# Polarization Considerations in Lidar Measurements 

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## Polarization of light

The pattern traced out in time by the tip of the electric field vector, for light travelling towards the observer


Linear
Two components in phase


Circular
Two equal components $90^{\circ}$ out of phase


Elliptical
Arbitrary amplitudes and/or phase

## Stokes Vectors

The Stokes vector S = [I, Q, U, V] completely describes the polarization state of a beam of light.

I describes the total intensity,
$Q$ describes the intensity on the $x$ and $y$ axes,
$U$ describes the intensity on the $+45^{\circ}$ and $-45^{\circ}$ axes, and
$V$ describes the intensity that is right-hand circular (RHC) or left-hand circular (LHC).

Stokes vectors are often normalized such that the first element $I$ is equal to unity.

## Stokes Vectors (Cont.)

## Examples:

$S=[1,1,0,0$, d describes a beam of light linearly polarized along the $x$-axis.
$S=[1,0,0,0]$ describes light that is completely unpolarized.

## Mueller Matrices

Anything that modifies the polarization state of a beam of light (such as an optical element or a scattering event), can be represented by a $4 \times 4$ matrix known as the Mueller Matrix.

The Mueller matrix for backscattering by spherical particles is

$$
M=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{array}\right]
$$

## Example Calculation

What happens when unpolarized light hits a linear polarizer?
$\frac{1}{2}\left[\begin{array}{llll}1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{array}\right]\left[\begin{array}{l}1 \\ 0 \\ 0 \\ 0\end{array}\right]=\left[\begin{array}{l}\text { Resulting Stokes vector : } \\ {\left[\begin{array}{l}1 / 2 \\ 1 / 2 \\ 0 \\ 0\end{array}\right]<\text { polarization }}\end{array}\right.$

One-half of the intensity comes through, linearly polarized

## Our Legacy

$$
P(R)=\underbrace{P_{0} \frac{c \tau}{2} A \eta \frac{O(R)}{R^{2}}}_{\text {instrumental/geometrical }} \underbrace{}_{\text {atmospheric }} \beta(R) \exp \left[-2 \int_{0}^{R} \alpha(r) d r\right]
$$

$$
\begin{gathered}
P_{\|}(R)=P_{0} \frac{c \tau}{2} A \eta_{\|} \frac{O(R)}{R^{2}} \beta_{\|}(R) \exp \left[-2 \int_{0}^{R} \alpha_{\|}(r) d r\right] \\
P_{\perp}(R)=P_{0} \frac{c \tau}{2} A \eta_{\perp} \frac{O(R)}{R^{2}} \beta_{\perp}(R) \exp \left[-2 \int_{0}^{R} \alpha_{\perp}(r) d r\right]
\end{gathered}
$$

Depolarization Ratio $\delta=\frac{P_{\perp}}{P_{\|}}=\frac{\beta_{\perp}}{\beta_{\|}}$
[Adapted from Schotland et al. (1971); notation from Wandinger (2006)]

## The First Problem - Demons!




## The Second Problem

$$
\delta=\frac{P_{\perp}}{P_{\|}}
$$

"... before 1930 it was customary to denote the ratio $I_{I} / I_{r}$ of a partially plane-polarized beam as its depolarization, a term that had no direct relation to scattering theories." [van de Hulst (1957)]

Use of this parameter

1) obscures the physics, and
2) leads to the strange-looking term

$$
\left[\frac{1-\delta}{1+\delta}\right]
$$

## Why Not in LIDAR?

We didn't have the scattering matrix! There were some early efforts:

$$
F=\left[\begin{array}{cccc}
a_{1} & 0 & 0 & b_{5} \\
0 & a_{2} & 0 & 0 \\
0 & 0 & -a_{2} & 0 \\
b_{5} & 0 & 0 & a_{4}
\end{array}\right]
$$

(1984)

$$
F=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & \frac{1-\delta}{1+\delta} & 0 & 0 \\
0 & 0 & F_{33} & 0 \\
0 & 0 & 0 & F_{44}
\end{array}\right]
$$

(2000)

## The Scattering Matrix

The Mueller matrix for backscattering by random non-spherical particles was recently shown to be

$$
M_{a t m}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1-d & 0 & 0 \\
0 & 0 & d-1 & 0 \\
0 & 0 & 0 & 2 d-1
\end{array}\right]
$$

Note that this matrix has only one parameter, $d$, which is the fraction of the scattered light that is depolarized

## Linear Depolarization Lidar



The LIDAR
transmits linearly polarized light and it employs two receivers, with analyzers parallel and perpendicular to the transmitted polarization

