Polarization Lidar

Characterization of the atmosphere using polarization

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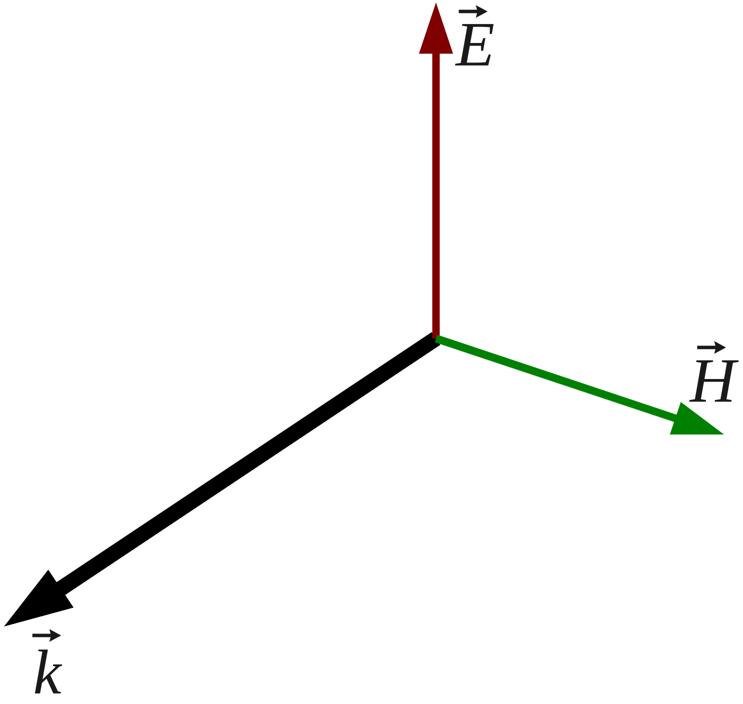
What is Polarization?

$$\vec{k}\cdot\vec{E}=0$$

$$\vec{k}\cdot\vec{H}=0$$

$$\vec{E}\cdot\vec{H}=0$$



What is Polarization?

For $\vec{k} \| \hat{z}$

$$\vec{E} = |E_x|\hat{x} + |E_y|e^{j\Gamma}\hat{y}$$

$$\vec{E} = \begin{bmatrix} |E_x| \\ |E_y|e^{j\Gamma} \end{bmatrix}$$

Linear Polarization

$$\Gamma = 0, \pi$$

Circular Polarization

$$\Gamma = \frac{\pi}{2}, -\frac{\pi}{2}$$

$$|E_x| = |E_y|$$

Elliptical Polarization

Everything Else

Stokes Vectors

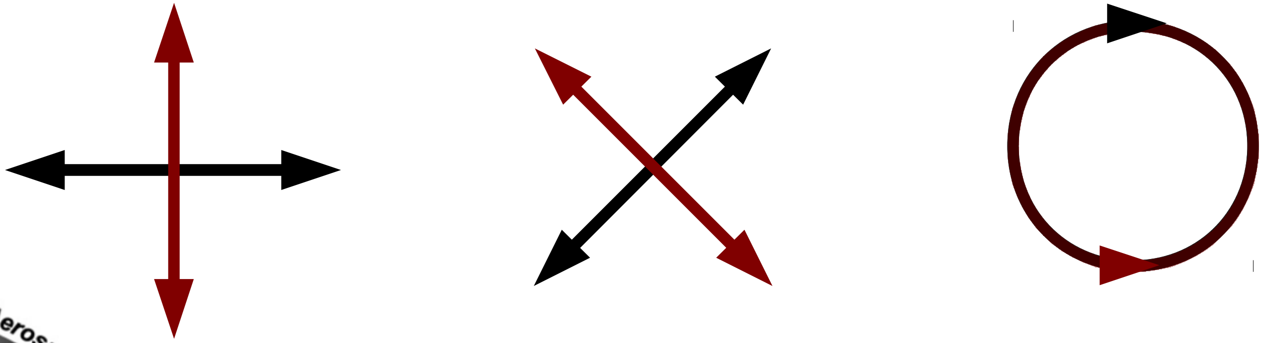
$$\vec{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

Total Intensity $|\vec{E}|^2$

Horizontal (+1) and Vertical (-1) Intensity

+45º (+1) and -45º (-1) Intensity

Left Hand Circular (+1) and Right Hand Circular (-1) Intensity



Stokes Vectors

Degree of Polarization (DOP)

$$DOP = \frac{\sqrt{Q^2 + U^2 + V^2}}{I} \leq 1$$

Unpolarized Light

$$\vec{S}_{unpol} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Mueller Matrices

4x4 Matrix that describes polarization optics

Three types of Polarization Matrices

Diattenuator – Polarization dependent efficiency

i.e. Horizontal Polarizer

Horizontal Polarized Input

$$\begin{bmatrix}1\\1\\0\\0\end{bmatrix}=\begin{bmatrix}0.5 & 0.5 & 0 & 0\\0.5 & 0.5 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}1\\1\\0\\0\end{bmatrix}$$

Vertical Polarized Input

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}=\begin{bmatrix}0.5 & 0.5 & 0 & 0\\0.5 & 0.5 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$$

Unpolarized Input

$$\begin{bmatrix}0.5\\0.5\\0\\0\end{bmatrix}=\begin{bmatrix}0.5 & 0.5 & 0 & 0\\0.5 & 0.5 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

Mueller Matrices

Three types of Polarization Matrices

Retarder – Polarization dependent phase shift

i.e. Horizontal Quarter Wave Plate

Horizontal Polarized Input

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

45° Polarized Input

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Unpolarized Input

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Mueller Matrices

Three types of Polarization Matrices
Depolarizer – reduces DOP

i.e. Total Depolarizer

Horizontal Polarized Input

$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}=\begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}1\\1\\0\\0\end{bmatrix}$$

