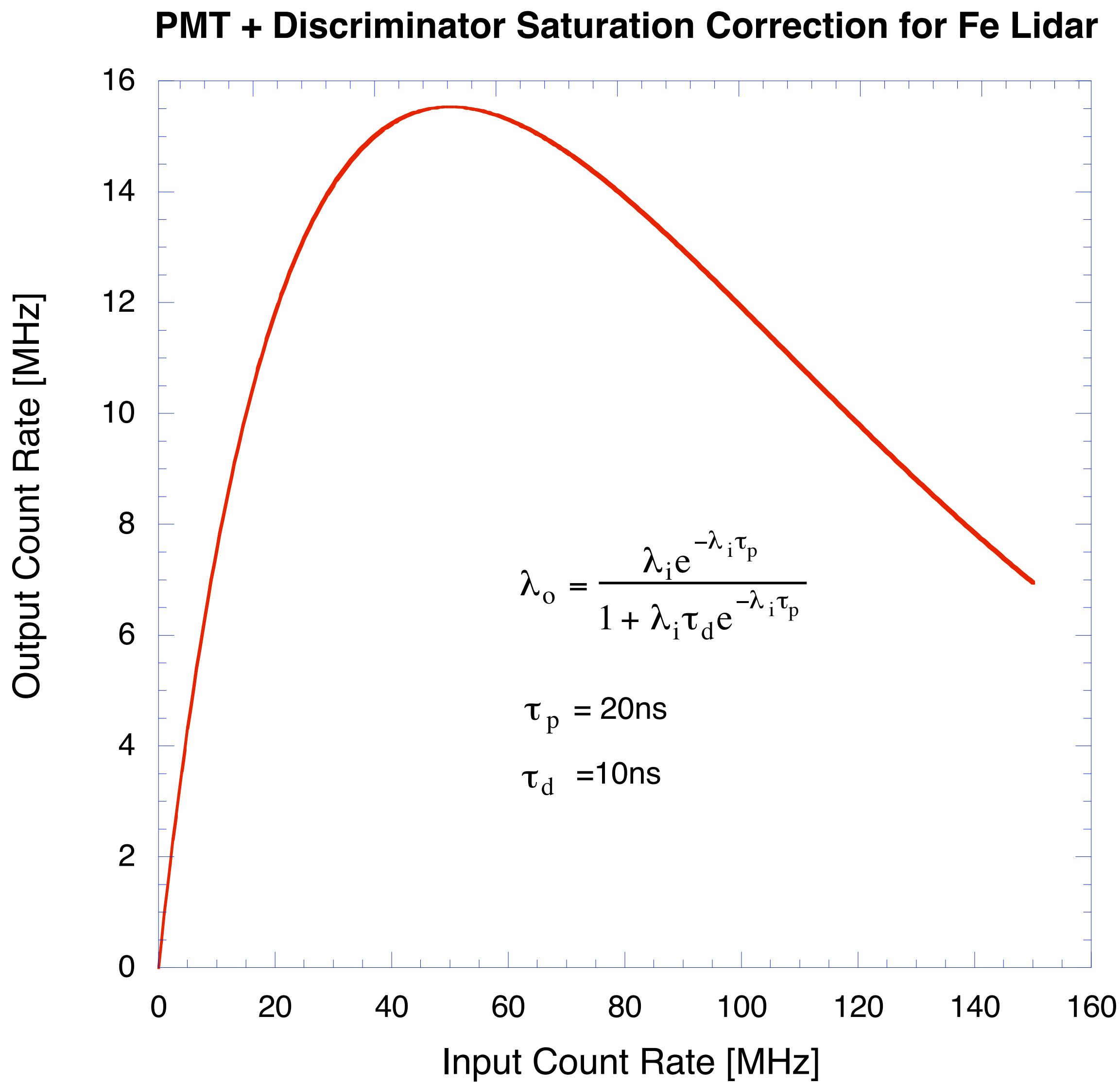
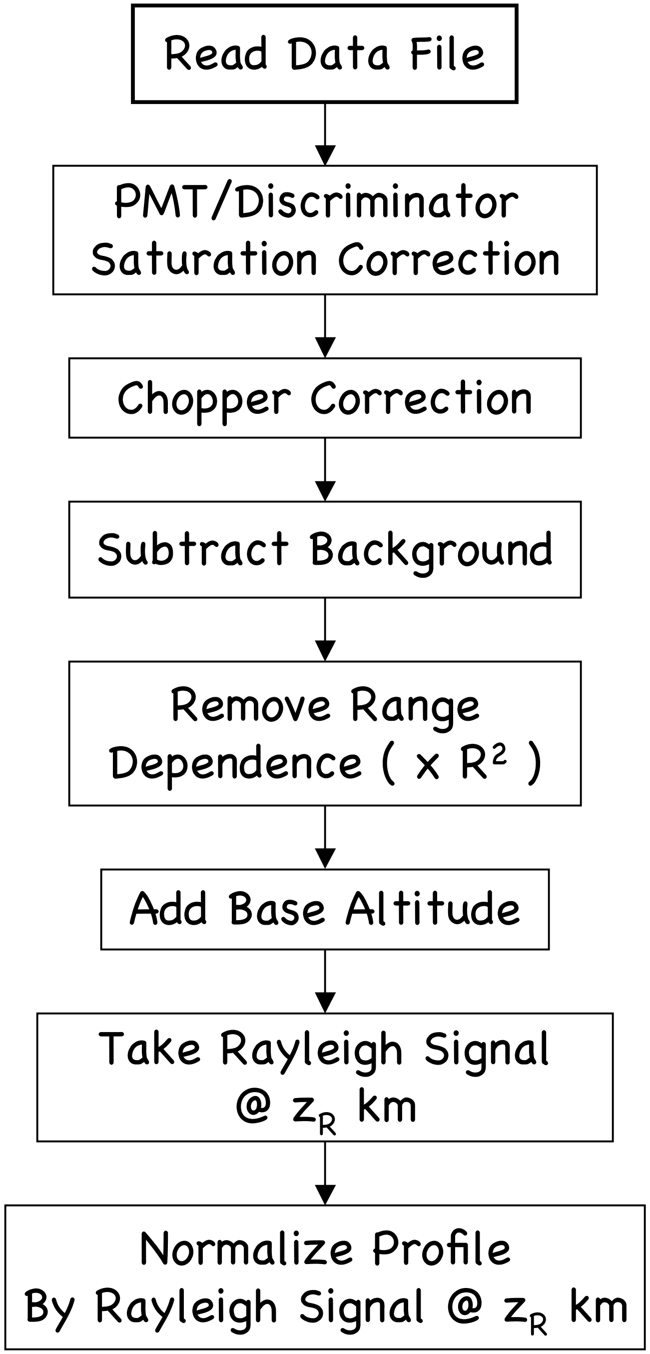
Lecture 14. Lidar Data Inversion (2)