## Analysis - Parallel

The stokes vector is found from

$$
\frac{1}{2}\left[\begin{array}{cccc}
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right]\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1-d & 0 & 0 \\
0 & 0 & d-1 & 0 \\
0 & 0 & 0 & 2 d-1
\end{array}\right]\left[\begin{array}{l}
1 \\
1 \\
0 \\
0
\end{array}\right]=\left[\begin{array}{c}
1-d / 2 \\
1-d / 2 \\
0 \\
0
\end{array}\right]
$$

Which can be decomposed to find

$$
\left[\begin{array}{c}
1-d / 2 \\
1-d / 2 \\
0 \\
0
\end{array}\right]=(1-d)\left[\begin{array}{l}
1 \\
1 \\
0 \\
0
\end{array}\right]+(d / 2)\left[\begin{array}{l}
1 \\
1 \\
0 \\
0
\end{array}\right]
$$

All of the polarized fraction plus onehalf of the unpolarized fraction of the received light passes through the parallel analyzer.

## Analysis - Perpendicular

The stokes vector is found from

$$
\frac{1}{2}\left[\begin{array}{cccc}
1 & -1 & 0 & 0 \\
-1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right]\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1-d & 0 & 0 \\
0 & 0 & d-1 & 0 \\
0 & 0 & 0 & 2 d-1
\end{array}\right]\left[\begin{array}{l}
1 \\
1 \\
0 \\
0
\end{array}\right]=(d / 2)\left[\begin{array}{c}
1 \\
-1 \\
0 \\
0
\end{array}\right]
$$

One-half of the unpolarized fraction of the received light passes through the perpendicular analyzer.

## Physical Description

Physically, a lidar measurement of linear depolarization has this simple description:

1) the lidar system transmits a beam of polarized laser light,
2) the atmospheric backscattering process leaves part of the received light in the original polarization state and randomly depolarizes the rest,
3) the parallel analyzer passes all of the light in the original state plus one-half of the unpolarized light, and
4) the perpendicular analyzer passes one-half of the unpolarized light.

## Finding $d$

We have shown that

$$
\begin{aligned}
& S_{\|} \propto(1-d / 2) \text { and } \\
& S_{\perp} \propto(d / 2)
\end{aligned}
$$

The depolarization parameter $d$ is therefore found from the LIDAR signals by using the equation

$$
d=\frac{2 S_{\perp}}{S_{\|}+S_{\perp}}
$$

## Correspondence with Legacy

Unpolarized fraction $\quad d=\frac{2 \delta}{1+\delta}$
Polarized fraction $1-d=\frac{1-\delta}{1+\delta}$
Example: CALIPSO researchers reported the relationship between depolarization and multiple scattering as

$$
\frac{1}{A_{S}}=\left(\frac{1+\delta}{1-\delta}\right)^{2}
$$

This relation should be rewritten as

$$
A_{S}=(1-d)^{2}
$$

The backscatter factor is equal to the square of the polarized fraction of the received light!

## Comparison of $\delta$ to $d$



## Correspondence with Legacy (2)

$$
\begin{aligned}
& P_{\|}(R)=P_{0} C_{\|} \frac{c \tau}{2} A \eta \frac{O(R)}{R^{2}} f_{\|}(d) \beta(R) \exp \left[-2 \int_{0}^{R} \alpha(r) d r\right] \\
& P_{\perp}(R)=P_{0} C_{\perp} \frac{c \tau}{2} A \eta \frac{O(R)}{R^{2}} f_{\perp}(d) \beta(R) \exp \left[-2 \int_{0}^{R} \alpha(r) d r\right]
\end{aligned}
$$

For linear polarization, $\quad f_{\|}(d)=(1-d / 2)$ and $f_{\perp}(d)=(d / 2)$
so

$$
\beta_{\|}=(1-d / 2) \beta \text { and } \beta_{\perp}=(d / 2) \beta
$$

## Why Not Keep Legacy Notation?

1. Mixes instrumental and atmospheric parameters;
2. Continued use of legacy risks continued misinterpretation;
3. Different set of equations required for each type of lidar;
4. Parameter $d$ should appear in lidar equations.

## Other sources of depolarization



1. Multiple Scattering

Multiple scattering requires grazing incidence, which depolarizes the laser light.

## Other sources of depolarization

2. Mirrors


Mirrors in scanning systems require incidence at varying angles, causing varying reflectance.

## Other sources of depolarization


3. Beamsplitters

The ratios of the beam intensities depend on the polarization state of the incident beam.

## Other sources of depolarization

4. Oriented Plates

Hex plates in cirrus are oriented, acting like tiny mirrors with high reflectance and no depolarization

## Conclusions



1. Polarization-sensitive lidar is a powerful technique.
2. The lidar legacy is misleading and confusing.
3. Lidar should be harmonized with scattering theory and optical physics.
4. There are several sources of depolarization, both atmospheric and instrumental.
5. Everything should be analyzed with Stokes vectors and Mueller matrices.