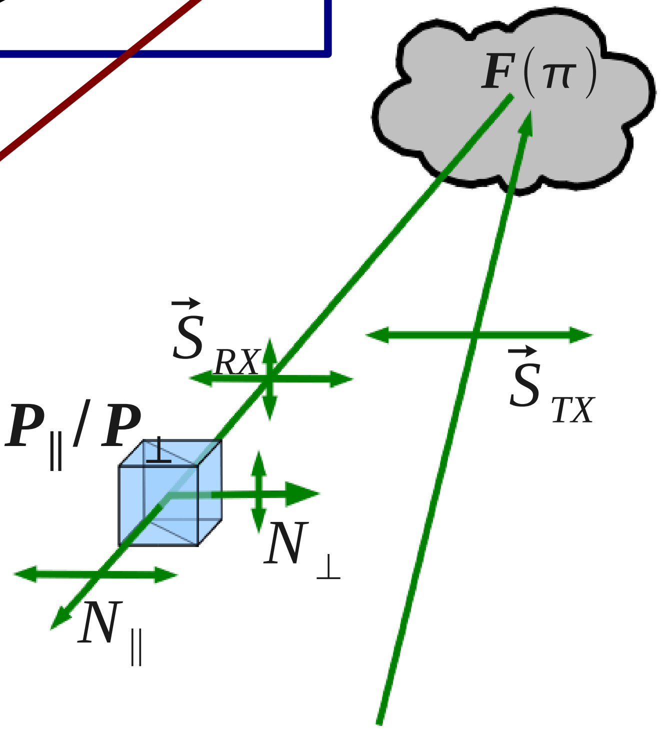
Stokes Vector Lidar Equation (SVLE)

Scatterer

$$\vec{S}_{RX}(z)=\boldsymbol{M}_{RX}\left[\left(G(z)\,\Delta z\,\frac{A}{z^{2}}\right)\boldsymbol{T}_{atm}(z)\,\boldsymbol{F}(\vec{k}_{i},\vec{k}_{s},z)\,\boldsymbol{T}_{atm}(z)\,\boldsymbol{M}_{TX}\vec{S}_{TX}+\vec{S}_{B}\right]$$

Transmission

Optical System



Measured Intensity/Photon Counts

$$N_{m}(z)=\vec{o}\,\boldsymbol{P}_{m}\vec{S}_{RX}(z)$$

$$\vec{o}=\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}$$

All matrices are Mueller matrices that can be analyzed using Lu-Chipman Decomposition

Phase Matrix Characterization

Requires we transmit 4 different polarizations and measure the resulting Stokes vectors to obtain the full matrix

- Horizontal
- Vertical
- 45º
- LHC

$$\vec{C}_1 = 0.5\left(\vec{S}_{RXh} + \vec{S}_{RXv}\right)$$
$$\vec{C}_2 = 0.5\left(\vec{S}_{RXh} - \vec{S}_{RXv}\right)$$
$$\vec{C}_3 = \vec{S}_{RX45} - \vec{C}_1$$
$$\vec{C}_4 = \vec{S}_{RXlhc} - \vec{C}_1$$

$$\boldsymbol{F}\left(\vec{k}_i, -\vec{k}_i, z\right) = \left[\vec{C}_1 \quad \vec{C}_2 \quad \vec{C}_3 \quad \vec{C}_4\right]$$

Stokes Vector Measurement

Each Stokes vector measurement requires 6 polarization measurements

$$\vec{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} N_H + N_V \\ N_H - N_V \\ N_{+45} - N_{-45} \\ N_{LHC} - N_{RHC} \end{bmatrix}$$

24 Intensity/Photon Count measurements are required to fully characterized the scattering phase matrix!

Scattering Phase Matrix

Backscattering matrix of randomly oriented scatterers

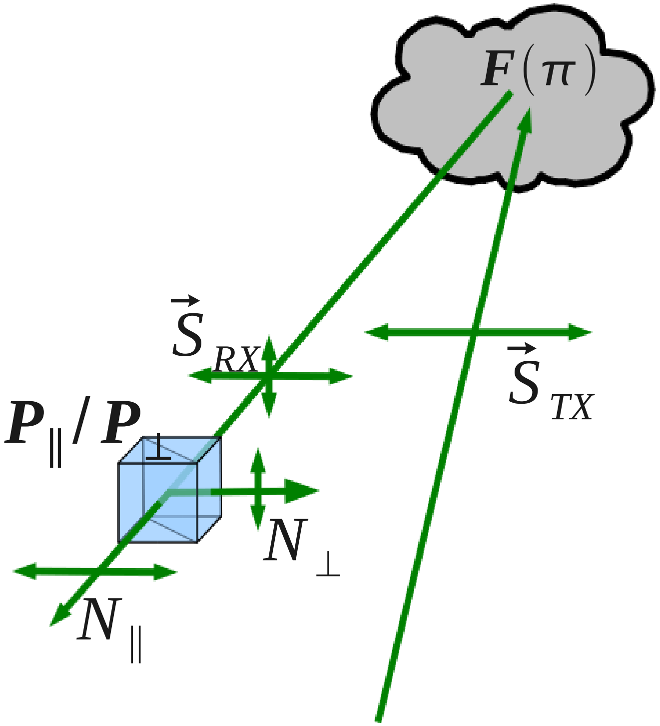
$$\boldsymbol{F}(\vec{k}_i, -\vec{k}_i, z) = \beta \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-d & 0 & 0 \\ 0 & 0 & d-1 & 0 \\ 0 & 0 & 0 & 2d-1 \end{bmatrix}$$

Conventional Polarization Lidar Technique

$$\begin{bmatrix} 1 \\ 1-d \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-d & 0 & 0 \\ 0 & 0 & d-1 & 0 \\ 0 & 0 & 0 & 2d-1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$N_{\parallel} = \vec{o} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1-d \\ 0 \\ 0 \end{bmatrix} = 1 - \frac{d}{2}$$

$$N_{\perp} = \vec{o} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1-d \\ 0 \\ 0 \end{bmatrix} = \frac{d}{2}$$