- Review of Preprocess
- Main Process Procedure to Derive T and V_R Using Ratio Doppler Technique
- Derivations of n_c from narrowband resonance Doppler lidar
- Derivation of β
- Derivation of n_c from broadband resonance lidar
- Summary

(By Chihoko Yamashita)

Answer for Question…
Nonlinearity of PMT + Discriminator





Review of Preprocess Procedure for Na Doppler Lidar

- Read data: for each set, and calculate T, W, and n for each set
- PMT/Discriminator saturation correction
- Chopper correction
- Background estimate and subtraction
- Range-dependence removal (not altitude)
- Base altitude adjustment
- Take Rayleigh signal @ z_R (Rayleigh fit or Rayleigh sum)
- Rayleigh normalization

$$\boxed{N_N(\lambda,z) = \frac{N_S(\lambda,z) - N_B}{N_S(\lambda,z_R) - N_B}\frac{z^2}{z_R^2}}$$

Solutions to Lidar Equation

- Lidar equation for pure Rayleigh backscattering

$$N_S(\lambda,z_R)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_R(\pi,\lambda)n_R(z_R)\right]\Delta z\left(\frac{A}{{z_R}^2}\right)T_a^{\,2}(\lambda,z_R)\big(\eta(\lambda)G(z_R)\big)+N_B$$

- Lidar equation for resonance fluorescence

$$N_S(\lambda,z)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_{eff}(\lambda,z)n_c(z)R_B(\lambda)+\sigma_R(\pi,\lambda)n_R(z)\right]\Delta z\left(\frac{A}{4\pi z^2}\right)$$
$$\times\left(T_a^2(\lambda)T_c^{\,2}(\lambda,z)\right)\big(\eta(\lambda)G(z)\big)+N_B$$

$$\Downarrow$$

$$n_c(z)=\left[\frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{{z_R}^2}-\frac{n_R(z)}{n_R(z_R)}\right]\cdot\frac{4\pi\sigma_R(\pi,\lambda)n_R(z_R)}{\sigma_{eff}(\lambda)R_B(\lambda)T_c^{\,2}(\lambda,z)}$$

Constituent Density

- Normalized Photon Count to the density estimation

$$n_c(z) = \left[\frac{N_S(\lambda,z) - N_B}{N_R(\lambda,z_R) - N_B} \cdot \frac{z^2}{z_R^{\,2}} - \frac{n_R(z)}{n_R(z_R)} \right] \cdot \frac{4\pi\sigma_R(\pi,\lambda) n_R(z_R)}{\sigma_{eff}(\lambda) R_B(\lambda) T_c^{\,2}(\lambda,z)}$$

Normalized Photon Count
From the preprocess

Temperature and wind dependent

→ we need to estimate the temperature and wind first to estimate the density

Basic Clue: Ratio Computation

- From physics, we calculate the ratios of R_T and R_W as

$$\boxed{R_T = \frac{\sigma_{eff}(f_+,z)+\sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}} \qquad \boxed{R_W = \frac{\sigma_{eff}(f_+,z)-\sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}}$$

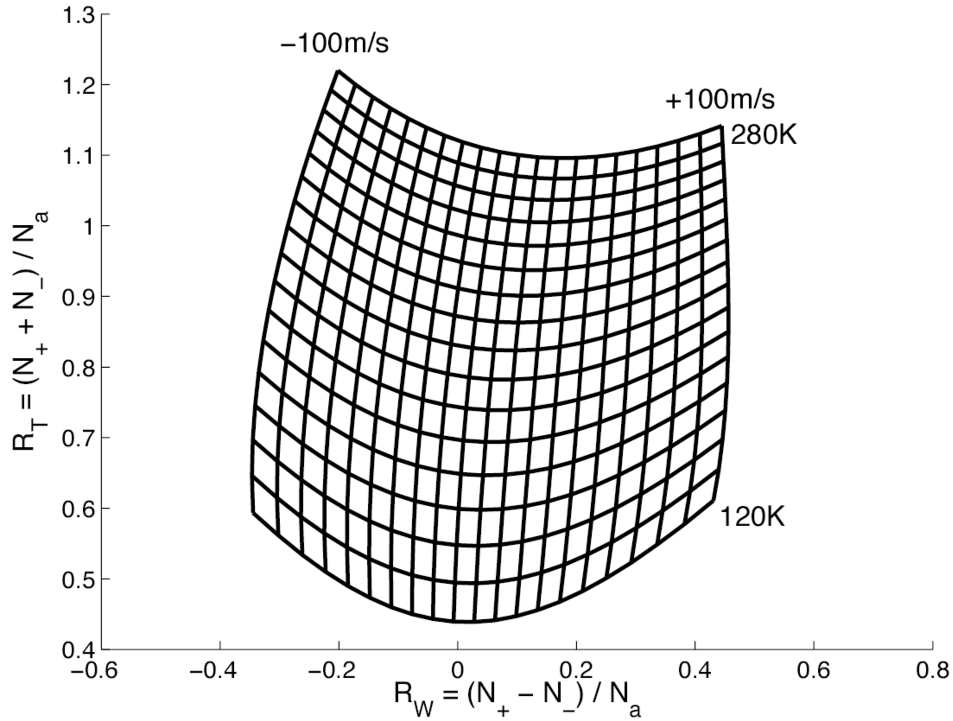
- From actual photon counts, we calculate the ratios as

