# Lecture 05. Fundamentals of Lidar Remote Sensing (3)

❑ General Illustration of lidar equation

❑ Overview of physical processes in lidar

❑ Elastic and in-elastic scattering vs polarization

❑ Absorption and differential absorption

❑ Fluorescence and resonance fluorescence

❑ Comparison of backscatter cross section

❑ Doppler effect and Boltzmann distribution

❑ Summary

Illustration of LIDAR Equation
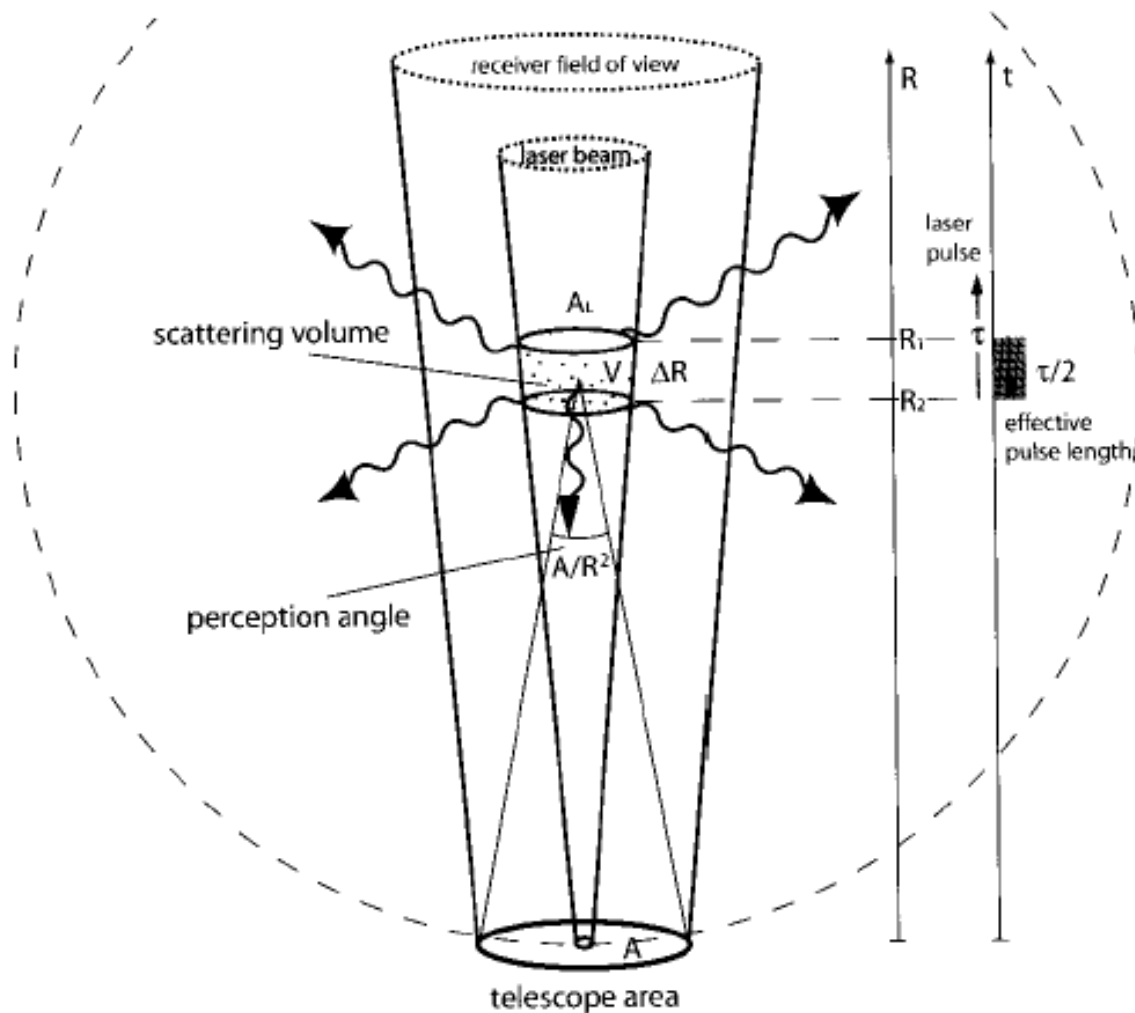
-- Courtesy of Ulla Wandinger [Introduction to Lidar]

General Lidar Equation in β and α

$$N_S(\lambda, R) = \left[\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right]\left[\beta(\lambda, \lambda_L, \theta, R)\Delta R\right]\left(\frac{A}{R^2}\right)$$

$$\cdot \exp\left[-\int_0^R \alpha(\lambda_L, r')dr'\right]\exp\left[-\int_0^R \alpha(\lambda, r')dr'\right]\left[\eta(\lambda, \lambda_L)G(R)\right] + N_B$$

β is the volume scatter coefficient
α is the extinction coefficient

Volume scatter coefficient $\quad \beta(\lambda, \lambda_L, R) = \sum_i \left[\frac{d\sigma_i(\lambda_L)}{d\Omega} n_i(R) p_i(\lambda)\right]$

Transmission $\quad T(\lambda_L, R)T(\lambda, R) = \exp\left[-\left(\int_0^R \alpha(\lambda_L, r)dr + \int_0^R \alpha(\lambda, r)dr\right)\right]$

