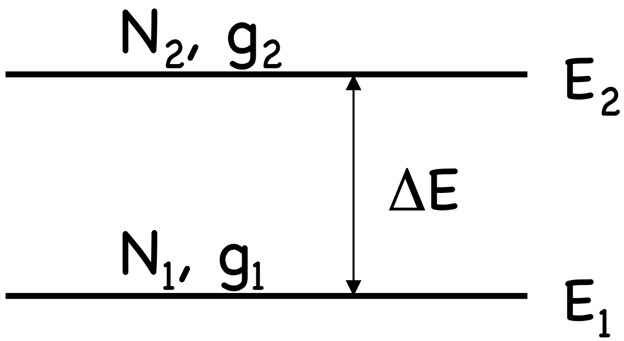
- Review physical processes in lidar equation
- Example calculation in physical processes
- Solution for scattering form lidar equation
- Solution for fluorescence form lidar equation
- Solution for differential absorption lidar equation
- Solution for resonance fluorescence lidar
- Solution for Rayleigh and Mie lidar
- Summary

Scattering by Molecules in Atmosphere



Boltzmann Distribution

- Maxwell-Boltzmann distribution is the law of particle population distribution according to energy levels (under thermodynamic equilibrium)



$$\boxed{\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\left\{-\left(E_2 - E_1\right)/k_B T\right\}}$$

$$\Downarrow$$

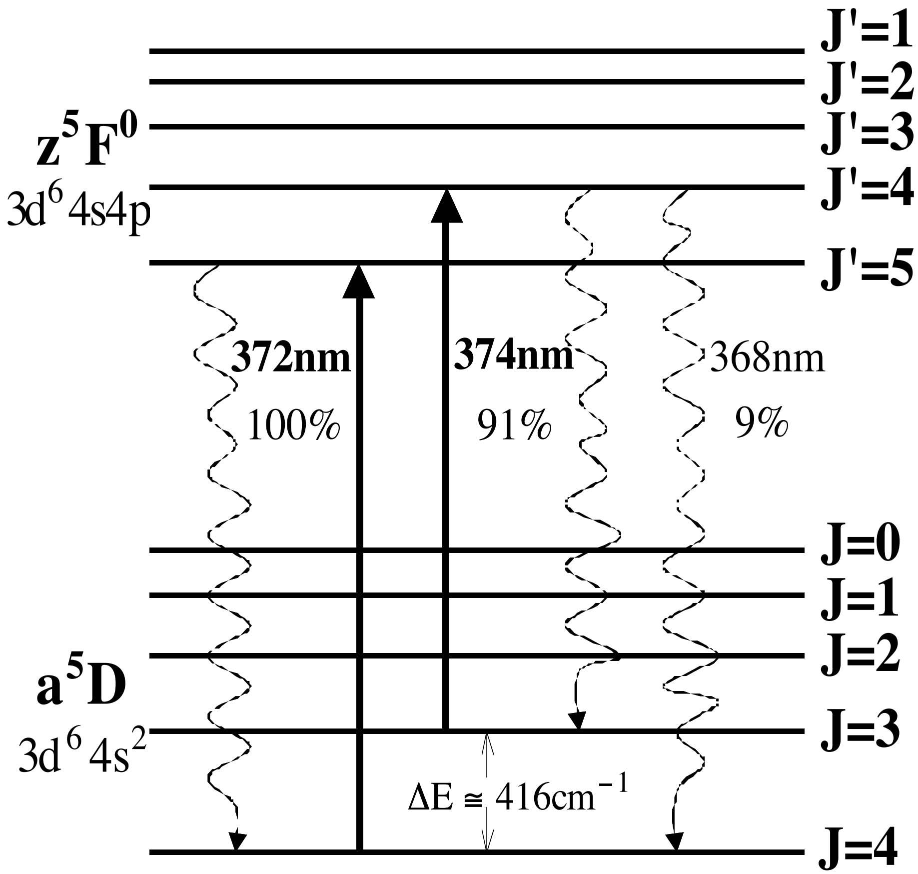
$$\boxed{T = \frac{\Delta E / k_B}{\ln\left(\frac{g_2}{g_1} \cdot \frac{N_1}{N_2}\right)}}$$

$$\boxed{\frac{N_k}{N} = \frac{g_k \exp(-E_k / k_B T)}{\sum_i g_i \exp(-E_i / k_B T)}}$$