$$\boxed{\begin{aligned} R_T &= \frac{N_{Norm}(f_+,z)+N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)} \\ &= \frac{\left(\frac{N_S(f_+,z)-N_B}{N_S(f_+,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_+,z)}-\frac{n_R(z)}{n_R(z_R)}\right)+\left(\frac{N_S(f_-,z)-N_B}{N_S(f_-,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_-,z)}-\frac{n_R(z)}{n_R(z_R)}\right)}{\frac{N_S(f_a,z)-N_B}{N_S(f_a,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_a,z)}-\frac{n_R(z)}{n_R(z_R)}} \end{aligned}}$$

$$\boxed{\begin{aligned} R_W &= \frac{N_{Norm}(f_+,z)-N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)} \\ &= \frac{\left(\frac{N_S(f_+,z)-N_B}{N_S(f_+,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_+,z)}-\frac{n_R(z)}{n_R(z_R)}\right)-\left(\frac{N_S(f_-,z)-N_B}{N_S(f_-,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_-,z)}-\frac{n_R(z)}{n_R(z_R)}\right)}{\frac{N_S(f_a,z)-N_B}{N_S(f_a,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_a,z)}-\frac{n_R(z)}{n_R(z_R)}} \end{aligned}}$$

Main Process Procedure

- Compute Doppler calibration curves from physics



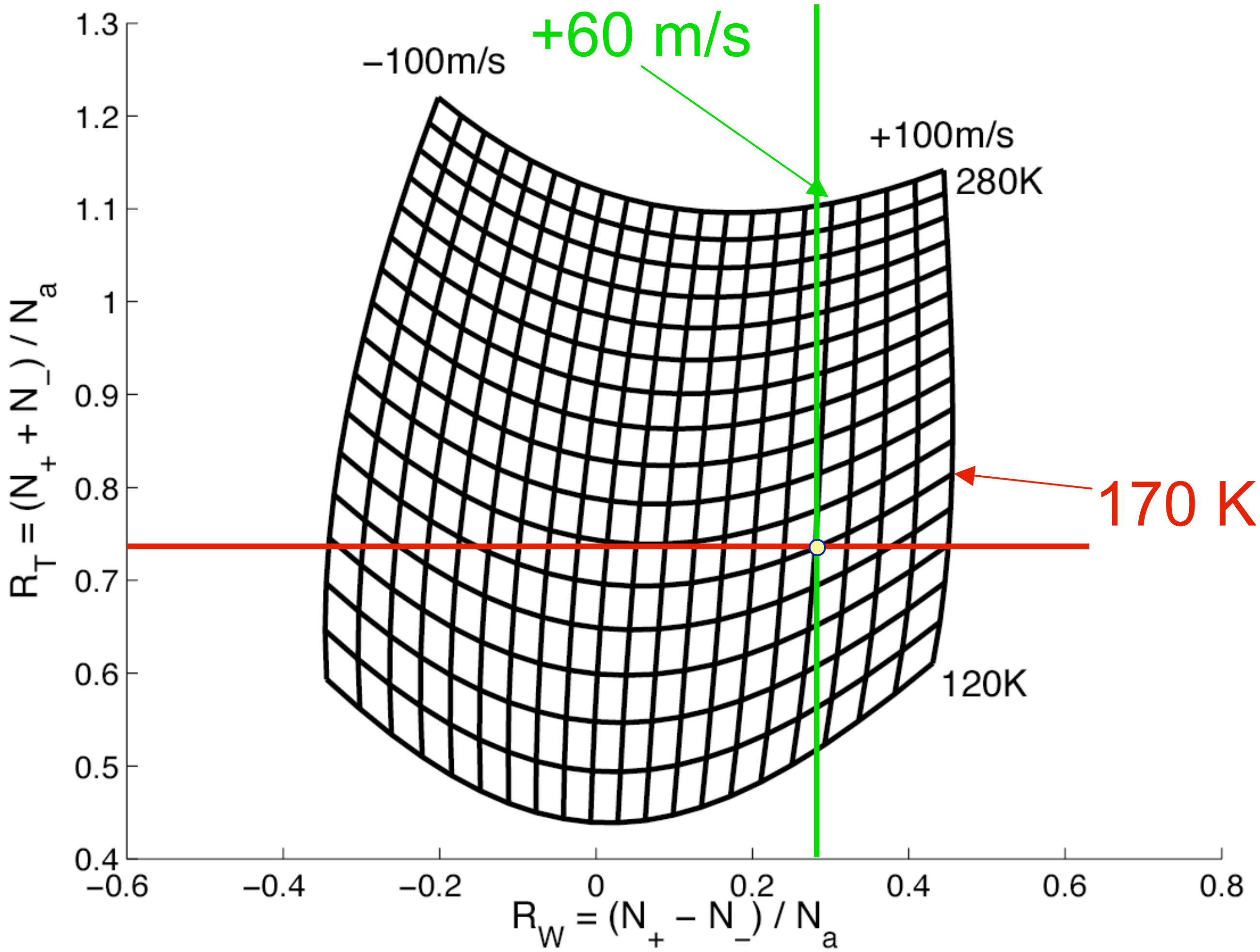
$$R_W = \frac{\sigma_{eff}(f_+, z) - \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

$$R_T = \frac{\sigma_{eff}(f_+, z) + \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

$$\sigma_{\mathrm{eff}}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{e}}} \frac{e^2 f}{4\epsilon_0 m_{\mathrm{e}} c} \sum_{n=1}^{6} A_n \exp\left(- \frac{\left[\nu_n - \nu\left(1 - \frac{v_{\mathrm{R}}}{c}\right)\right]^2}{2\sigma_{\mathrm{e}}^2} \right)$$