Overview of Lidar Physical Processes

❑ Interaction between light and objects

(1) Scattering (instantaneous elastic & inelastic): Mie, Rayleigh, Raman

(2) Absorption and differential absorption

(3) Laser induced fluorescence

(4) Resonance fluorescence

(5) Doppler shift and Doppler broadening

(6) Boltzmann distribution

(7) Reflection from target or surface

❑ Light propagation in atmosphere or medium: transmission vs extinction

Extinction = Scattering + Absorption

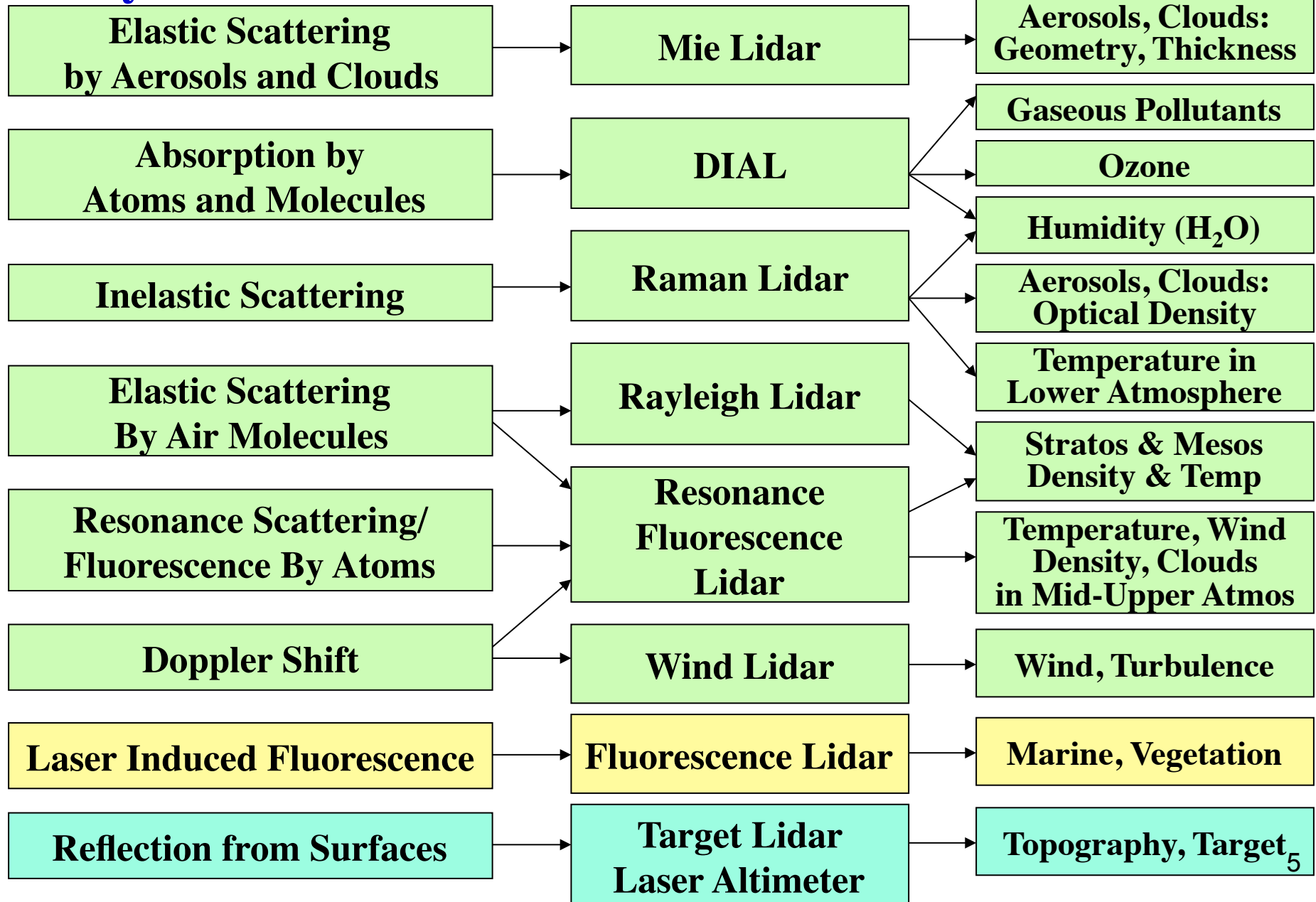$$T(\lambda,R) = \exp\left[-\int_0^R \alpha(\lambda,r)\,dr\right]$$

$$\alpha(\lambda,R) = \sum_i \left[\sigma_{i,ext}(\lambda)n_i(R)\right]$$

Physical Process	**Device**	**Objective**
Elastic Scattering by Aerosols and Clouds	Mie Lidar	Aerosols, Clouds: Geometry, Thickness
Absorption by Atoms and Molecules	DIAL	Gaseous Pollutants
		Ozone
		Humidity (H_2O)
Inelastic Scattering	Raman Lidar	Aerosols, Clouds: Optical Density
Elastic Scattering By Air Molecules	Rayleigh Lidar	Temperature in Lower Atmosphere
Resonance Scattering/ Fluorescence By Atoms	Resonance Fluorescence Lidar	Stratos & Mesos Density & Temp
		Temperature, Wind Density, Clouds in Mid-Upper Atmos
Doppler Shift	Wind Lidar	Wind, Turbulence
Laser Induced Fluorescence	Fluorescence Lidar	Marine, Vegetation
Reflection from Surfaces	Target Lidar Laser Altimeter	Topography, Target

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Elastic and Inelastic Scattering

❑ Elastic scattering:

scattering with no apparent change of wavelength

Sum of elastic scattering from atmospheric molecules and elastic scattering from aerosol particles and cloud droplets