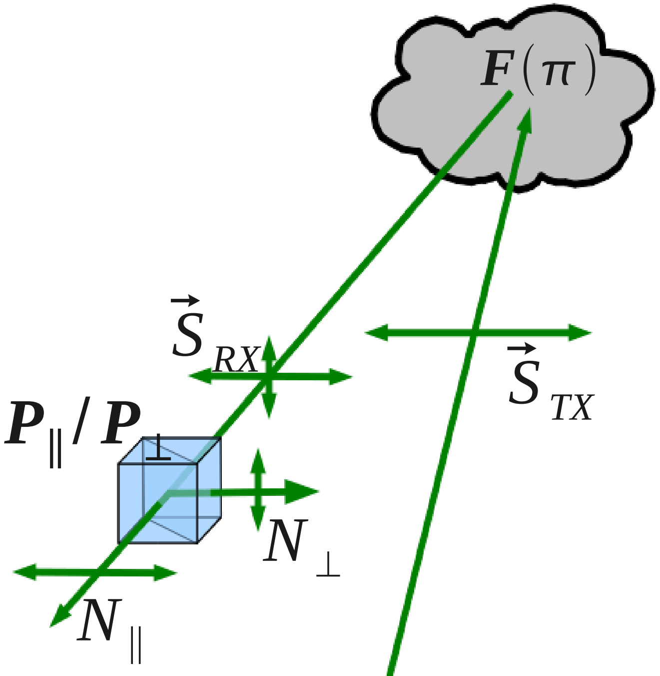


Conventional Polarization Lidar Technique

Linear: $$d = \frac{2N_{\perp}}{N_{\perp} + N_{\parallel}}$$

Circular: $$d = \frac{N_{\perp}}{N_{\perp} + N_{\parallel}}$$

Arbitrary: $$d = \frac{4N_{\perp}}{(N_{\perp} + N_{\parallel})(3 - \cos 4\Gamma)}$$



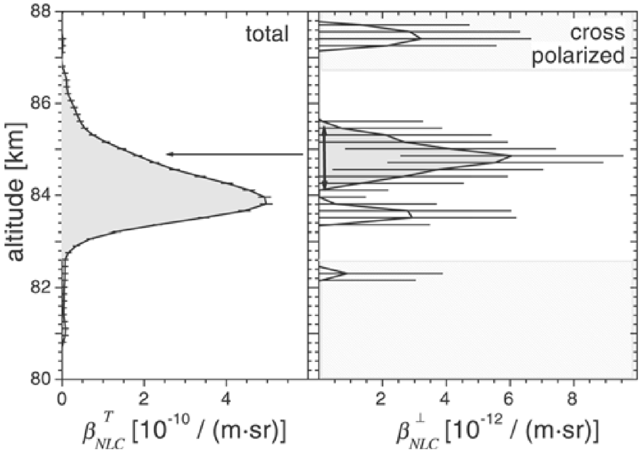
- No matter what polarization is used, the matrix can be characterized
- Results are independent of the system's polarization of operation

Information in Depolarization

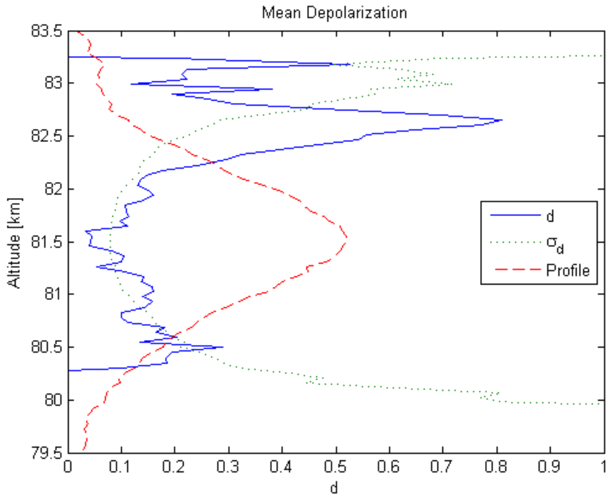
- Spherical scatterers do not depolarize (*d* = 0)
- Depolarization provides distinction in particle
  - Shape
  - Index of refraction
  - Size
- This is used for
  - Particle shape, size index, density retrievals
  - Thermodynamic phase of water (ice or liquid)
  - Polar stratospheric cloud characterization
  - Identification of dust, volcanic ash and other particulate constituents of the atmosphere

Polar Mesospheric Cloud Particles

- Form in the Mesopause at an altitude of 83 km
- Particle shape impacts
    - Aerodynamic Properties
    - Surface Area-to-Volume ratio
    - Growth and Sublimation rates
    - Area for heterogeneous chemistry



G. Baumgarten, K.H. Fricke, "Investigation of the shape of noctilucent cloud particles by polarization lidar technique" Geophys. Res. Lett.. 2002.

