Lecture 27. Lidar Simulation:
Range-Resolved and Error Analysis

- Review the overview of lidar simulation and error analysis (from Lecture 9)
- Lidar simulation contents and steps
- Range-resolved lidar simulation
- Error analysis for photon noise
- Summary

Lidar Simulation and Error Analysis

- What are the lidar simulation and error analysis and why?

- Analogy to atmospheric modeling, the lidar simulation and error analysis are lidar modeling to integrate complicated lidar remote sensing processes together.

- The basis for lidar simulation and error analysis is the lidar theory, spectroscopy, and measurement principles.

- Four Major Goals of Lidar Simulation and Error Analysis

1. Estimate of expected lidar returns (signal level and shape)
2. Analysis of expected measurement precision & resolution, i.e., errors (uncertainty) introduced by photon noise and trade-off among lidar parameters (e.g., off-zenith angle determination)
3. Analysis of expected measurement accuracy & precision (caused by uncertainty in system parameters) and lidar measurement sensitivity to atomic, lidar, and atmospheric parameters
4. Forward model to test data retrieval code and metrics

Lidar Simulation and Error Analysis

❑ Merits, Functions, or Applications of each aspect

1. Estimate of expected lidar returns (signal level and shape)

- Show what signals you can expect to see on real systems;
- Assess the potential of a lidar system;
- Comparison to actual signals to help diagnose lidar efficiency and other system problems, including reality check

2. Analysis of expected measurement precision & resolution, i.e., errors (uncertainty) introduced by photon noise

- Help determine needed laser power, receiver aperture, system efficiency, filter width, FOV, etc
- Help determine whether daytime measurement is doable
- Trade-off between resolution and precision
- Determination of optimum lidar parameters

Lidar Simulation and Error Analysis

- ❑ Merits, Functions, or Applications of each aspect

3. Analysis of expected measurement accuracy & precision (caused by uncertainty in system parameters) and lidar measurement sensitivity to atomic, lidar, and atmospheric parameters

- Help define requirements on lidar system parameters, like frequency accuracy, linewidth, stability, etc.
- Provide a guideline to lidar system development
- Help determine measurement accuracy (bias)

4. Forward model to test data retrieval code and metrics

- Test data retrieval code and its sensitivity to noise
- Help compare different metrics to minimize cross-talk between different measurement errors

Lidar Simulation Contents and Steps

- Three major levels of lidar simulation are

1. Envelope estimation (non-range-resolved): integrated photon returns from an entire layer or region

2. Range-resolved simulation: photon returns from different ranges

3. Range-resolved and spectral-resolved simulation: photon returns from different ranges and distribution in spectrum at each range

- Other factors to consider:

1) Background (MODTRAN code is a good resource),

2) Noise (Poisson distribution),

3) Geometry, 4) Polarization, 5) ……

- Three main steps for range-resolved lidar simulation

(1) Initialization: Define constants and parameters

(2) Simulation of photon counts vs. altitude (range)

\- Rayleigh, resonance fluorescence, background, aerosol, noise

(3) Computation of signal-to-noise ratio (SNR) for further error analysis

Range-Resolved Simulation: Initialization

- Initialization: to define constants and parameters

1) Fundamental constants: c, h, Q_e, M_e, ...
2) Atomic parameters: resonance wavelengths, frequencies, strength, oscillator strength, A_{ki}, degeneracy factors, isotopes, abundance, ...

or Molecular parameters: CO_2 structure, etc.

3) Lidar transmitter and receiver parameters: pulse energy, linewidth, frequency, repetition rate, telescope diameter, R, T, η_{QE}, T_{IF}, ...
4) User-controlled parameters: integration time (shots number), bin width Δz, pointing up or down, base altitude, pointing angle, model choice, ...
5) Atmospheric parameters (taken from model, e.g., MSIS00): number density, pressure, temperature, transmission, ...
6) Na/K/Fe layer parameters: distribution, Z_0, σ_{rms}, Abdn, ...