Rayleigh scattering and Mie scattering

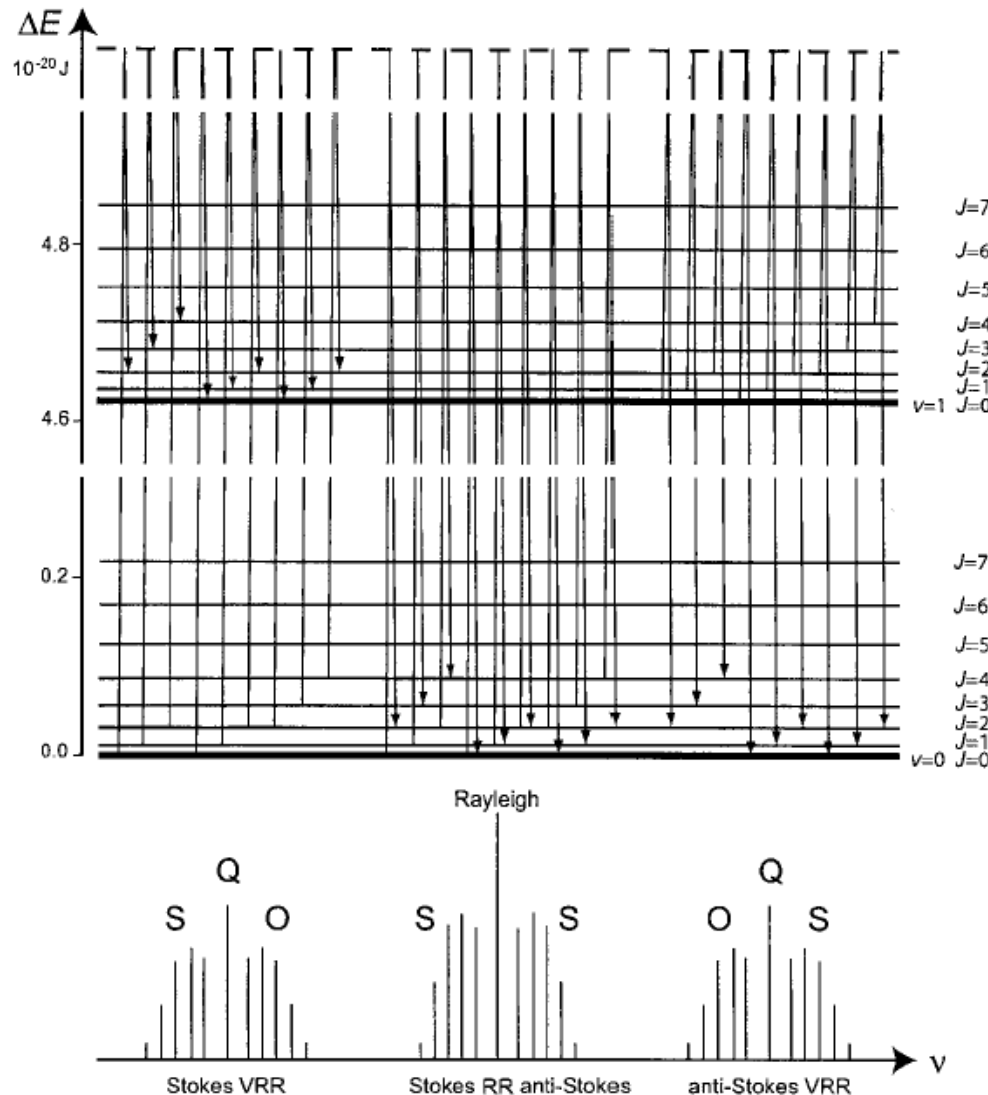(There are confusions in definitions of these scatterings.)

❑ Inelastic scattering:

scattering with apparent change of wavelength

Raman scattering

Pure rotational Raman and vibration-rotational Raman

Definitions for Different Scatterings

Virtual Energy Levels

Real Energy Levels

Real Energy Levels

$$\Delta v = 0, \qquad \Delta J = 2$$
$$\Delta v = \pm 1, \qquad \Delta J = 0, \pm 2$$

Fig. 9.1. Vibration–rotation energy levels of the N_2 molecule, Raman transitions, and resulting spectrum.

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Rayleigh (Molecular) Scattering

❑ Rayleigh scattering is referred to the elastic scattering from atmospheric molecules (particle size is much smaller than the wavelength), i.e., scattering with no apparent change of wavelength, although still undergoing Doppler broadening and Doppler shift.

❑ However, depending on the resolution of detection, Rayleigh scattering can consist of the Cabannes scattering (really elastic scattering from molecules) and pure rotational Raman scattering.
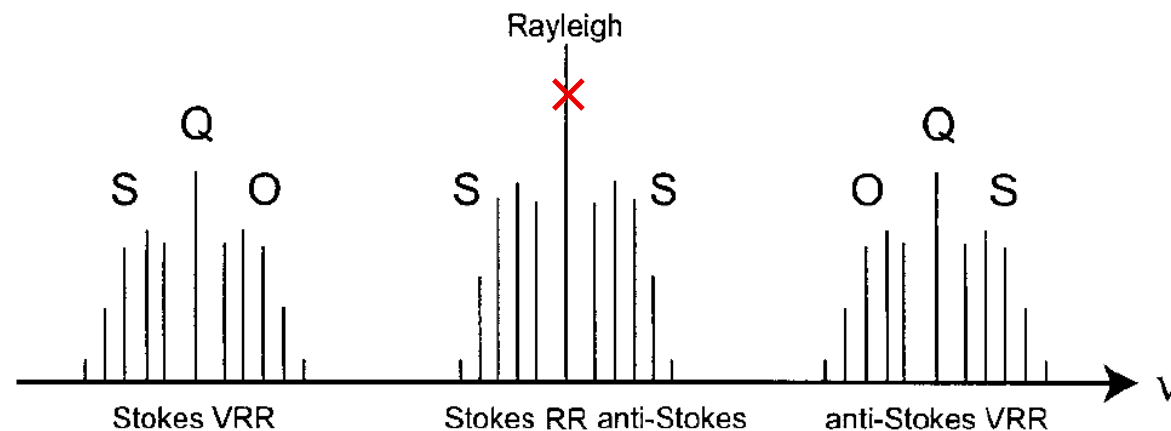
Pure Rotational Raman   Rayleigh   Cabannes Line