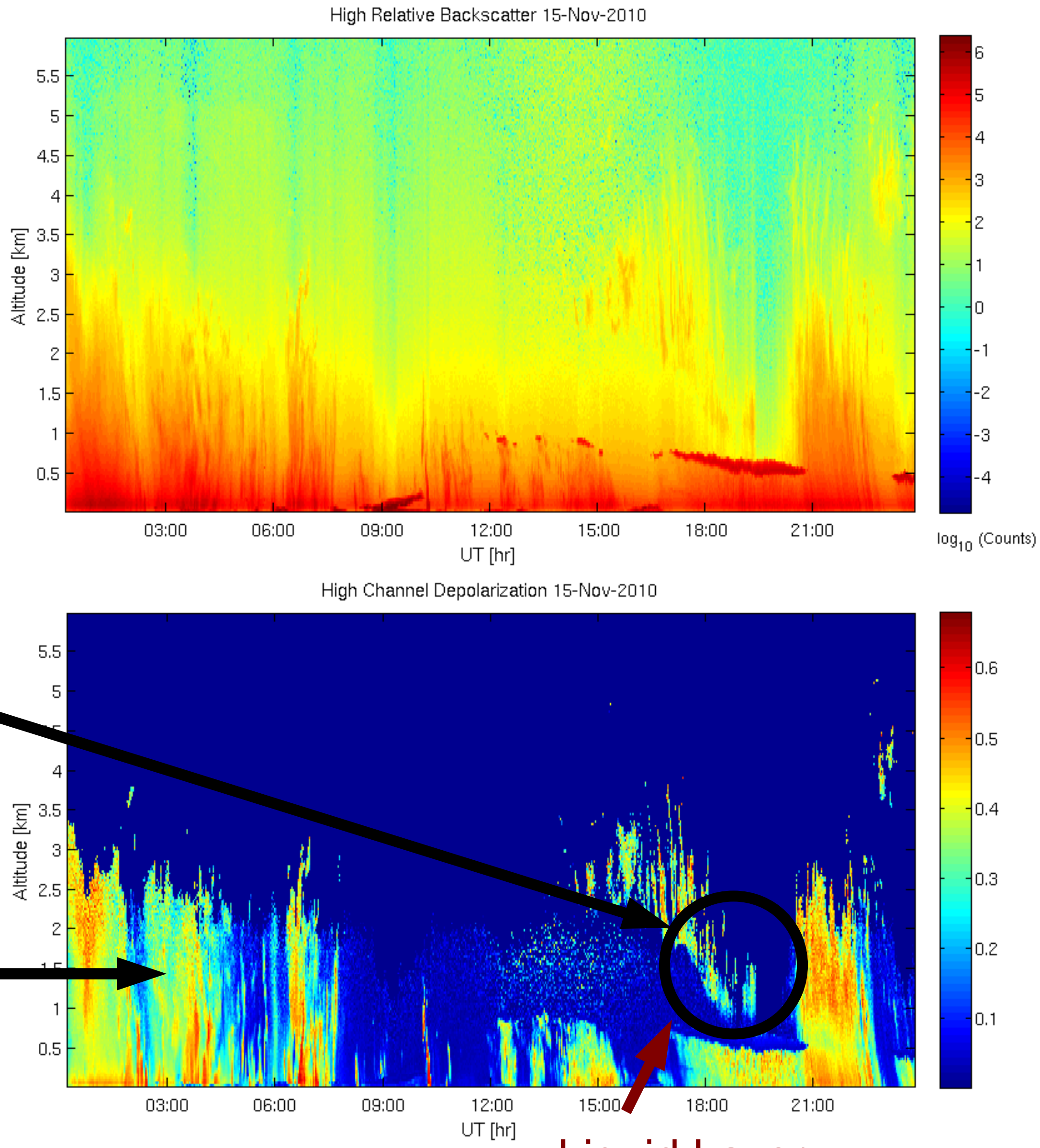


PMC particles are small compared to optical wavelengths, so *d* is expected to be less than 0.03

M. Hayman, J.P. Thayer, "Lidar Polarization Measurements of PMCs," J. Atmos. Sol. Terr. Phys. 2010.

Lidar Depolarization Data

Provides information about cloud phase



Seeder-Feeder Cloud

Snow/Ice Precipitation

Liquid Layer

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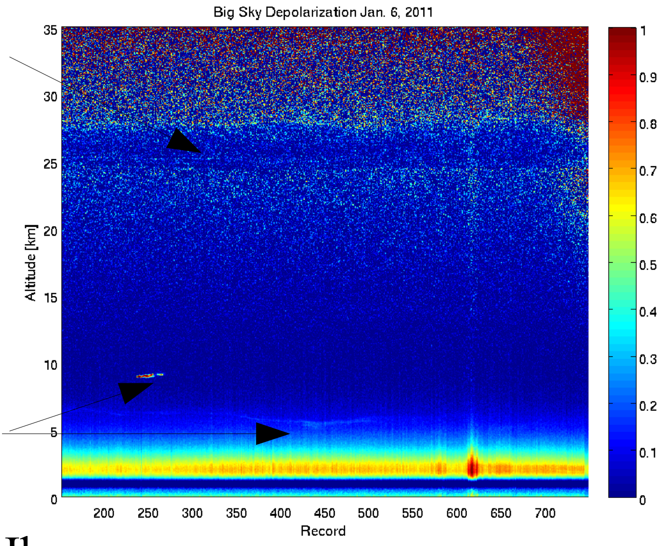
Lidar Depolarization Data

Polar Stratospheric Clouds

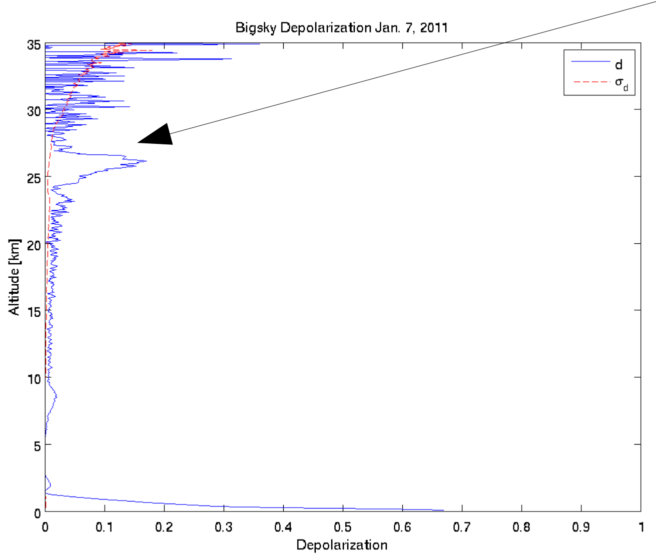
- Type I: nitric acid trihydrate (NAT)
  - Ia: Small spherical particles
  - Ib: Larger aspherical particles
- Type II: Water

