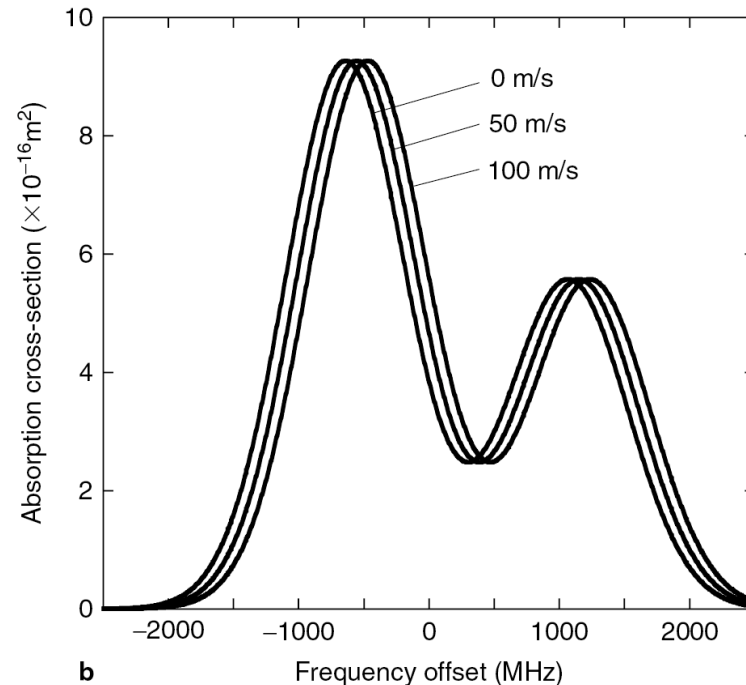
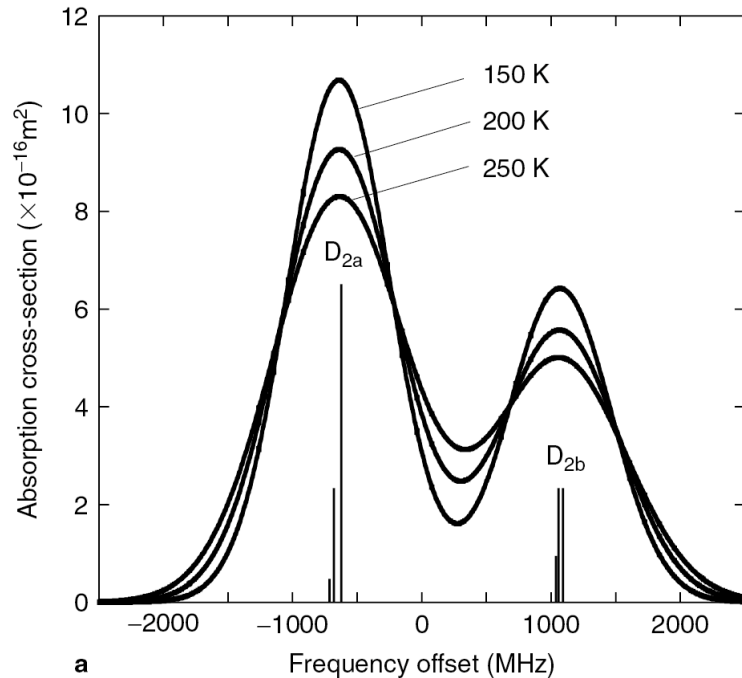


Lecture 12. Temperature Lidar (2)

Doppler Ratio Technique

- ❑ Review Doppler Technique
- ❑ Scanning technique vs. ratio technique
- ❑ Principle of Doppler ratio technique
- ❑ Comparison of calibration curves
- ❑ Other resonance fluorescence Doppler lidars
- ❑ Summary