N_1 and N_2 – particle populations on energy levels E_1 and E_2
g_1 and g_2 – degeneracy for energy levels E_1 and E_2, $\Delta E = E_2 - E_1$
k_B – Boltzmann constant, T – Temperature, N – total population

Population Ratio ⇨ Temperature

Boltzmann Technique



Atomic Fe Energy Level

[Gelbwachs, 1994; Chu et al., 2002]

Example: Fe Boltzmann

$$\frac{N(J=4)}{N(J=3)} = \frac{g_1}{g_2}\exp\{\Delta E/k_B T\}$$

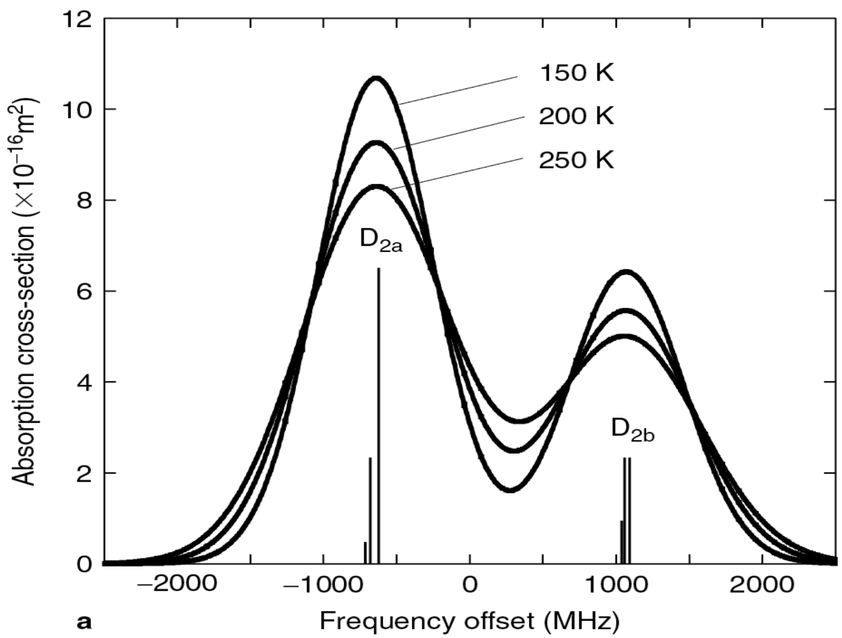
$$\boxed{\begin{aligned} g_1 &= 2*4+1=9 \\ g_2 &= 2*3+1=7 \\ \Delta E &= 416(cm^{-1}) \\ &= hc\times 416\times 100(J) \\ \Delta E/k_B &= 598.43K \end{aligned}}$$

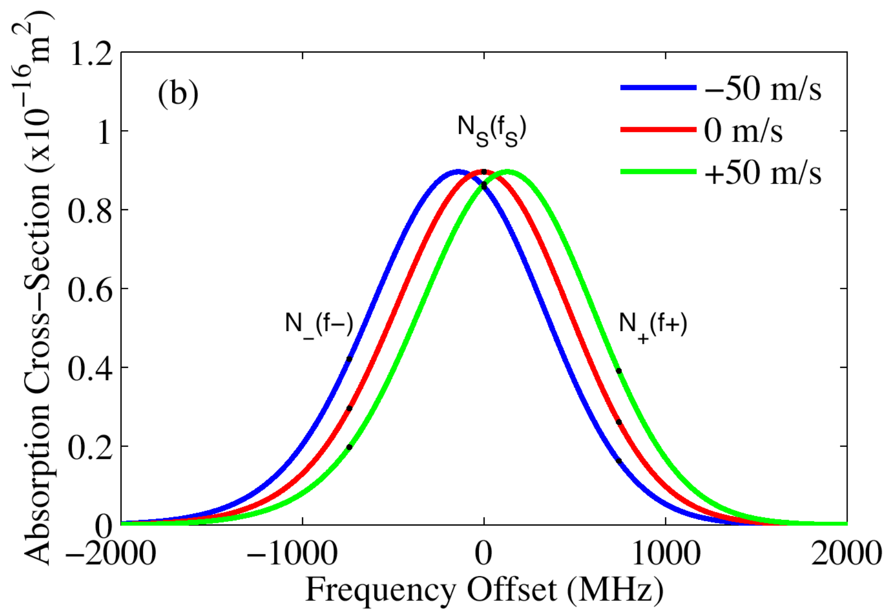
$$For \quad T=200,$$

$$\frac{N(J=4)}{N(J=3)} = \frac{9}{7}e^{598.43/200} = 25.6$$

Doppler Shift and Broadening

- **Doppler Technique** - Doppler linewidth broadening and Doppler frequency shift are temperature-dependent and wind-dependent, respectively (applying to both Na, K, Fe resonance fluorescence and molecular scattering)