Stokes VRR   Stokes RR anti-Stokes   anti-Stokes VRR

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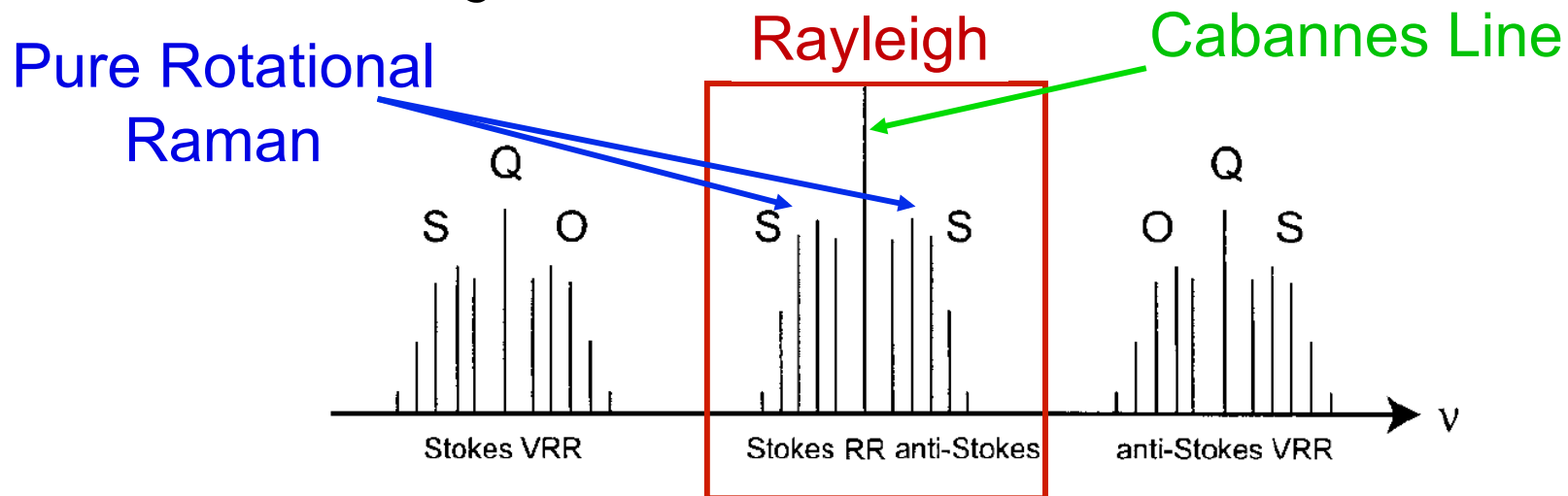
Raman Scattering

❑ Raman scattering is an inelastic scattering with rotational quantum state or vibration–rotational quantum state change as the result of scattering.

❑ Separation between Cabannes line and the nearest pure rotational Raman (O_2 and N_2) is about 10 cm^{-1} = 300 GHz.

❑ For vibration–rotational Raman, Q branch is not a single line, but consists of many lines with very small separations, < 0.1 cm^{-1} = 3 GHz, due to the different splitting of the rotational levels in the lower and upper vibrational levels.

Mie (Aerosol/Cloud) Scattering

❑ Strictly speaking, Mie scattering is an elastic scattering from spherical particles [Mie, 1908], which includes the solution of Rayleigh scattering.

❑ However, in lidar field, Mie scattering is referred to the elastic scattering from spherical particles whose size is comparable to or larger than the wavelength.

❑ Furthermore, Mie scattering is generalized to elastic scattering from overall aerosol particles and cloud droplets, i.e., including non-spherical particles.

❑ To precisely calculate the scattering from non-spherical particles, Mie scattering theory has to be replaced by non-spherical particle scattering theories. This is a complicated issue in the elastic lidar field.

Polarization in Scattering

❑ According to Mie theory, backscattering from spherical particles does not change the polarization state of the radiation. The backscattered light has polarization parallel to that of the transmitted beam (usually linearly polarized).

❑ As long as the particles are small compared to the wavelength, the actual particle shape does not play a major role for the scattering properties as (many) theories for non-spherical scattering show.

❑ Large non-spherical particles lead to a depolarization of backscattered radiation, i.e., partial backscattered light has polarization perpendicular to that of the transmitted beam.

❑ Multiple scattering can cause depolarization. Inhomogeneous refraction index can also lead to depolarization.

Rayleigh Backscatter Coefficient

❑ Precise equations should be obtained from Rayleigh theory and experimental measurements of parameters.

❑ In lidar field, a common practice is to use the equation

$$\beta_{Rayleigh}(\lambda, z, \theta = \pi) = 2.938 \times 10^{-32} \frac{P(z)}{T(z)} \cdot \frac{1}{\lambda^{4.0117}} \left(m^{-1} sr^{-1} \right)$$

where P is the atmosphere pressure in mbar and T is the temperature in Kelvin at altitude z, λ is the wavelength in meter, and $\beta_{Rayleigh}$ is the backscatter coefficient (angular).

❑ Total scatter coefficient β_T has relationship:

$$\beta(\theta) = \frac{\beta_T}{4\pi} P(\theta) = \frac{\beta_T}{4\pi} \times 0.7629 \times (1 + 0.9324 \cos^2 \theta)$$

Rayleigh Backscatter Cross Section

❑ It is also common in lidar field to calculate the Rayleigh backscatter cross section using the following equation

$$\frac{d\sigma_m(\lambda)}{d\Omega} = 5.45 \cdot \left(\frac{550}{\lambda}\right)^4 \times 10^{-32}\left(m^2 sr^{-1}\right)$$

where λ is the wavelength in nm.