Main Process Procedure

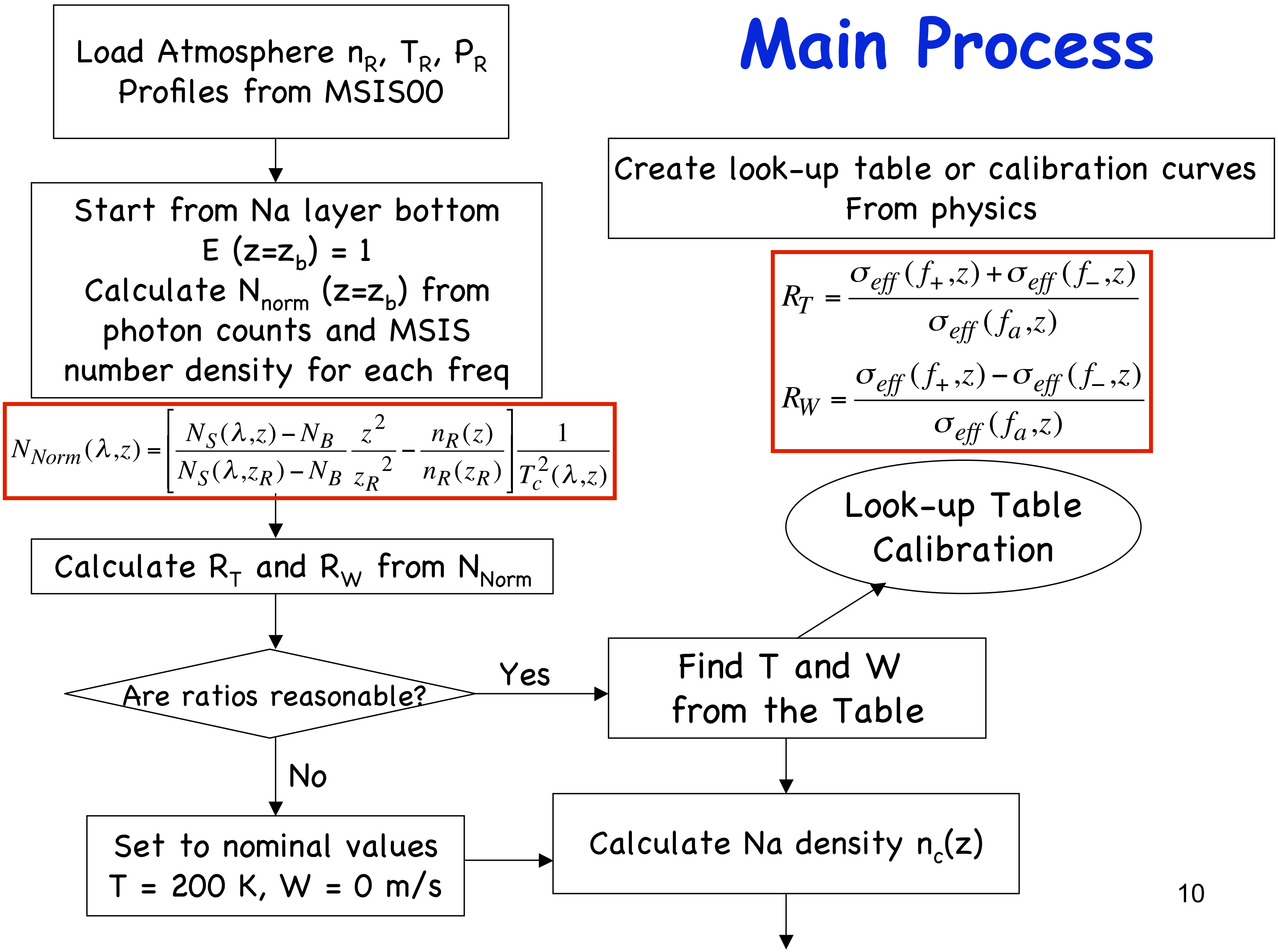
- Compute actual ratios R_T and R_W from photon counts
- Look up these two ratios on the calibration curves to infer the corresponding Temperature and Wind from isoline/isogram.