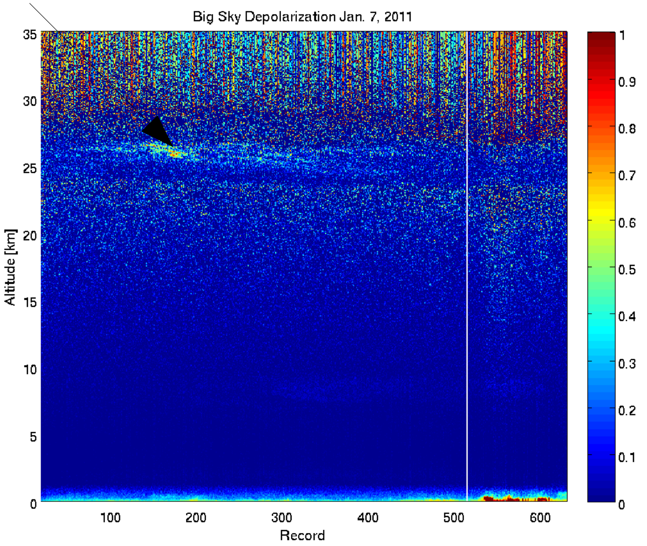
Type Ia



Ice clouds

Type Ib



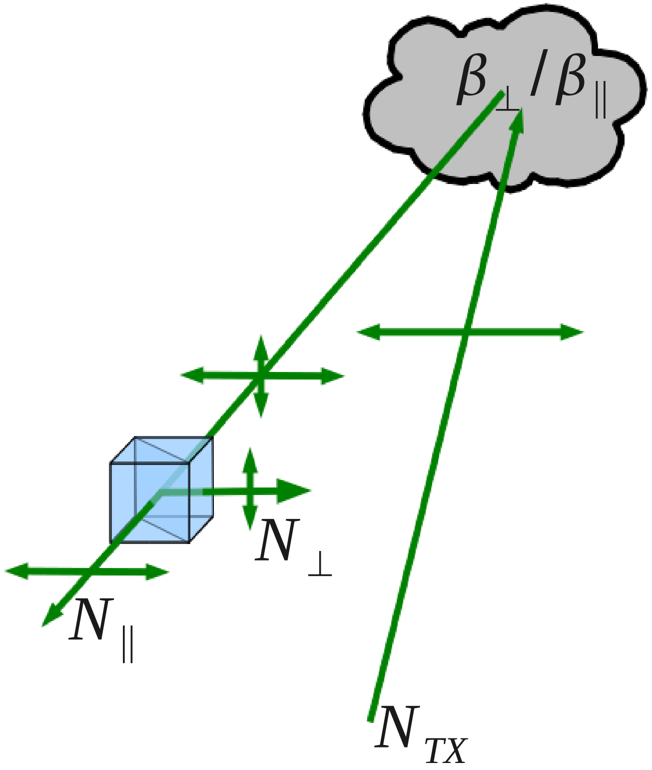


Conventional Polarization Lidar Terminology

Polarization Ratio

Conventionally polarization data is reported as the ratio of the parallel and perpendicular channels

$$\delta \equiv \frac{\beta_{\perp}}{\beta_{\|}} = \frac{N_{\perp}}{N_{\|}}$$



The data product is dependent on the system's polarization mode of operation

Linear

$$\delta_L = \frac{d}{2-d}$$

Circular

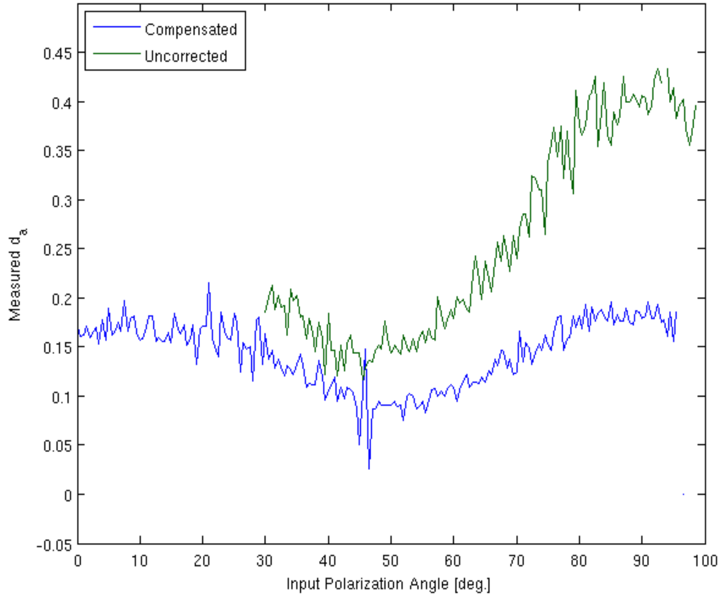
$$\delta_C = \frac{d}{1-d}$$

Arbitrary

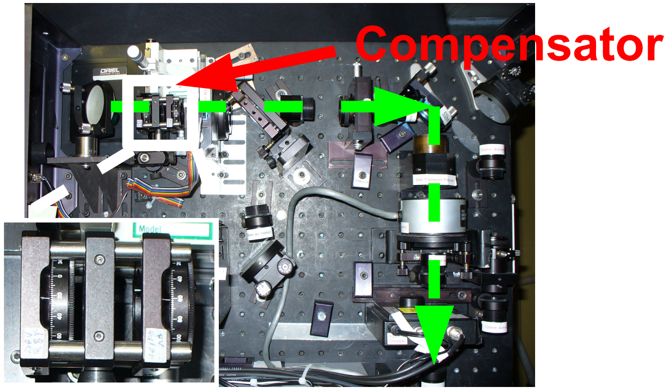
$$\delta_A = \frac{d(3-\cos 4\Gamma)}{4-d(3-\cos 4\Gamma)}$$

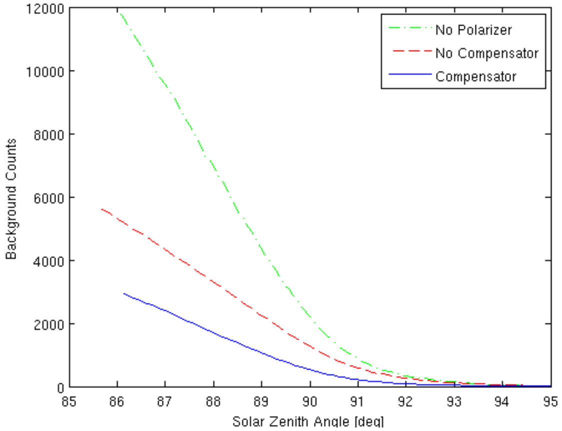
System Corrections

- Mirrors, beamsplitters and filters can change the received polarization state
- Full system characterization performed in operation
- Apply Lu-Chipman Mueller Decomposition
    - Retarding effects are canceled with two quarter wave plates and one half wave plate



$$\vec{S}_{Sca} \longrightarrow \boxed{M_{RX}=M_R M_\Delta M_D} \longrightarrow \boxed{M_R^{-1}} \longrightarrow \vec{S}_{RX}$$





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Software Correction

All of the following error sources can be folded into a single term:

- Partial Polarization of Transmitter
- Polarization Misalignment
- Receiver Depolarization
- Receiver Retardance