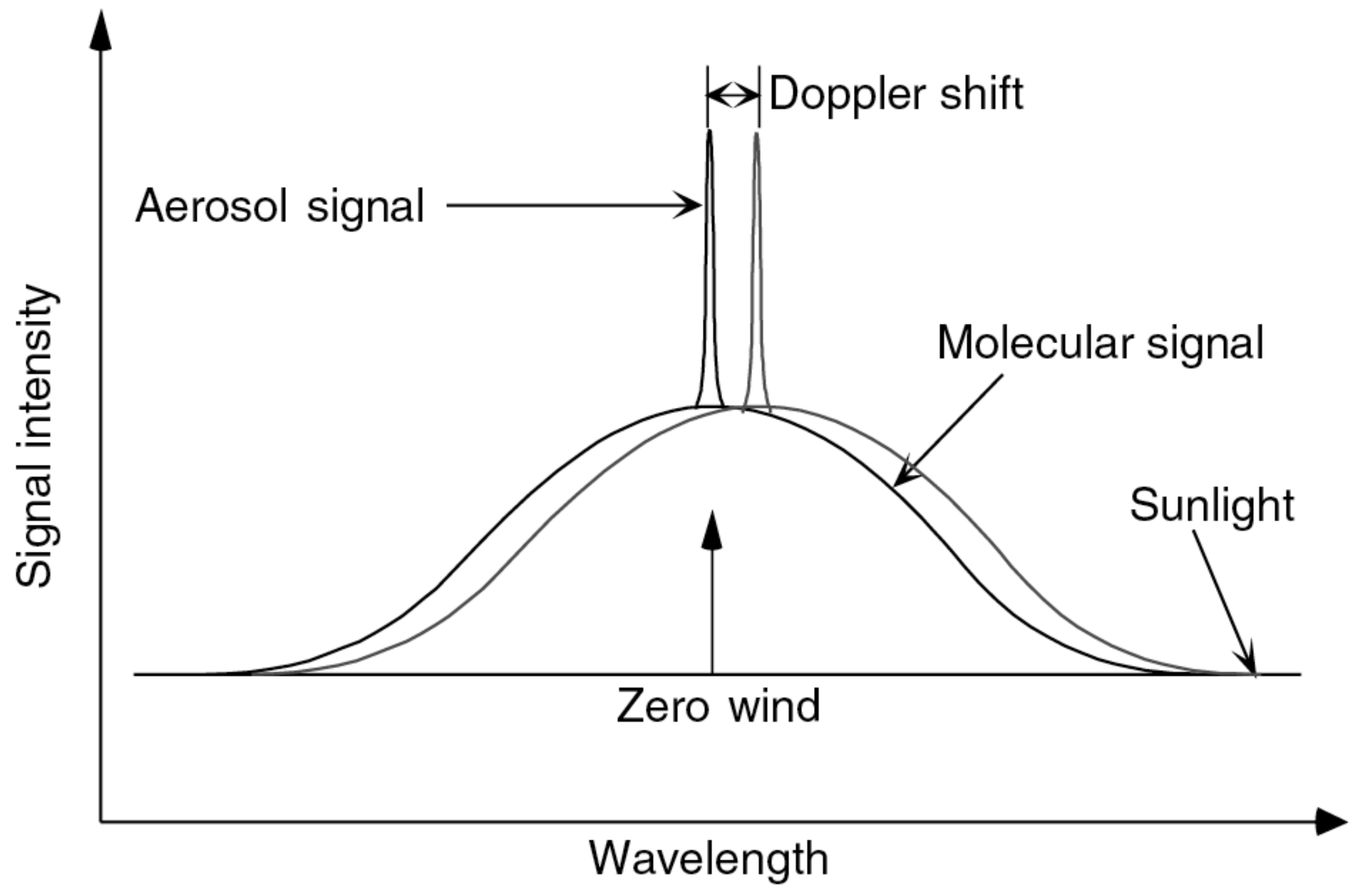




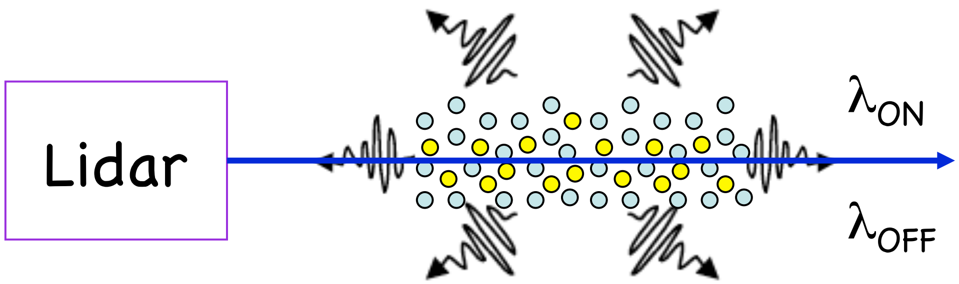
$$\sigma_{rms} = \frac{\omega_0}{c}\sqrt{\frac{k_B T}{M}} = \frac{1}{\lambda_0}\sqrt{\frac{k_B