❑ For K lidar, λ = 770 nm, ⇨ 1.42 x 10^{-32} m²sr⁻¹

❑ For Na lidar, λ = 589 nm, ⇨ 4.14 x 10^{-32} m²sr⁻¹

❑ For Fe lidar, λ = 372 nm, ⇨ 2.60 x 10^{-31} m²sr⁻¹

❑ For Rayleigh lidar, λ = 532 nm, ⇨ 6.22 x 10^{-32} m²sr⁻¹

Scattering Form of Lidar Equation

❑ Rayleigh, Mie, and Raman scattering processes are instantaneous scattering processes, so there are no finite relaxation effects involved, but infinitely short duration.

❑ For Rayleigh and Mie scattering, there is no frequency shift when the atmospheric particles are at rest. The lidar equation is written as

$$N_S(\lambda, R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)(\beta(\lambda, R)\Delta R)\left(\frac{A}{R^2}\right)T^2(\lambda, R)(\eta(\lambda)G(R)) + N_B$$

❑ For Raman scattering, there is a large frequency shift. Raman lidar equation may be written as

$$N_S(\lambda, R) = \left(\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right)(\beta(\lambda, \lambda_L, R)\Delta R)\left(\frac{A}{R^2}\right)(T(\lambda_L, R)T(\lambda, R))(\eta(\lambda, \lambda_L)G(R)) + N_B$$
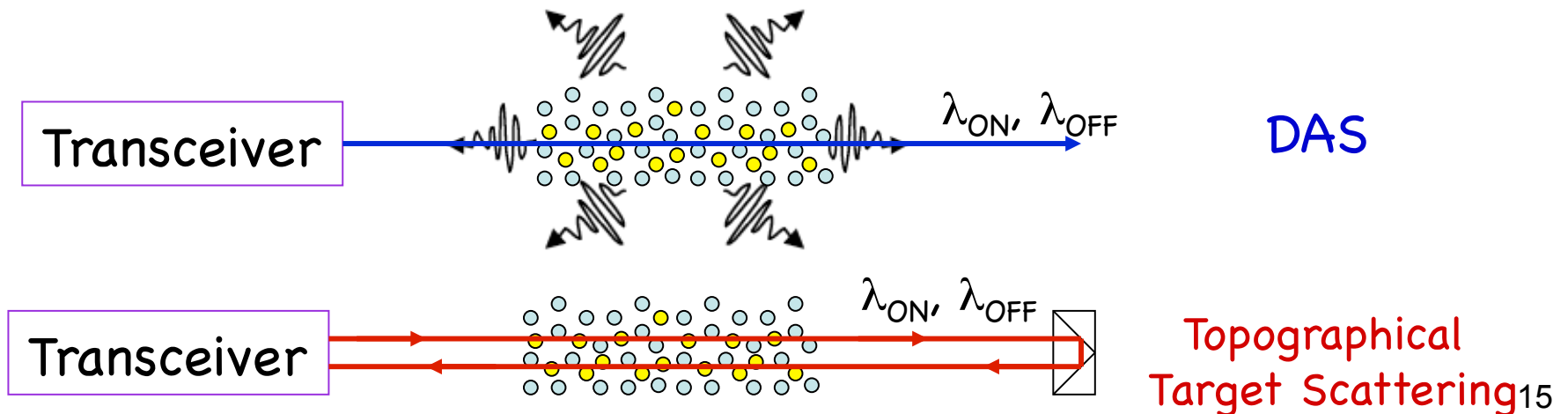
$$\lambda \neq \lambda_L, \quad p_i(\lambda) \neq 1, \quad p_i(\lambda) < 1$$

$$T(\lambda, R) = \exp\left[-\int_0^R \alpha(\lambda, r)dr\right]$$

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Differential Absorption

❑ For the molecular species to be detected by DIAL, usually they have strong absorption, however, nearly none resonance fluorescence due to strong relaxation processes other than fluorescence (e.g., frequent collisions with surrounding atmosphere molecules can make molecules decay from excited states to ground state without giving fluorescence).

❑ Thus, in the lidar equation for DIAL, the influence of molecular species is in the extinction (atmosphere transmission) part, not in the backscatter part. In other words, the molecular absorption contributes to the extinction of light when incident light and scattered light propagate through atmosphere, while the return signals are from the scattering of laser light by air molecules and aerosols.

Transceiver $\lambda_{ON}, \lambda_{OFF}$ DAS

Transceiver $\lambda_{ON}, \lambda_{OFF}$ Topographical Target Scattering

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Differential Absorption/Scattering Form of Lidar Equation

❑ For the laser with wavelength λ_{on} on the molecular absorption line

$$N_S(\lambda_{on},R) = N_L(\lambda_{on})\left[\beta_{sca}(\lambda_{on},R)\Delta R\right]\left(\frac{A}{R^2}\right)\exp\left[-2\int_0^z \overline{\alpha}(\lambda_{on},r')dr'\right]$$

$$\times \exp\left[-2\int_0^z \sigma_{abs}(\lambda_{on},r')n_c(r')dr'\right]\left[\eta(\lambda_{on})G(R)\right] + N_B$$

❑ For the laser with wavelength λ_{off} off the molecular absorption line

$$N_S(\lambda_{off},R) = N_L(\lambda_{off})\left[\beta_{sca}(\lambda_{off},R)\Delta R\right]\left(\frac{A}{R^2}\right)\exp\left[-2\int_0^z \overline{\alpha}(\lambda_{off},r')dr'\right]$$

$$\times \exp\left[-2\int_0^z \sigma_{abs}(\lambda_{off},r')n_c(r')dr'\right]\left[\eta(\lambda_{off})G(R)\right] + N_B$$

❑ Differential absorption cross-section

$$\Delta\sigma_{abs}(R) = \sigma_{abs}(\lambda_{ON},R) - \sigma_{abs}(\lambda_{OFF},R)$$

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Resonance Fluorescence

❑ In the middle and upper atmosphere, there exist some metal atoms and atomic ions. They have large absorption cross section, and quenching is not a problem in that region. Therefore, laser tuned to the resonance frequency of the absorption lines can excite resonance fluorescence from these atoms and ions.