Range-Resolved Simulation: N(R) or N(z)

- Photon counts vs. altitude: Sum of the following terms

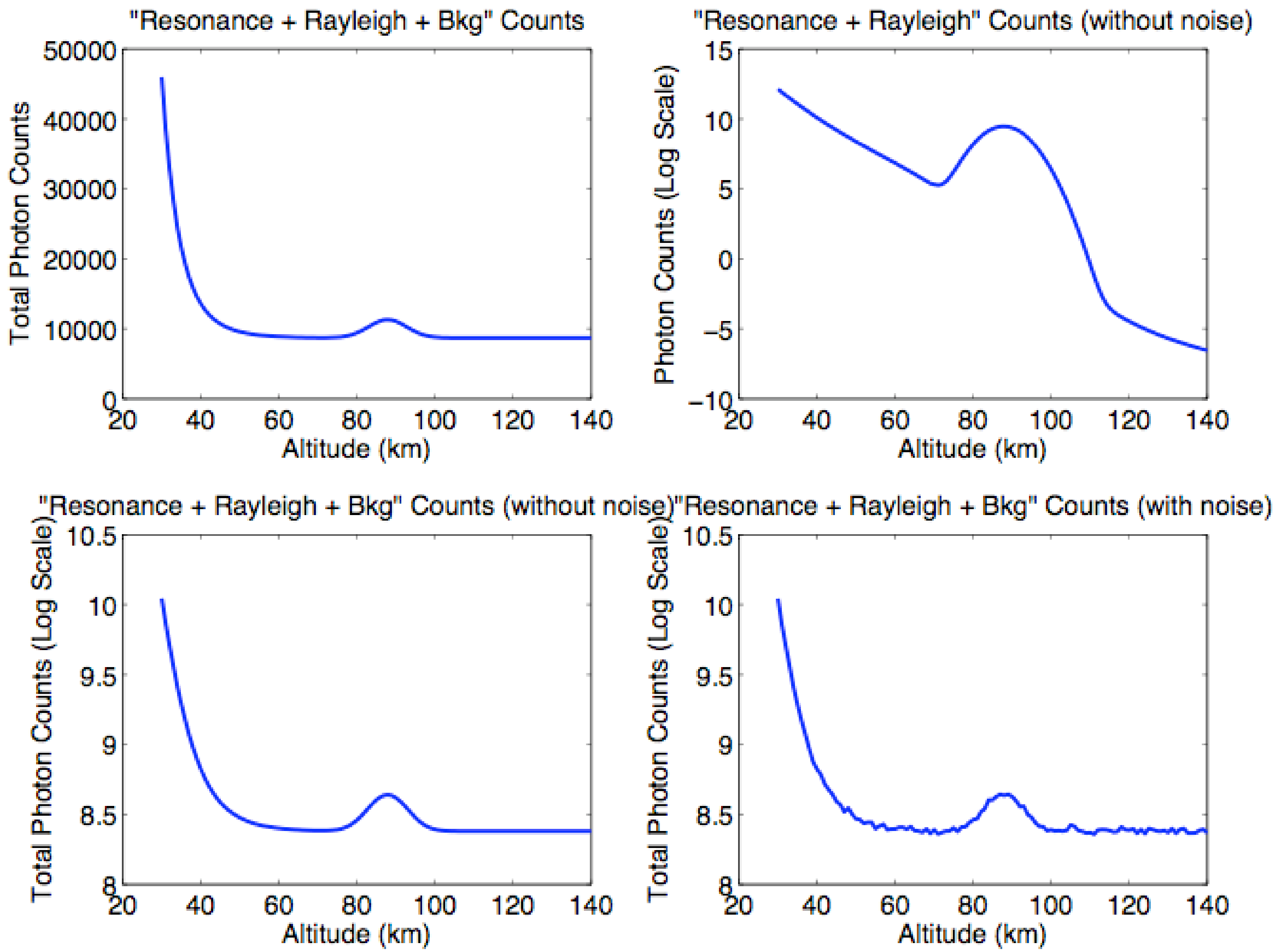
1) Rayleigh scattering signals: take number density distribution profile n(z) from a model, e.g., MSIS00, set bin width $\Delta z = 0.5$ km, compute $\sigma_R \cdot n(z) \cdot \Delta z$ or further considering the pointing angle, then follow the normal simulation procedure for each bin
2) Resonance fluorescence signals: compute the Na/K/Fe density distribution profile from Z_0, σ_{rms}, Abdn, compute effective cross-section σ_{eff} from atomic and laser spectroscopy, compute $\sigma_{eff} \cdot n_{Na}(z) \cdot \Delta z$ or further considering the pointing angle, consider the transmission (extinction) caused by atomic absorption, follow the normal simulation procedure for each bin
3) Aerosol signals: usually in specific regions
4) Background counts: scale from real measurements or use MODTRAN code to compute background (needing lidar parameters, like FOV, filter function, etc.)
5) Noise: Poisson distribution from photon counts $\Delta N(z) = \sqrt{N(z)}$

Range-Resolved Simulation: N(R) or N(z)

- What would you do if your lidar is used to study non-resonance interactions?