T}{M}}$$

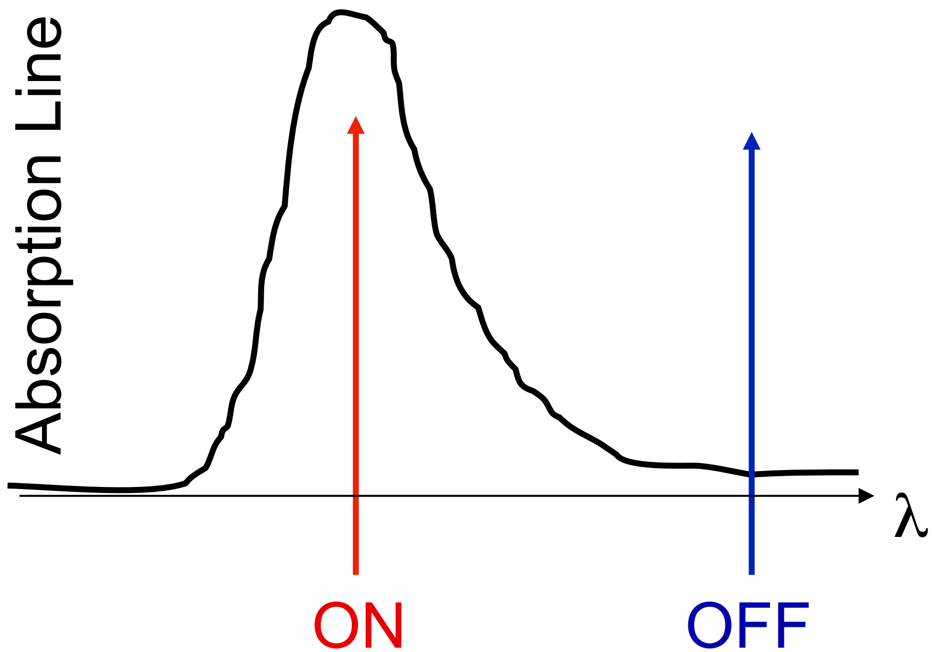
$$\Delta\omega = \omega - \omega_0 = -\vec{k}\cdot\vec{v} = -\omega_0 \frac{v\cos\theta}{c}$$