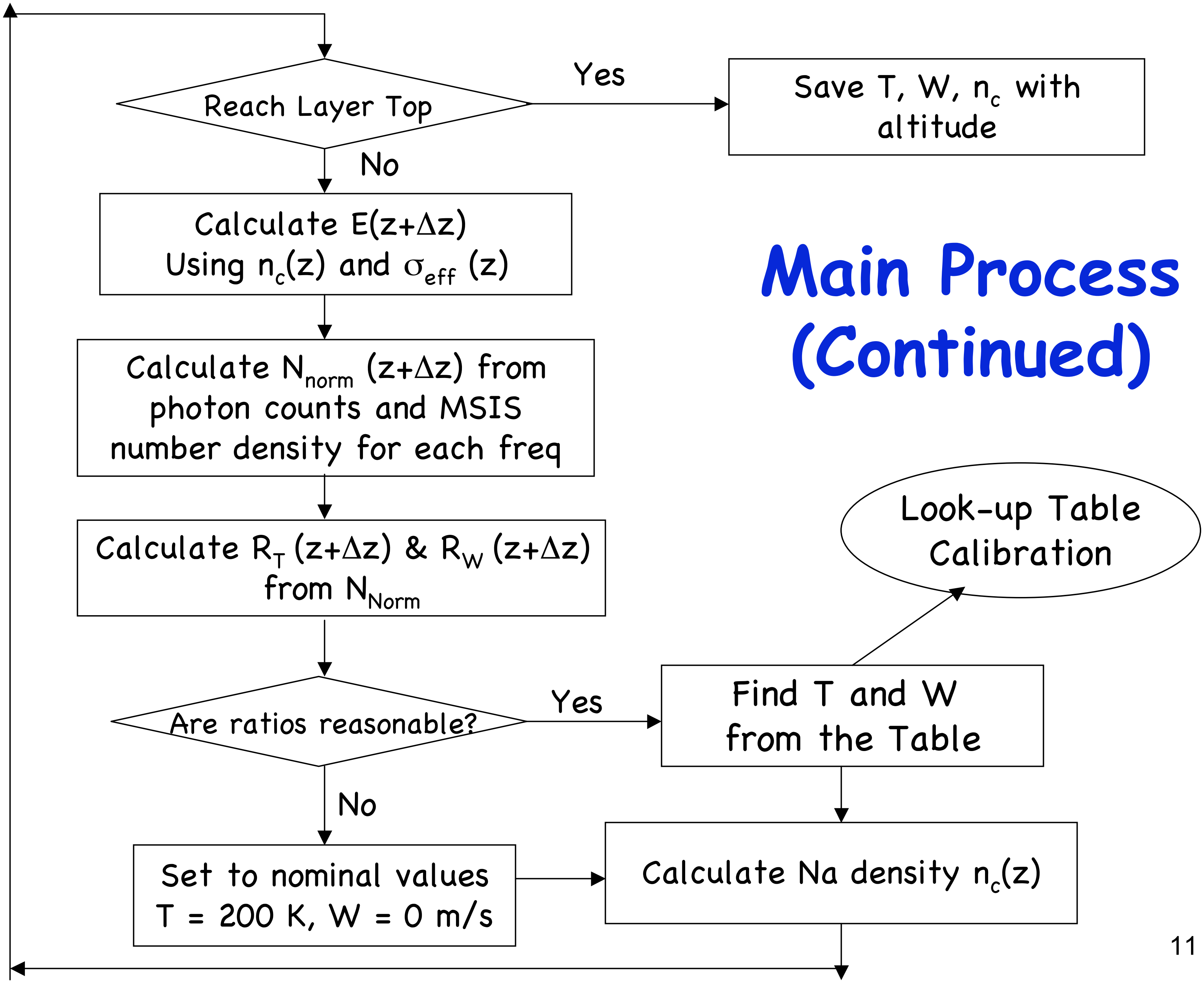
Main Ideas to Derive Na T and W

- In the ratio technique, Na number density is cancelled out. So we have two ratios R_T and R_W that are independent of Na density but both dependent on T and W.
- The idea is to derive temperature and radial wind from these two ratios first, and then derive Na number density using computed temperature and wind at each altitude bin.
- To derive T and W from R_T and R_W, the basic idea is to use look-up table or iteration methods to derive them: (1) compute R_T and R_W from physics point-of-view to generate the table or calibration curves, (2) compute R_T and R_W from actual photon counts, (3) check the table or calibration curves to find the corresponding T and W. (4) If R_T and R_W are out of range, then set to nominal T and W.
- However, because the Na extinction coefficient is involved, the upper bins are related to lower bins, and extinction coefficient is related to Na density and effective cross-section. The solution is to start from the bottom of the Na layer.

Main Process



Main Process (Continued)



Derivation of Extinction

- The extinction can be derived from

$$\boxed{T_c(\lambda,z)=\exp\left(-\int_{z_{bottom}}^{z}\sigma_{eff}(\lambda,z)n_c(z)\,\mathrm{d}z\right)=\exp\left(-\sum_{z_{bottom}}^{z}\sigma_{eff}(\lambda,z)n_c(z)\Delta z\right)}$$

- The effective cross-section

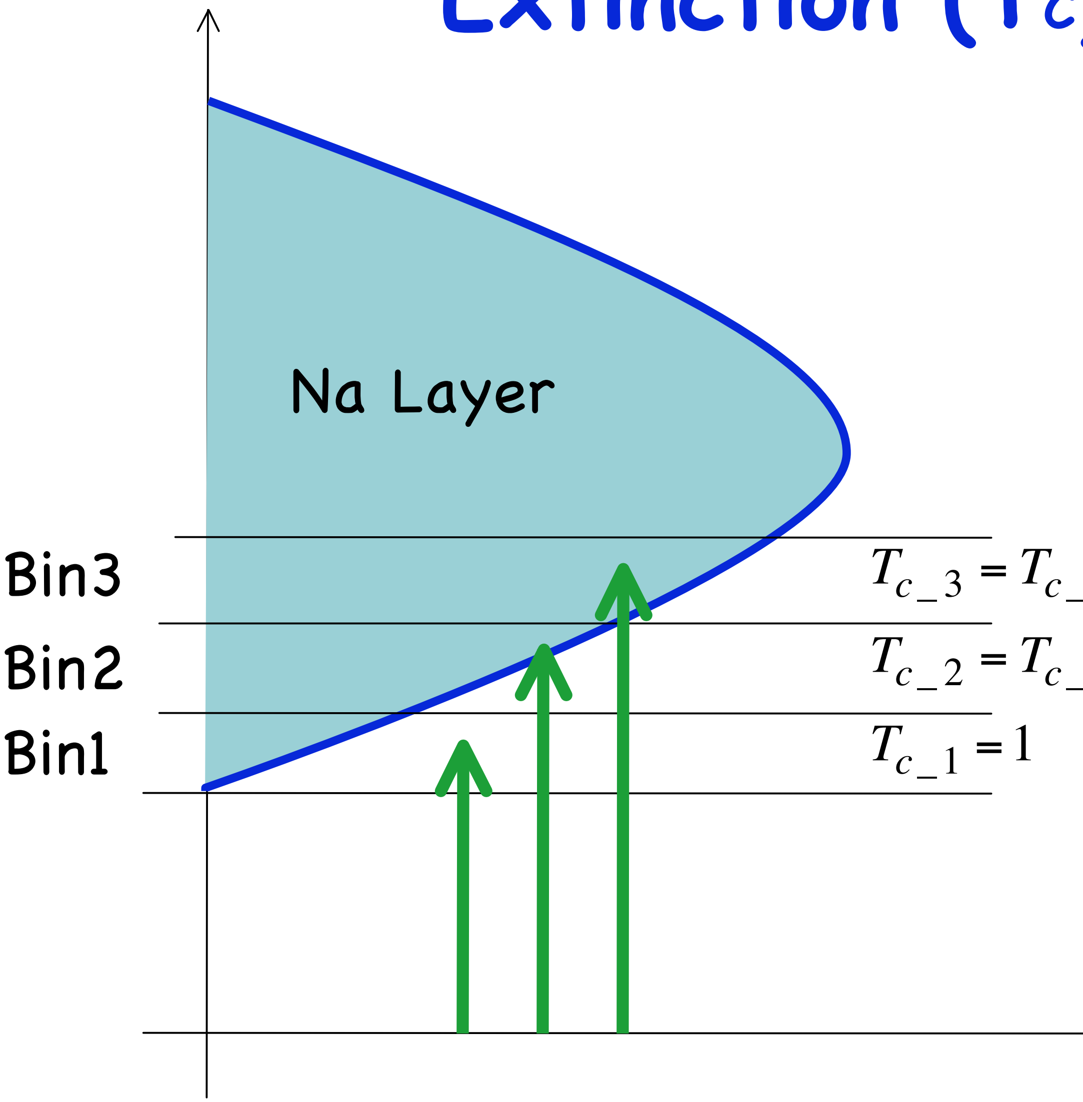
$$\boxed{\sigma_{\mathrm{eff}}(\nu)=\frac{1}{\sqrt{2\pi}\sigma_{\mathrm{e}}}\frac{e^2 f}{4\epsilon_0 m_{\mathrm{e}}c}\sum_{n=1}^{6}A_n\exp\left(-\frac{\left[\nu_n-\nu\left(1-\frac{v_{\mathrm{R}}}{c}\right)\right]^2}{2\sigma_{\mathrm{e}}^2}\right)}$$