❑ Resonance fluorescence is a two-step process: absorption first, and then spontaneous emission. Therefore, there is finite time delay between these two steps – radiative lifetime.

$$h\nu \qquad h\nu = E_2 - E_1$$

$$E_2$$

$$E_1$$

❑ Due to frequent collisions it is hard to obtain resonance fluorescence in lower atmosphere.

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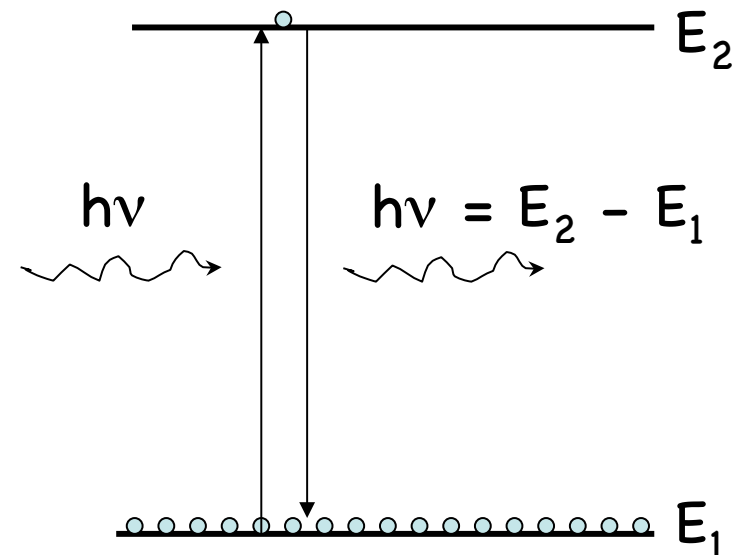
Laser Induced Fluorescence

❑ In contrast to free atoms and molecules, solids and liquids exhibit broad absorption and emission spectra because of the strong intermolecular interactions.

❑ A fixed frequency laser can be used for the excitation due to the broad absorption.

❑ Following the excitation, there is a very fast (ps) radiationless relaxation down to the lowest sub-level of the excited state, where the molecules remain for a typical excited-state fluorescence lifetime.

❑ The decay then occurs to different sub-levels of the ground state giving rise to a distribution of fluorescence light, which reflect the lower-state level distribution.

❑ Fixing the excitation wavelength, we can obtain fluorescence spectra. While fixing the detection channel and varying the excitation wavelength, an excitation spectrum can be recorded.

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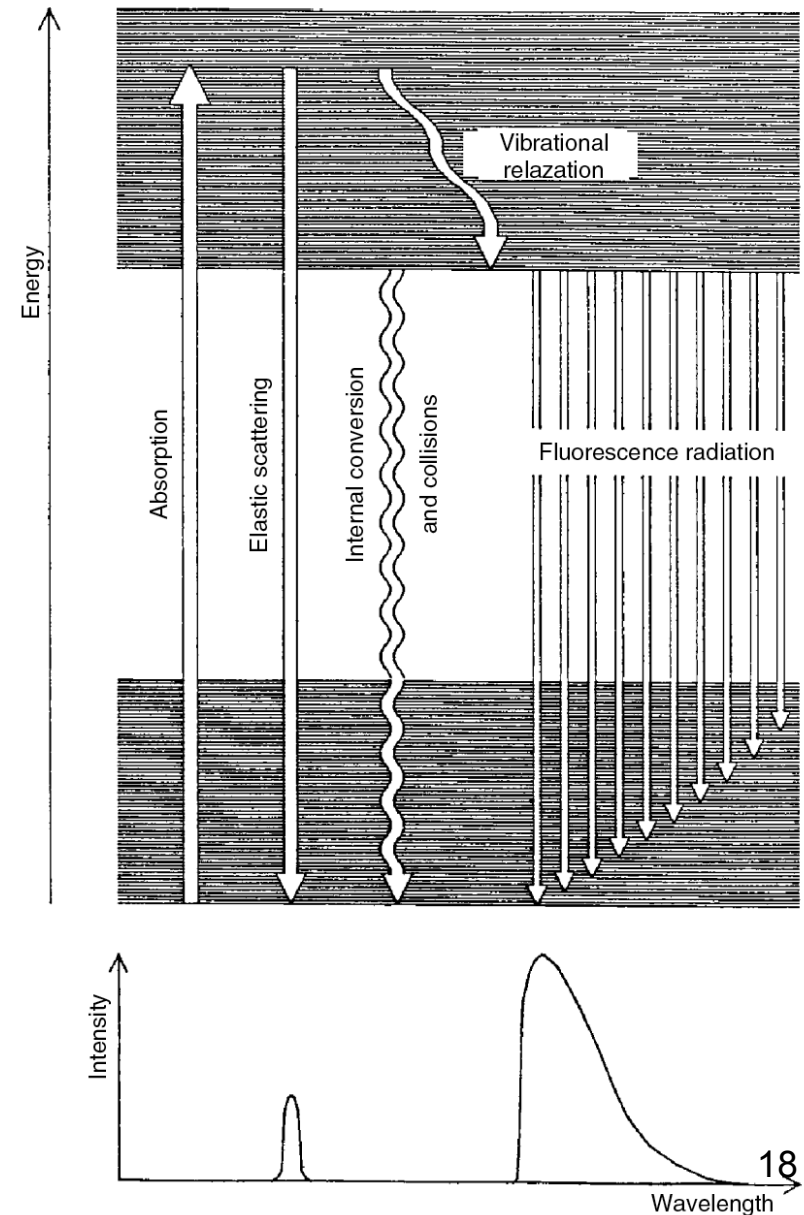
Fluorescence Form of Lidar Equation

❑ Resonance fluorescence and laser-induced-fluorescence are NOT instantaneous processes, but have delays due to the radiative lifetime of the excited states.