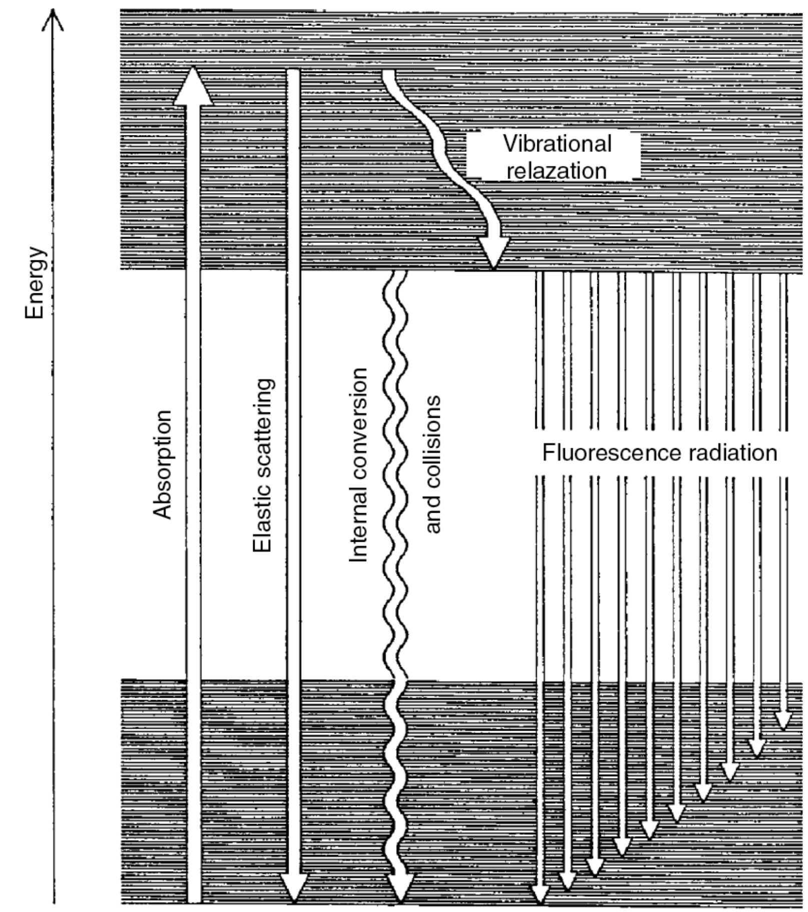
Doppler Shift and Broadening

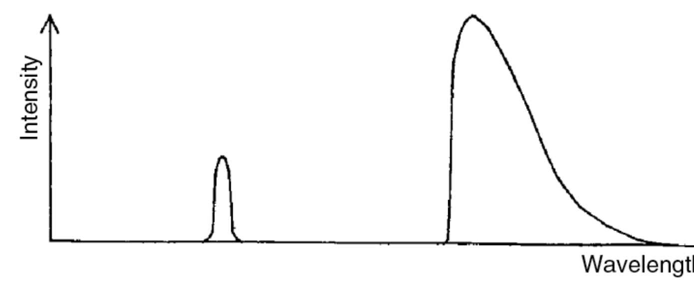


Absorption and Fluorescence









Backscatter Cross-Section Comparison

Physical Process	Backscatter Cross-Section	Mechanism
Mie (Aerosol) Scattering	10^{-8} - 10^{-10} cm^2sr^{-1}	Two-photon process Elastic scattering, instantaneous
Atomic Absorption and Resonance Fluorescence	10^{-13} cm^2sr^{-1}	Two single-photon process (absorption and spontaneous emission) Delayed (radiative lifetime)
Molecular Absorption	10^{-19} cm^2sr^{-1}	Single-photon process
Fluorescence from molecule, liquid, solid	10^{-19} cm^2sr^{-1}	Two single-photon process Inelastic scattering, delayed (lifetime)
Rayleigh Scattering (Wavelength Dependent)	10^{-27} cm^2sr^{-1}	Two-photon process Elastic scattering, instantaneous
Raman Scattering (Wavelength Dependent)	10^{-30} cm^2sr^{-1}	Two-photon process Inelastic scattering, instantaneous

Rayleigh Backscatter Coefficient

$$\beta_{Rayleigh}(\lambda, z, \theta = \pi) = 2.938 \times 10^{-32} \frac{P(z)}{T(z)} \cdot \frac{1}{\lambda^{4.0117}} \left(m^{-1} sr^{-1}\right)$$

P in mbar and T in Kelvin at altitude z, λ in meter.

$$\beta(\theta) = \frac{\beta_T}{4\pi} P(\theta) = \frac{\beta_T}{4\pi} \times 0.7629 \times (1 + 0.9324 \cos^2 \theta)$$

Rayleigh Backscatter Cross Section

$$\frac{d\sigma_m(\lambda)}{d\Omega} = 5.45 \cdot \left(\frac{550}{\lambda}\right)^4 \times 10^{-32} \left(m^2 sr^{-1}\right)$$

where λ is the wavelength in nm.