Ready to estimate the constituent density

$$\boxed{\sigma_e=\sqrt{\sigma_D^2+\sigma_L^2}}$$

$$\boxed{n_c(z)=\left[\frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R{}^2}-\frac{n_R(z)}{n_R(z_R)}\right]\cdot\frac{4\pi\sigma_R(\pi,\lambda)n_R(z_R)}{\sigma_{eff}(\lambda)R_B(\lambda)T_c{}^2(\lambda,z)}}$$

A Key Step to Main Process Extinction (T_c)



$$T_{c\_3} = T_{c\_2} \times \exp(-\sigma_{eff\_2} \cdot n_{c\_2} \cdot \Delta z)$$

$$T_{c\_2} = T_{c\_1} \times \exp(-\sigma_{eff\_1} \cdot n_{c\_1} \cdot \Delta z)$$

$$T_{c\_1} = 1$$

Main Process Step 1: Starting Point

1. Extinction (Tc) at the bottom of Na layer is 1
2. Calculate the normalized photon count for each frequency

$$\boxed{N_{Norm}(\lambda,z) = \left[\frac{N_S(\lambda,z)-N_B}{N_S(\lambda,z_R)-N_B}\frac{z^2}{{z_R}^2} - \frac{n_R(z)}{n_R(z_R)}\right]\frac{1}{T_c^2(\lambda,z)}}$$

3. Based on the normalized photon counts, you get R_T and R_W

$$\boxed{R_T = \frac{N_{Norm}(f_+,z)+N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}} \qquad \boxed{R_W = \frac{N_{Norm}(f_+,z)-N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}}$$

4. Estimate the temperature and wind using the calibration curves

Main Process Step 2: Bin-by-Bin Procedure

5. Calculate the effective cross section using temperature and wind derived
6. Using the effective cross-section and Tc = 1 (at the bottom), calculate the Na density.

$$n_c(z) = \left[\frac{N_S(\lambda,z) - N_B}{N_R(\lambda,z_R) - N_B} \cdot \frac{z^2}{z_R^{\,2}} - \frac{n_R(z)}{n_R(z_R)} \right] \cdot \frac{4\pi\sigma_R(\pi,\lambda) n_R(z_R)}{\sigma_{eff}(\lambda) R_B(\lambda) T_c^{\,2}(\lambda,z)}$$

7. From effective cross-section and Na density, calculate the extinction for the next bin.

$$T_c(\lambda,z) = \exp\left(-\int_{z_{bottom}}^{z} \sigma_{eff}(\lambda,z) n_c(z)\,\mathrm{d}z\right) = \exp\left(-\sum_{z_{bottom}}^{z} \sigma_{eff}(\lambda,z) n_c(z)\Delta z\right)$$

Na Density Derivation

- The Na density can be inferred from the peak freq signal

$$n_{Na}(z) = \frac{N_{norm}(f_a, z)}{\sigma_a} 4\pi n_R(z_R)\sigma_R = \frac{N_{norm}(f_a, z)}{\sigma_a} 4\pi \times 2.938 \times 10^{-32} \frac{P(z_R)}{T(z_R)} \cdot \frac{1}{\lambda^{4.0117}}$$

- The Na density can be inferred from a weighted average of all three frequency signals.
- The weighted effective cross-section is

$$\sigma_{eff\_wgt} = \sigma_a + \alpha\sigma_+ + \beta\sigma_-$$

where α and β are chosen so that

$$\frac{\partial \sigma_{eff\_wgt}}{\partial T} = 0; \qquad \frac{\partial \sigma_{eff\_wgt}}{\partial v_R} = 0$$

- The Na density is then calculated by

$$n_{Na}(z) = 4\pi n_R(z_R)\sigma_R \frac{N_{norm}(f_a, z) + \alpha N_{norm}(f_+, z) + \beta N_{norm}(f_-, z)}{\sigma_a + \alpha\sigma_+ + \beta\sigma_-}$$

Process Procedure for β

Read Data File

↓

PMT Saturation Correction

↓

Chopper Correction

↓

Subtract Background

↓

Remove Range Dependence (x R^2)