1) Rayleigh scattering signals: you will always have Rayleigh signals no matter what lidar you use
2) Non-resonance fluorescence: Raman, differential absorption, fluorescence, aerosol, or reflection signals - need to get some rough distribution, cross-section, reflectivity, etc. information
3) Aerosol signals: playing a role in both backscatter coefficient and atmospheric extinction, also needing some distribution information - could be what you want to detect or just as background or noise
4) Background counts: you will always have some
5) Noise: you will always have some

Example: Resonance + Rayleigh + Background + Photon Noise



Range-Resolved Simulation: SNR

- Computation of signal-to-noise ratio (SNR) from simulation results

$$SNR(z) = \frac{N_{Signal}(z)}{\Delta N_{Signal}(z)}$$

- However, we must consider how the N(z) is obtained

$$N_{Signal}(z) = N_{Total}(z) - N_B$$

$$\Delta N_{Signal}(z) = \Delta N_{Total}(z) - \Delta N_B$$

$$\left(\Delta N_{Signal}(z)\right)_{rms} = \sqrt{\left(\Delta N_{Total}(z)\right)^2 + \left(\Delta N_B\right)^2}$$

$$\left(\Delta N_{Signal}(z)\right)_{rms} = \sqrt{N_{Total}(z) + \left(\Delta N_B\right)^2} \approx \sqrt{N_{Total}(z)}$$

Error Analysis for Photon Noise

- In this lecture we will deal with the uncertainty caused by photon noise, and in the next lecture we will systematically discuss errors.
- Photon noise causes the uncertainty in the measured photon counts, then the photon count uncertainty will lead to the uncertainty in the temperature and wind ratios R_T and R_W, which will result in errors in the inferred temperature and wind T and W. -- Error propagation procedure
- How would we derive the T and W errors from the photon noise? - Through the use of differentials of the corresponding ratios R_T and R_W
- For 3-frequency technique, the relative errors of R_T and R_W are

$$\frac{\Delta R_T}{R_T} = \frac{\left(1+\frac{1}{R_T}\right)^{1/2}}{\left(N_{f_a}\right)^{1/2}}\left[1+\frac{B}{N_{f_a}}\frac{\left(1+\frac{2}{R_T{}^2}\right)}{\left(1+\frac{1}{R_T}\right)}\right]^{1/2}$$

$$\frac{\Delta R_W}{R_W} = \frac{\left(1+\frac{1}{R_W}\right)^{1/2}}{\left(N_{f_+}\right)^{1/2}}\left[1+\frac{B}{N_{f_+}}\frac{\left(1+\frac{1}{R_W{}^2}\right)}{\left(1+\frac{1}{R_W}\right)}\right]^{1/2}$$

where B is the background photon count.

Derivation of Measurement Errors

- We use the temperature error derivation for 3-freq Na lidar as an example to explain the error analysis procedure.

- For 3-frequency technique, we have the temperature ratio

$$R_T = \frac{\sigma_{eff}(f_+) + \sigma_{eff}(f_-)}{\sigma_{eff}(f_a)} = \frac{N(f_+) + N(f_-)}{N(f_a)}$$

- Through this ratio R_T or further through the effective cross-section, the temperature T is an implicit function of R_T, laser frequencies f_a, f_+, f_-, laser linewidth σ_L, radial wind, etc. Each parameter could have some uncertainty or error, leading to errors in the measured temperature.

- Therefore, the temperature error is given by the derivatives

$$\Delta T = \frac{\partial T}{\partial R_T}\Delta R_T + \frac{\partial T}{\partial f_a}\Delta f_a + \frac{\partial T}{\partial f_\pm}\Delta f_\pm + \frac{\partial T}{\partial \sigma_L}\Delta \sigma_L + \frac{\partial T}{\partial v_R}\Delta v_R$$

Error Derivation: Error Propagation

- The root-mean-square (rms) temperature error is given by

$$(\Delta T)_{rms} = \sqrt{\left(\frac{\partial T}{\partial R_T}\Delta R_T + \frac{\partial T}{\partial f_a}\Delta f_a + \frac{\partial T}{\partial f_\pm}\Delta f_\pm + \frac{\partial T}{\partial \sigma_L}\Delta \sigma_L + \frac{\partial T}{\partial v_R}\Delta v_R\right)^2}$$