❑ The lidar equation in fluorescence form is given by

$$N_S(\lambda,R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda,R)n_c(z)R_B(\lambda)\Delta R\right)\left(\frac{A}{4\pi R^2}\right)\left(T_a^2(\lambda,R)T_c^2(\lambda,R)\right)\left(\eta(\lambda)G(R)\right)+N_B$$

❑ Here, $T_C(R)$ is the transmission caused by the constituent absorption.

$$T_C(R) = \exp\left(-\int_{R_{bottom}}^{R}\sigma_{eff}(\lambda,r')n_c(r')\mathrm{d}r'\right) = \exp\left(-\int_{R_{bottom}}^{R}\alpha_c(\lambda,r')\mathrm{d}r'\right)$$

❑ Here, $\alpha(\lambda,R)$ is the extinction coefficient caused by the absorption.
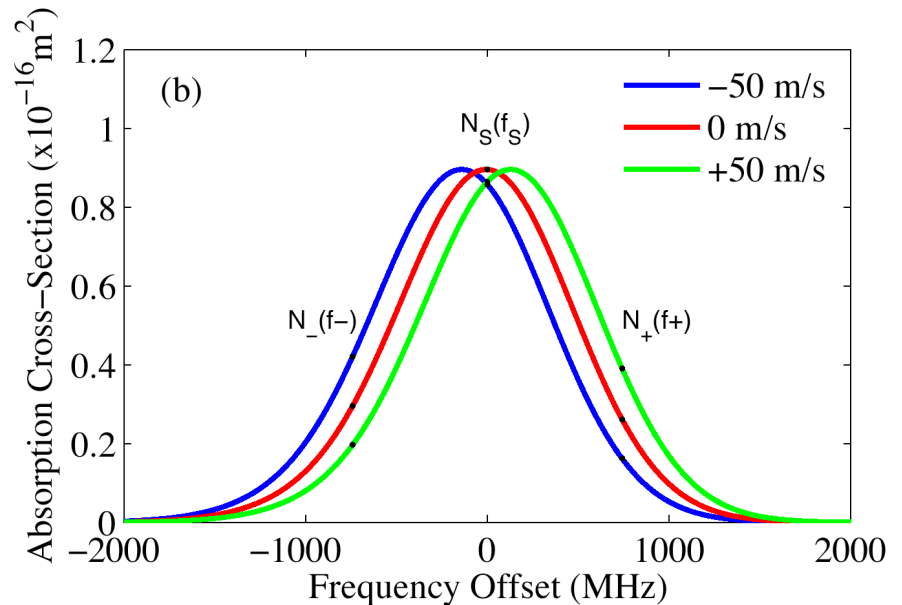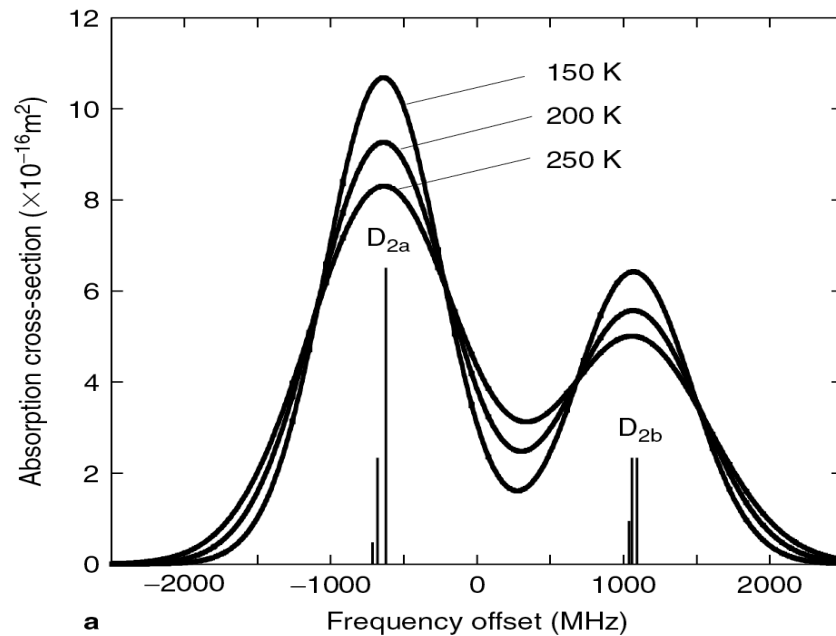
$$\alpha_c(\lambda,R) = \sigma_{eff}(\lambda,R)n_c(R)$$

Backscatter Cross-Section Comparison

Physical Process	Backscatter Cross-Section	Mechanism
Mie (Aerosol) Scattering	$10^{-8} - 10^{-10}$ cm^{2}sr^{-1}	Two-photon process Elastic scattering, instantaneous
Resonance Fluorescence	10^{-13} cm^{2}sr^{-1}	Two single-photon process (absorption and spontaneous emission) Delayed (radiative lifetime)
Molecular Absorption	10^{-19} cm^{2}sr^{-1}	Single-photon process
Fluorescence from molecule, liquid, solid	10^{-19} cm^{2}sr^{-1}	Two single-photon process Inelastic scattering, delayed (lifetime)
Rayleigh Scattering	10^{-27} cm^{2}sr^{-1}	Two-photon process Elastic scattering, instantaneous
Raman Scattering	10^{-30} cm^{2}sr^{-1}	Two-photon process Inelastic scattering, instantaneous