- For Rayleigh lidar, λ = 532 nm, ⇨ 6.22×10^{-32} $m^2 sr^{-1}$

Absorption Cross-Section for Atoms

Atomic absorption cross section σ_{abs}

for Na 589-nm D2 line is about 10^{-15} m^2

for Fe 372-nm line is about 10^{-16} m^2

How do you derive the backscatter cross section?

$$\frac{d\sigma_m}{d\Omega}(\theta = \pi) = \frac{\sigma_{abs}}{4\pi} = \frac{10^{-15} m^2}{4\pi}$$

$$= 7.95 \times 10^{-16} m^2 sr^{-1} = 7.95 \times 10^{-12} cm^2 sr^{-1}$$

Polarization in Scattering

- Depolarization can be resulted from

(1) Non-spherical particle shape (true for both aerosol/cloud and atmosphere molecules)

(2) Inhomogeneous refraction index

(3) Multiple scattering inside particle

- The range-resolved linear depolarization ratio is defined from lidar or optical observations as

$$\boxed{\delta(R) = \frac{P_\perp}{P_{||}} = \frac{I_\perp}{I_{||}}}$$

where P and I are the light power and intensity detected, respectively.

- According to Gary Gimmestad, the following definition is misleading:

$$\delta(R) = \left[\beta_\perp(R)/\beta_{||}(R)\right]\exp\left(T_{||} - T_\perp\right)$$

β and T are the backscattering coefficients and atmospheric transmittances.

Review Lidar Equation

- General lidar equation with angular scattering coefficient

$$N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

- General lidar equation with total scattering coefficient

$$N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta_T(\lambda,\lambda_L,R)\Delta R\right]\cdot\frac{A}{4\pi R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

- General lidar equation in angular scattering coefficient β and extinction coefficient α form

$$N_S(\lambda,R) = \left[\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right]\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\left(\frac{A}{R^2}\right)$$
$$\cdot\exp\left[-\int_0^R \alpha(\lambda_L,r')dr'\right]\exp\left[-\int_0^R \alpha(\lambda,r')dr'\right]\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

Specific Lidar Equations

- Lidar equation for Rayleigh lidar

$$N_S(\lambda,R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\beta(\lambda,R)\Delta R\right)\left(\frac{A}{R^2}\right)T^2(\lambda,R)\left(\eta(\lambda)G(R)\right) + N_B$$

- Lidar equation for resonance fluorescence lidar

$$N_S(\lambda,R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda,R)n_c(z)R_B(\lambda)\Delta R\right)\left(\frac{A}{4\pi R^2}\right)\left(T_a^2(\lambda,R)T_c^2(\lambda,R)\right)\left(\eta(\lambda)G(R)\right) + N_B$$

- Lidar equation for differential absorption lidar

$$N_S(\lambda_{on}^{off},R) = N_L(\lambda_{on}^{off})\left[\beta_{sca}(\lambda_{on}^{off},R)\Delta R\right]\left(\frac{A}{R^2}\right)\exp\left[-2\int_0^z \overline{\alpha}(\lambda_{on}^{off},r')dr'\right]$$
$$\times\exp\left[-2\int_0^z \sigma_{abs}(\lambda_{on}^{off},r')n_c(r')dr'\right]\left[\eta(\lambda_{on}^{off})G(R)\right] + N_B$$

General Lidar Equation

Assumptions: independent and single scattering

$$N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

- N_S - expected photon counts detected at λ and distance R;
- 1st term - number of transmitted laser photons;
- 2nd term - probability that a transmitted photon is scattered by the scatters into a unit solid angle at angle θ;
- 3rd term - probability that a scatter photon is collected by the receiving telescope;
- 4th term - light transmission during light propagation from laser source to distance R and from distance R to receiver;
- 5th term - overall system efficiency;
- 6th term - background and detector noise.

More in General Lidar Equation

$$N_S(\lambda,R) = \left[\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right]\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

N_S (R) - expected received photon number from a distance R
P_L - transmitted laser power, λ_L - laser wavelength
Δt - integration time,
h - Planck's constant, c - light speed
β(R) - volume scatter coefficient at distance R for angle θ,
ΔR - thickness of the range bin
A - area of receiver,
T(R) - one way transmission of the light from laser source to distance R or from distance R to the receiver,
η - system optical efficiency,
G(R) - geometrical factor of the system,
N_B - background and detector noise photon counts.