↓

Add Base Altitude

↓

Take Rayleigh Signal @ z_R km

↓

Normalize Profile By Rayleigh Signal @ z_R km

↓

Load Atmosphere n_R, T, P Profiles from MSIS00

↓

Calculate $R(z)$ and $\beta(z)$

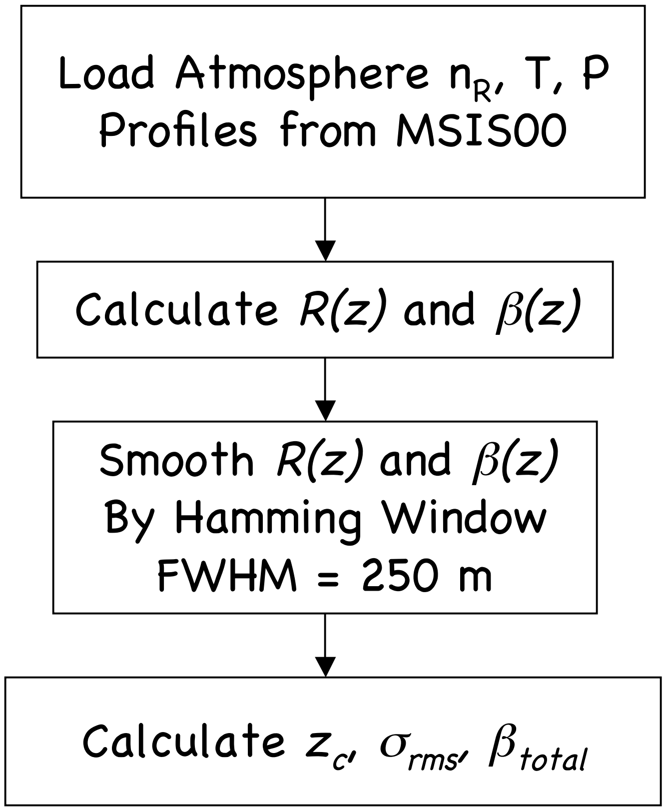
↓

Smooth $R(z)$ and $\beta(z)$ By Hamming Window FWHM = 250 m

↓

Calculate z_c, σ_{rms}, β_{total}

Process Procedure for β of PMC



$$R = \frac{\left[N_S(z) - N_B\right]\cdot z^2}{\left[N_S(z_{RN}) - N_B\right]\cdot z_{RN}^2}\cdot\frac{n_R(z_{RN})}{n_R(z)}$$

$$\beta_{PMC}(z) = \left[\frac{\left[N_S(z) - N_B\right]\cdot z^2}{\left[N_S(z_{RN}) - N_B\right]\cdot z_{RN}^2} - \frac{n_R(z)}{n_R(z_{RN})}\right]\cdot\beta_R(z_{RN})$$

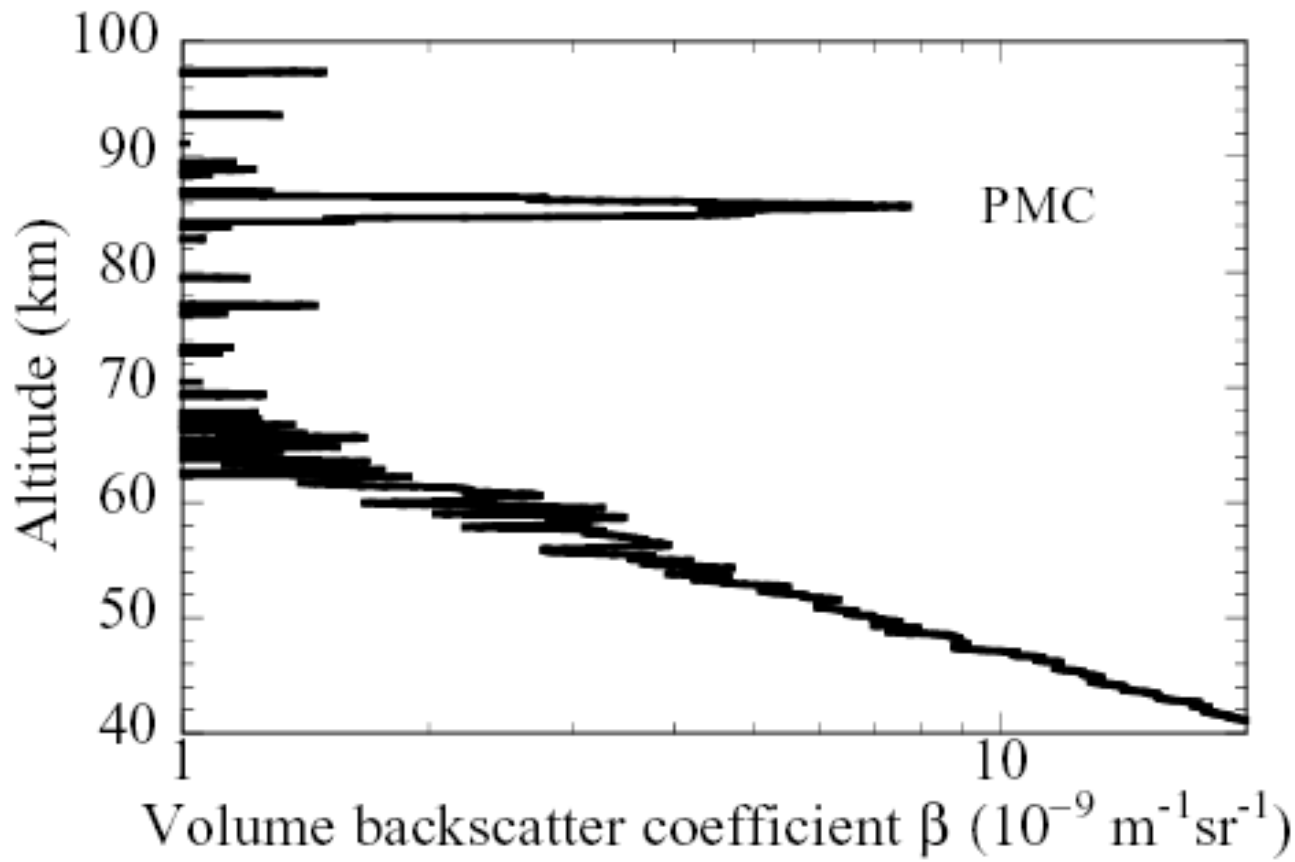
$$\beta_R(z_{RN},\pi) = \frac{\beta}{4\pi}P(\pi) = 2.938\times 10^{-32}\frac{P(z_{RN})}{T(z_{RN})}\cdot\frac{1}{\lambda^{4.0117}}$$