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Doppler Shift and Broadening

❑ Doppler Technique - Doppler linewidth broadening and Doppler frequency shift are temperature-dependent and wind-dependent, respectively (applying to both Na, K, Fe resonance fluorescence and molecular scattering)
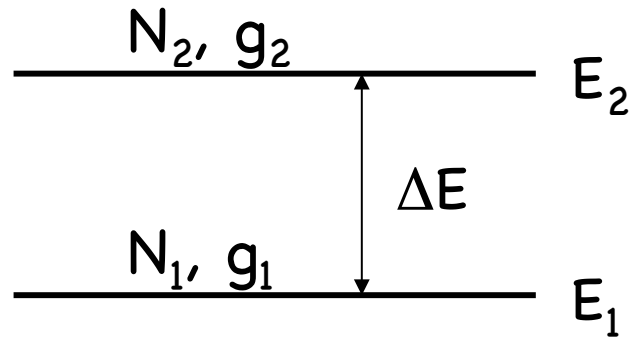

$$\sigma_{rms} = \frac{\omega_0}{c}\sqrt{\frac{k_B T}{M}} = \frac{1}{\lambda_0}\sqrt{\frac{k_B T}{M}}$$

$$\Delta\omega = \omega - \omega_0 = -\vec{k}\cdot\vec{v} = -\omega_0\frac{v\cos\theta}{c}$$

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Boltzmann Distribution

❑ Boltzmann distribution is the law of particle population distribution according to energy levels

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\{-(E_2 - E_1)/k_B T\}$$

$$T = \frac{\Delta E / k_B}{\ln\left(\dfrac{g_2}{g_1} \cdot \dfrac{N_1}{N_2}\right)}$$

$$\frac{N_k}{N} = \frac{g_k \exp(-E_k/k_B T)}{\sum_i g_i \exp(-E_i/k_B T)}$$

N_1 and N_2 – particle populations on energy levels E_1 and E_2
g_1 and g_2 – degeneracy for energy levels E_1 and E_2, $\Delta E = E_2 - E_1$
k_B – Boltzmann constant, T - Temperature, N – total population

Population Ratio ⇨ Temperature

Reflection from Target or Surface

❑ Specular reflection from water or ice or target surface;

❑ Laser non-specular reflection from ground;

❑ Laser non-specular reflection from target surface;

❑ Solar reflectance from ground or target surface.

LASER ALTIMETER INSTRUMENT

R

INSTRUMENT NADIR

φ

TARGET SURFACE

Δφ$_i$

Z$_i$

S$_i$

LASER FOOTPRINT
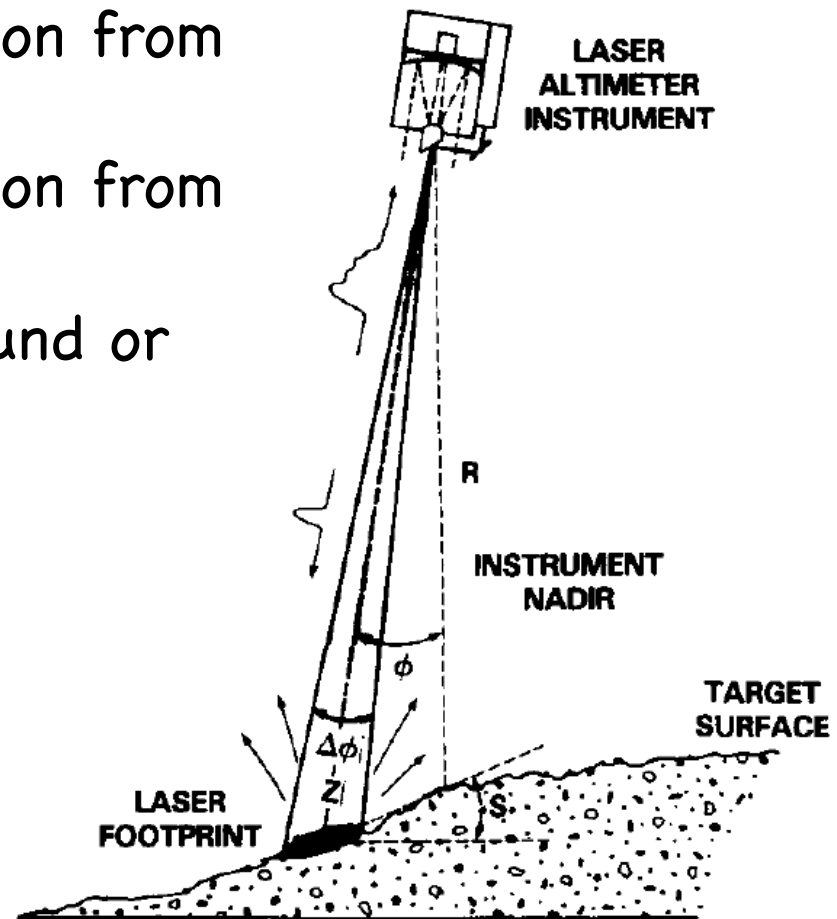
Summary

❑ Numerous physical processes are involved in lidars.

❑ Main physical processes include elastic and inelastic scattering, absorption and differential absorption, resonance fluorescence, laser induced fluorescence, Doppler effect, Boltzmann distribution, and reflection from target or surface.

❑ Understanding these processes precisely is the key to successful lidar simulations and applications.

❑ Lidar equation may change its form to best fit for each particular physical process and lidar application.

Chapter 3 for elastic scattering and polarization
Chapter 4 for differential absorption
Chapters 5 & 7 for resonance fluorescence, Boltzmann, Doppler
Chapter 6 for laser-induced fluorescence

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