- If the error sources are independent from each other, then the means of cross terms are zero. Then we have

$$(\Delta T)_{rms} = \sqrt{\left(\frac{\partial T}{\partial R_T}\Delta R_T\right)^2 + \left(\frac{\partial T}{\partial f_a}\Delta f_a\right)^2 + \left(\frac{\partial T}{\partial f_\pm}\Delta f_\pm\right)^2 + \left(\frac{\partial T}{\partial \sigma_L}\Delta \sigma_L\right)^2 + \left(\frac{\partial T}{\partial v_R}\Delta v_R\right)^2}$$

- The above error equation indicates that many laser parameters and radial wind errors could affect the inferred temperature because they all influence the effective cross sections. In the meantime, photon noise can cause uncertainty in the ratio R_T, resulting in temperature error.

Error Derivation: Implicit Differentiation

- How to derive the error coefficients, like $\frac{\partial T}{\partial R_T}, \frac{\partial T}{\partial f_a}, etc.$?
- We may use the implicit differentiation through R_T as below:

$$\Delta T = \Delta R_T \left(\frac{\partial R_T}{\partial R_T} \Big/ \frac{\partial R_T}{\partial T} \right) + \Delta f_a \left(\frac{\partial R_T}{\partial f_a} \Big/ \frac{\partial R_T}{\partial T} \right) + \Delta f_\pm \left(\frac{\partial R_T}{\partial f_\pm} \Big/ \frac{\partial R_T}{\partial T} \right)$$
$$+ \Delta \sigma_L \left(\frac{\partial R_T}{\partial \sigma_L} \Big/ \frac{\partial R_T}{\partial T} \right) + \Delta \mathrm{v_R} \left(\frac{\partial R_T}{\partial \mathrm{v_R}} \Big/ \frac{\partial R_T}{\partial T} \right)$$

- For the photon-noise induced temperature error,

$$\Delta T = \frac{1}{\partial R_T / \partial T} \cdot \Delta R_T = \frac{R_T}{\partial R_T / \partial T} \cdot \frac{\Delta R_T}{R_T}$$

- The relative error of R_T has been given in terms of measured signal and background photon counts on the previous slide.

Derivation of Error Coefficients

- The temperature error coefficient can be derived numerically

$$\frac{R_T}{\partial R_T / \partial T} = \frac{R_T}{[R_T(T+\delta T) - R_T(T)]/\delta T}$$

- Two approaches to derive the above numerical solution:

(1) One way is to use the equation of R_T in terms of cross sections. You don't have to go through the entire simulation process each time when you change the temperature, but just calculate the R_T from the effective cross section.

$$R_T = \frac{\sigma_{eff}(f_+,T) + \sigma_{eff}(f_-,T)}{\sigma_{eff}(f_a,T)}$$

(2) Another way is to use the equation of R_T in terms of photon counts, and then go through the entire simulation procedure to re-compute R_T for each new temperature. This method is more universal than the first approach, because not all cases could have a R_T written in terms of pure physical cross sections.

$$R_T = \frac{N(f_+,T) + N(f_-,T)}{N(f_a,T)}$$

Temperature Error Due to Photon Noise

- Integrating above equations together, we obtain the equation for the temperature error caused by photon noise as below:

$$\Delta T = \frac{R_T}{\partial R_T / \partial T} \cdot \frac{\Delta R_T}{R_T} = \frac{R_T}{[R_T(T+\delta T) - R_T(T)]/\delta T} \cdot \frac{\left(1 + \frac{1}{R_T}\right)^{1/2}}{\left(N_{f_a}\right)^{1/2}} \left[1 + \frac{B}{N_{f_a}} \frac{\left(1 + \frac{2}{R_T{}^2}\right)}{\left(1 + \frac{1}{R_T}\right)}\right]^{1/2}$$

- The photon counts in the above equation can be written in terms of signal to noise ratio (SNR), if it is more convenient or desirable for some analyses.

Summary

- Lidar simulation and error analysis are the "lidar modeling". It is an integration of complicated lidar remote sensing procedure.
- The key is still our understanding of the lidar theory and the physical interactions between the laser light and the objects you want to study. Only when we clearly understand the interactions in the atmosphere and the entire lidar detection procedure could we do good lidar simulation and error analysis.
- Calculation of errors for ratio technique utilizes the differentiation of the metric ratios as described in the textbook. It works for both systematic and random errors.
- Reference our textbook: section 5.2.2.5.2
- Homework project #4 is on range-resolved lidar simulation and photon-noise error analysis.