Solution for Scattering Form Lidar Equation

- Scattering form lidar equation

$$N_S(\lambda,R) = \left[\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right]\cdot\left[\beta(\lambda,\lambda_L,R)\Delta R\right]\cdot\left[\frac{A}{R^2}\right]\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right] + N_B$$

- Solution for scattering form lidar equation

$$\beta(\lambda,\lambda_L,R) = \frac{N_S(\lambda,R) - N_B}{\left[\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right]\Delta R\left(\frac{A}{R^2}\right)\left[T(\lambda_L,R)T(\lambda,R)\right]\left[\eta(\lambda,\lambda_L)G(R)\right]}$$

Solution for Fluorescence Form Lidar Equation

- Fluorescence form lidar equation

$$N_S(\lambda,R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda,R)n_c(R)R_B(\lambda)\Delta R\right)\left(\frac{A}{4\pi R^2}\right)\left(T_a^2(\lambda,R)T_c^2(\lambda,R)\right)\left(\eta(\lambda)G(R)\right) + N_B$$

- Solution for fluorescence form lidar equation

$$n_c(R) = \frac{N_S(\lambda,R) - N_B}{\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda)R_B(\lambda)\Delta R\right)\left(\frac{A}{4\pi R^2}\right)\left(\eta(\lambda)T_a{}^2(\lambda,R)T_c^2(\lambda,R)G(R)\right)}$$

Differential Absorption/Scattering Form

- For the laser with wavelength λ_{on} on the molecular absorption line

$$N_S(\lambda_{on},R) = N_L(\lambda_{on})\left[\beta_{sca}(\lambda_{on},R)\Delta R\right]\left(\frac{A}{R^2}\right)\exp\left[-2\int_0^R \overline{\alpha}(\lambda_{on},r')dr'\right]$$
$$\times\exp\left[-2\int_0^R \sigma_{abs}(\lambda_{on},r')n_c(r')dr'\right]\left[\eta(\lambda_{on})G(R)\right] + N_B$$

- For the laser with wavelength λ_{off} off the molecular absorption line

$$N_S(\lambda_{off},R) = N_L(\lambda_{off})\left[\beta_{sca}(\lambda_{off},R)\Delta R\right]\left(\frac{A}{R^2}\right)\exp\left[-2\int_0^R \overline{\alpha}(\lambda_{off},r')dr'\right]$$
$$\times\exp\left[-2\int_0^R \sigma_{abs}(\lambda_{off},r')n_c(r')dr'\right]\left[\eta(\lambda_{off})G(R)\right] + N_B$$

Differential Absorption/Scattering Form

- The ratio of photon counts from these two channels is a function of the differential absorption and scattering:

$$\boxed{\begin{aligned}\frac{N_S(\lambda_{on},R)-N_B}{N_S(\lambda_{off},R)-N_B} &= \frac{N_L(\lambda_{on})\beta_{sca}(\lambda_{on},R)}{N_L(\lambda_{off})\beta_{sca}(\lambda_{off},R)}\,\frac{\eta(\lambda_{on})}{\eta(\lambda_{off})}\\ &\times\exp\left\{-2\int_0^R\left[\overline{\alpha}(\lambda_{on},r')-\overline{\alpha}(\lambda_{off},r')\right]dr'\right\}\\ &\times\exp\left\{-2\int_0^R\left[\sigma_{abs}(\lambda_{on},r')-\sigma_{abs}(\lambda_{off},r')\right]n_c(r')dr'\right\}\end{aligned}}$$

$$\boxed{\Delta\sigma=\sigma_{abs}(\lambda_{on})-\sigma_{abs}(\lambda_{off})}$$

Solution for Differential Absorption Lidar Equation

- Solution for differential absorption lidar equation

$$n_c(R) = \frac{1}{2\Delta\sigma}\frac{d}{dR}\left\{ \begin{array}{l} \ln\left[\dfrac{N_L(\lambda_{on})\beta_{sca}(\lambda_{on},R)}{N_L(\lambda_{off})\beta_{sca}(\lambda_{off},R)}\dfrac{\eta(\lambda_{on})}{\eta(\lambda_{off})}\right] \\ \qquad -\ln\left[\dfrac{N_S(\lambda_{on},R)-N_B}{N_S(\lambda_{off},R)-N_B}\right] \\ \qquad -2\int_0^R\left[\overline{\alpha}(\lambda_{on},r')-\overline{\alpha}(\lambda_{off},r')\right]dr' \end{array} \right\}$$

$$\Delta\sigma = \sigma_{abs}(\lambda_{on}) - \sigma_{abs}(\lambda_{off})$$

Solution for Resonance Fluorescence Lidar Equation

- Resonance fluorescence and Rayleigh lidar equations

$$N_S(\lambda,z)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda,z)n_c(z)R_B(\lambda)\Delta z\right)\left(\frac{A}{4\pi z^2}\right)\left(T_a{}^2(\lambda)T_c^2(\lambda,z)\right)\left(\eta(\lambda)G(z)\right)+N_B$$

$$N_R(\lambda,z_R)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_R(\pi,\lambda)n_R(z_R)\Delta z\right)\left(\frac{A}{z_R{}^2}\right)T_a{}^2(\lambda,z_R)\left(\eta(\lambda)G(z_R)\right)+N_B$$