$$z_c = \frac{\sum_i \beta_{PMC}(z_i)\cdot z_i}{\sum_i \beta_{PMC}(z_i)}$$

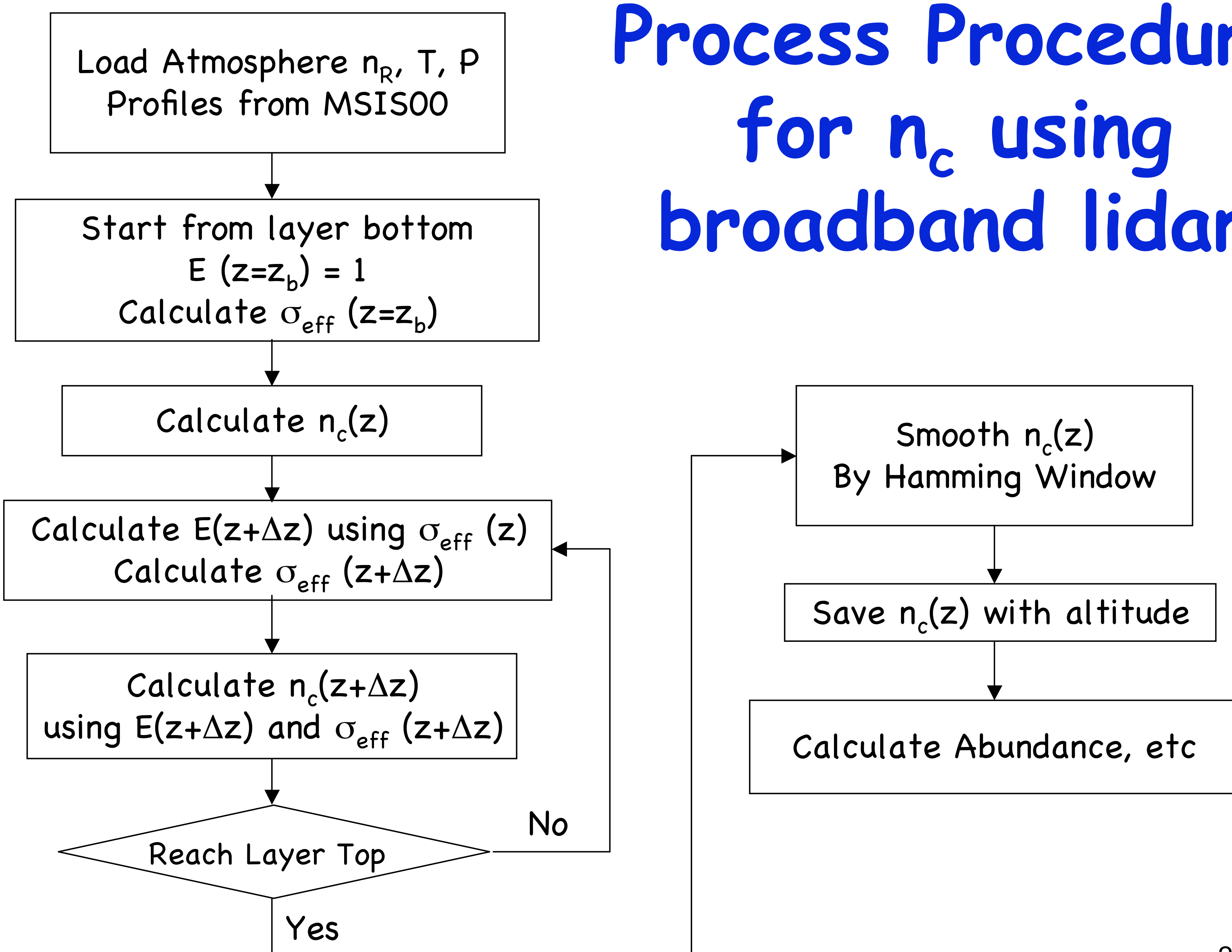
$$\sigma_{rms} = \sqrt{\frac{\sum_i (z_i - z_c)^2\,\beta_{PMC}(z_i)}{\sum_i \beta_{PMC}(z_i)}}$$

$$\beta_{total} = \int \beta_{PMC}(z)\,dz$$

Example Result: South Pole PMC



Process Procedure for n_c using broadband lidar



Process Procedure for n_c

- Computation of effective cross-section (concerning laser shape, assuming nominal T and W)
- Spatial resolution - binning or smoothing
- temporal resolution - integration

-- in order to improve SNR

- Extinction coefficient
- Calculate density
- Calculate abundance, peak altitude, etc.
- Show Na lidar data as examples in class

To Improve SNR

- In order to improve signal-to-noise ratio (SNR), we have to sacrifice spatial and/or temporal resolutions.
- Spatial resolution
  - binning
  - smoothing
- temporal resolution
  - integration

Summary

- The preprocess is to convert the raw photon counts to corrected and normalized photon counts in consideration of hardware properties and limitations.
- The process of T and V_R is to convert the normalized photon counts to T and V_R through iteration or looking-up table methods.
- The process of n_c is to convert the normalized photon counts to meaningful number density, in combination with prior acquired knowledge or model knowledge of certain atmosphere information or atomic/molecular spectroscopy.
- The data inversion procedure consists of three main processes: (1) preprocess, (2) process of T and V_R, (3) process of n_c and β, etc.