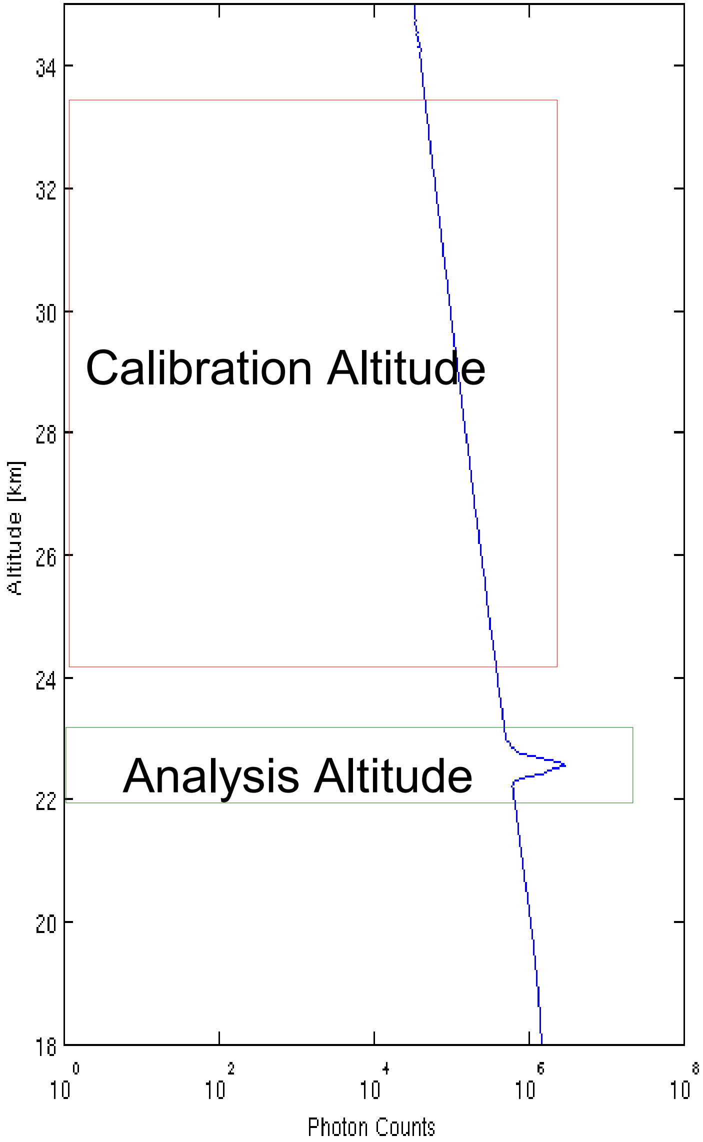
$$d(z) = 1 - \epsilon\left[1 - d_{atm}(z)\right]$$

A calibration altitude is used to solve for the error term.

$$\epsilon = \frac{1 - d(z_c)}{1 - d_M}$$

The error term is then used to produce an estimate of depolarization for all other altitudes

$$\hat{d}_{atm}(z) = 1 - \frac{1 - d(z)}{\epsilon}$$



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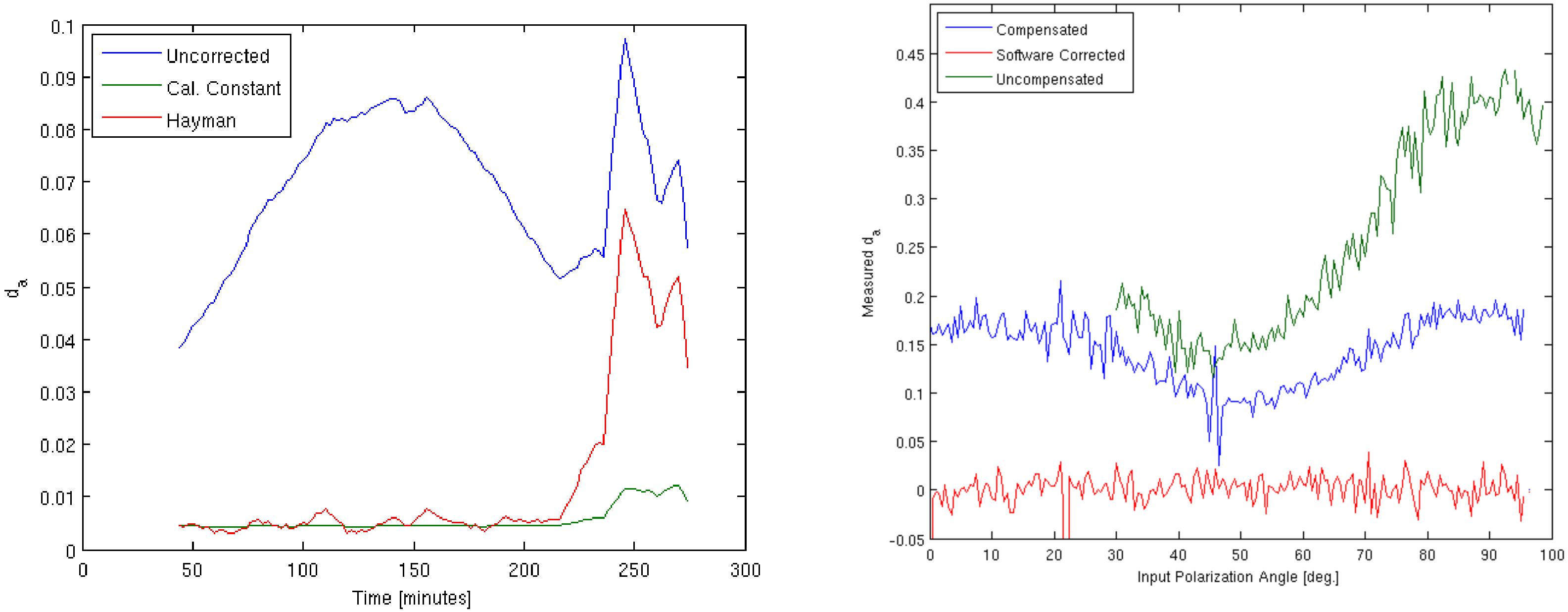
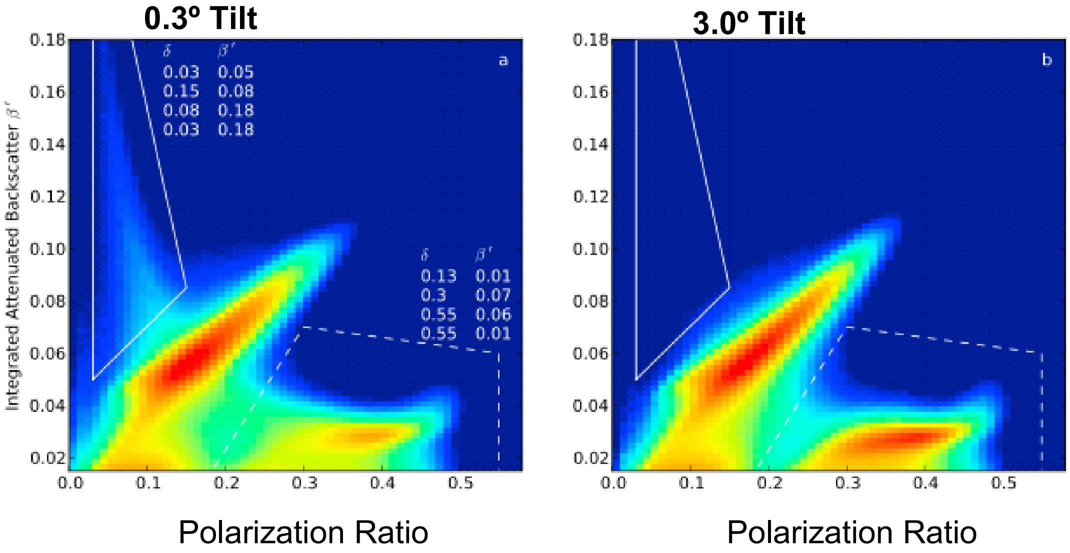
Software Correction



M. Hayman, J.P. Thayer, "Explicit Description of Polarization Coupling in Lidar Applications," Opt. Lett. **34** pp. 611-613 (2009).
M. Hayman, J.P. Thayer, "Lidar Polarization Measurements of PMCs," J. Atmos. Sol. Terr. Phys. 2010.