- Rayleigh normalization

$$\frac{n_c(z)}{n_R(z_R)}=\frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R{}^2}\cdot\frac{4\pi\sigma_R(\pi,\lambda)}{\sigma_{eff}(\lambda,z)R_B(\lambda)}\cdot\frac{\cancel{T_a{}^2}(\lambda,z_R)\cancel{G(z_R)}}{\cancel{T_a{}^2}(\lambda,z)T_c^2(\lambda,z)\cancel{G(z)}}$$

- Solution for resonance fluorescence

$$\boxed{n_c(z)=n_R(z_R)\frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R{}^2}\cdot\frac{4\pi\sigma_R(\pi,\lambda)}{\sigma_{eff}(\lambda,z)R_B(\lambda)}\cdot\frac{1}{T_c^2(\lambda,z)}}$$

Solution for Rayleigh and Mie Lidars

- Rayleigh and Mie (middle atmos) lidar equations

$$N_S(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\beta_R(z) + \beta_{aerosol}(z)\right)\Delta z\left(\frac{A}{z^2}\right)T_a^{\,2}(\lambda,z)\left(\eta(\lambda)G(z)\right) + N_B$$

$$N_R(\lambda,z_R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\beta_R(z_R)\Delta z\right)\left(\frac{A}{z_R^{\,2}}\right)T_a^{\,2}(\lambda,z_R)\left(\eta(\lambda)G(z_R)\right) + N_B$$

- Rayleigh normalization

$$\frac{\beta_R(z) + \beta_{aerosol}(z)}{\beta_R(z_R)} = \frac{N_S(\lambda,z) - N_B}{N_R(\lambda,z_R) - N_B} \cdot \frac{z^2}{z_R^{\,2}} \cdot \frac{\cancel{T_a^{\,2}(\lambda,z_R)}\cancel{G(z_R)}}{\cancel{T_a^{\,2}(\lambda,z)}\cancel{G(z)}}$$

- For Rayleigh scattering at z and z_R

$$\boxed{\frac{\beta_R(z)}{\beta_R(z_R)} = \frac{\sigma_R(z)n_{atm}(z)}{\sigma_R(z_R)n_{atm}(z_R)} = \frac{n_{atm}(z)}{n_{atm}(z_R)}}$$

Solution (Continued)

- Solution for Mie scattering in middle atmosphere

$$\boxed{\beta_{aerosol}(z) = \beta_R(z_R)\left[\frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R{}^2}-\frac{n_{atm}(z)}{n_{atm}(z_R)}\right]}$$

$$\boxed{\beta_R(\lambda,z_R,\pi) = 2.938\times10^{-32}\frac{P(z_R)}{T(z_R)}\cdot\frac{1}{\lambda^{4.0117}}\left(m^{-1}sr^{-1}\right)}$$

- Rayleigh normalization when aerosols not present

$$\frac{\beta_R(z)}{\beta_R(z_R)} = \frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R{}^2}\cdot\frac{\cancel{T_a{}^2(\lambda,z_R)}\cancel{G(z_R)}}{\cancel{T_a{}^2(\lambda,z)}\cancel{G(z)}}$$

- Solution for relative number density in Rayleigh lidar

$$\boxed{RND(z) = \frac{n_{atm}(z)}{n_{atm}(z_R)} = \frac{\beta_R(z)}{\beta_R(z_R)} = \frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R{}^2}}$$

Summary

- Solutions of lidar equation can be obtained by solving the lidar equation directly if all the lidar parameters and atmosphere conditions are well known.
- Solutions for three forms of lidar equations are shown: scattering form, fluorescence form, and differential absorption form.
- However, system parameters and atmosphere conditions may vary frequently and are NOT well known to experimenters.
- A good solution is to perform Rayleigh normalization to cancel out most of the system and atmosphere parameters so that the essential and known parts can be solved.