Detection of Oriented Ice Crystals

CALIOP Lidar on CALIPSO



Oriented Ice Crystals: Significant impact on radiative transfer.

- Specular scattering prevented other simultaneous cloud/aerosol studies
- Oriented scatterer detection lasted only 18 months
- Oriented ice crystal studies need backscatter signals in the same dynamic range as other clouds and aerosols

Noel, Chepfer, J. Geophys. Res., D00H23, 115 (2010).

Scattering Phase Matrix:

Linear Diattenuation

Oriented

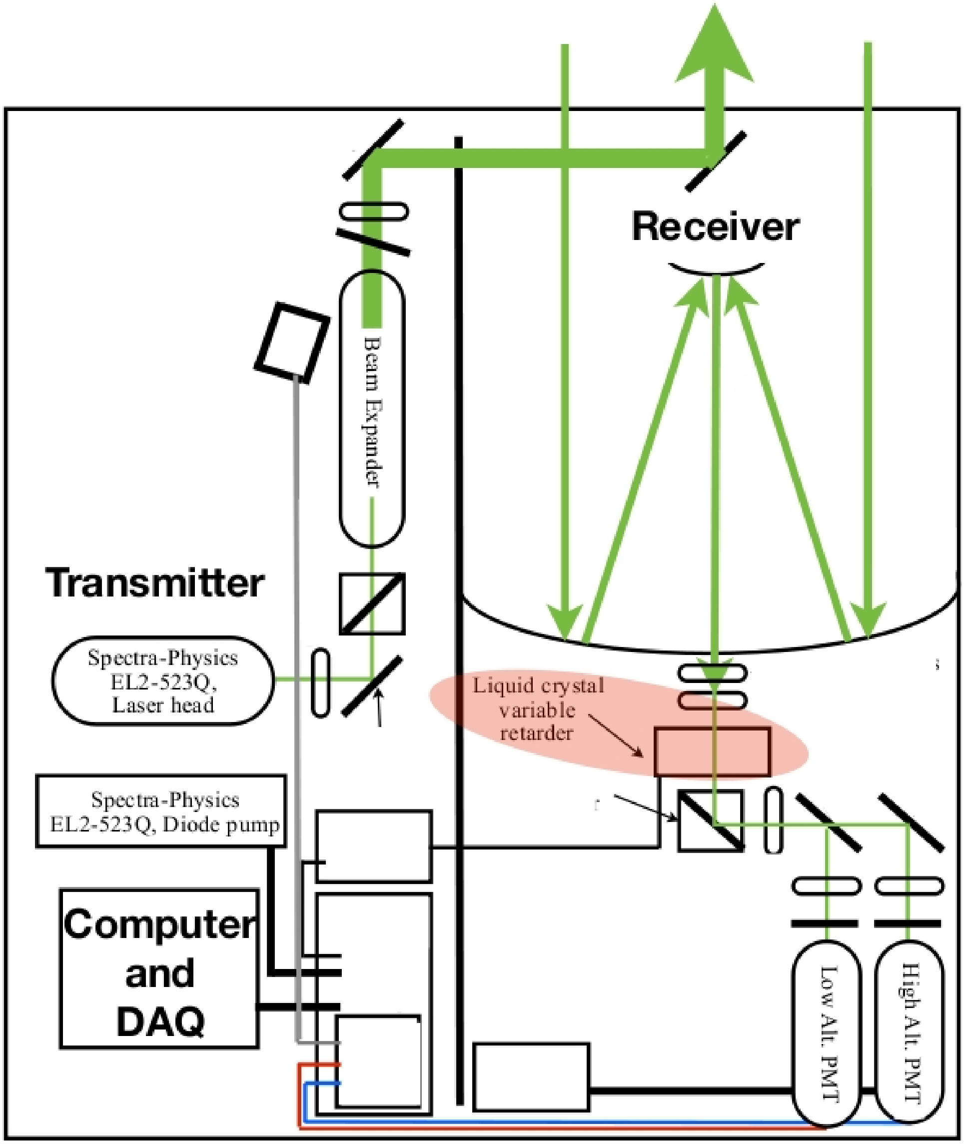
$$\boldsymbol{F}(\vec{k}_i, -\vec{k}_i, z) = \begin{bmatrix} f_{11} & f_{12} & 0 & 0 \\ f_{12} & f_{22} & 0 & 0 \\ 0 & 0 & f_{33} & f_{34} \\ 0 & 0 & -f_{34} & f_{44} \end{bmatrix}$$

Randomly Oriented

$$\boldsymbol{F}(\vec{k}_i, -\vec{k}_i, z) = \beta \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-d & 0 & 0 \\ 0 & 0 & d-1 & 0 \\ 0 & 0 & 0 & 2d-1 \end{bmatrix}$$

Cloud Aerosol Polarization And Backscatter Lidar (CAPABL)

- Deployed to Summit Camp, Greenland March 2010
- Transmits a single linear polarization
- Detects Three Polarizations using liquid crystal variable retarder
    - Parallel
    - Perpendicular
    - 45 degrees
- Two detectors for low and high altitude returns
- 30 m altitude resolution
- 5 sec temporal resolution
- 24/7 Operations with remote access and control



CAPABL Measurement Technique

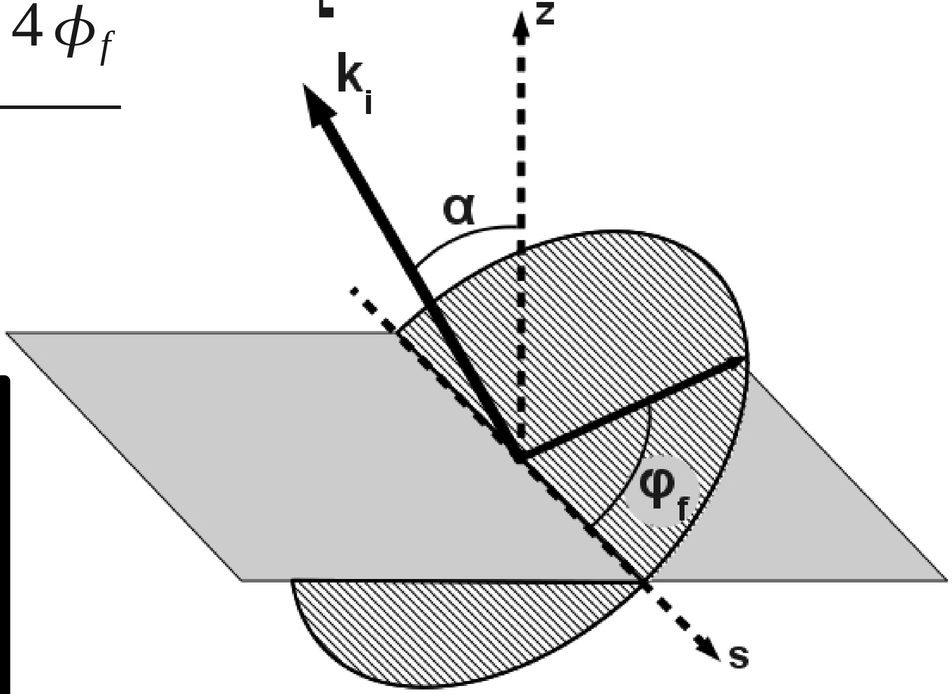
$$\begin{bmatrix} f_{11} & f_{12} & 0 & 0 \\ f_{12} & f_{22} & 0 & 0 \\ 0 & 0 & f_{33} & f_{34} \\ 0 & 0 & -f_{34} & f_{44} \end{bmatrix}$$

$$D_q = \frac{2N_{45}}{N_\perp + N_\parallel} - 1$$

$$D_q = \frac{f_{12} \sin \phi_f + \frac{1}{2}(f_{22} + f_{33}) \sin 4\phi_f}{f_{11} + f_{12} \cos 2\phi_f}$$

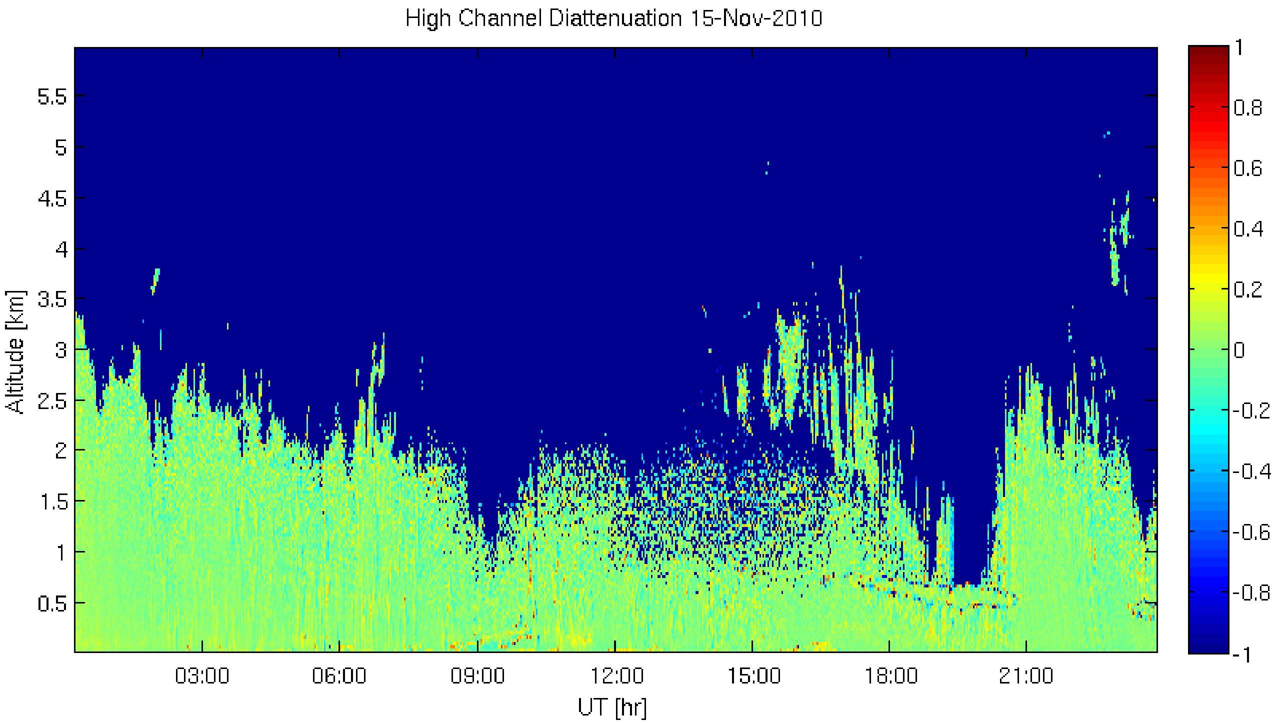
choose $\phi_f = \frac{\pi}{4}$

$$\boxed{\begin{gathered} D_q = \frac{f_{12}}{f_{11}} \\ \frac{N_\parallel - N_\perp}{N_\perp + N_\parallel} = \frac{f_{33}}{f_{11}} \end{gathered}}$$



Cloud Aerosol Polarization And Backscatter Lidar

- Deployed in March 2010
- Over a year of diattenuation data with instrument pointing zenith
  - Determine potential for false positives



CAPABL will be tilted this month to begin a campaign to identify oriented ice crystals

Conclusion

- Polarization lidar should be described in terms of Stokes vectors and Mueller matrices.
    - Results should be reported in terms of scattering matrix parameters
    - Mueller matrix descriptions of the instrument descriptions offer better solutions for system error in polarization measurements.
- Polarization can be used to study a number of particle properties relating to shape, index of refraction and size.
- Detection of linear diattenuation provides a means of identifying horizontally oriented ice crystals while providing backscatter signals in the same dynamic range as other clouds and aerosols.