Review Doppler Technique

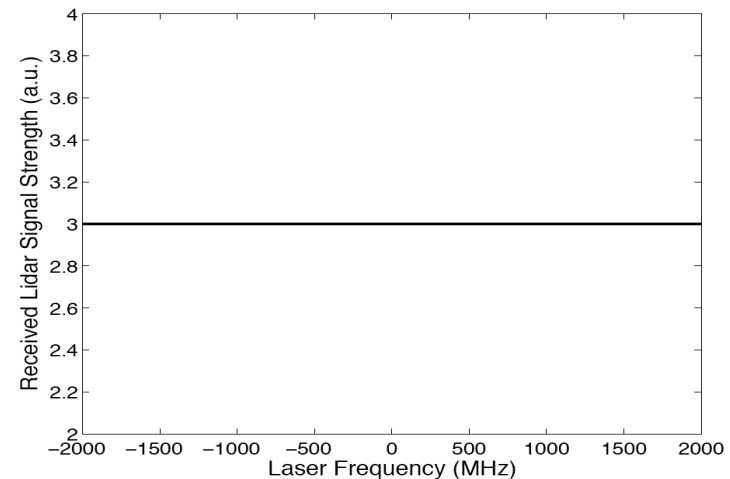
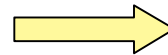
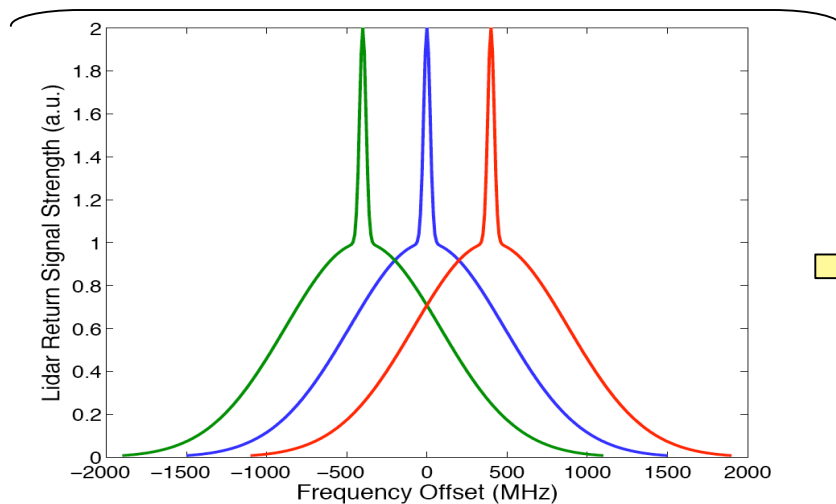
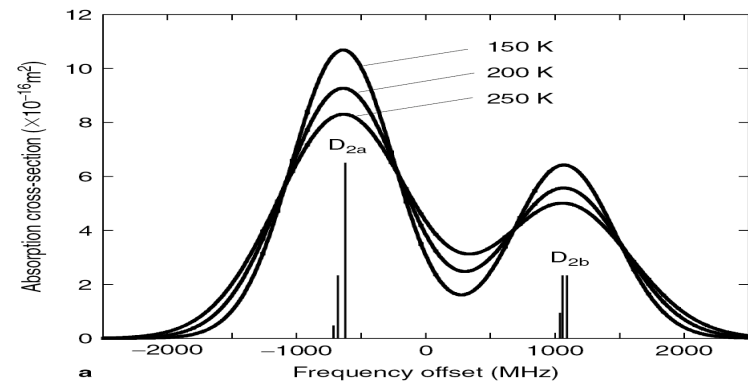
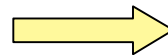
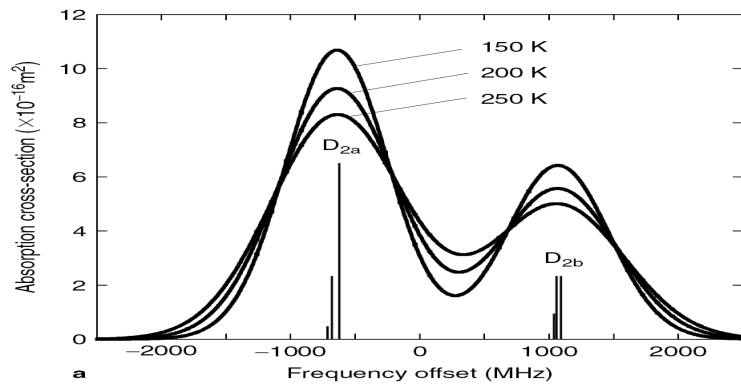


Temperature determined from Doppler broadening width
Radial wind determined from Doppler frequency shift

□ Resonance absorption experiences 1-time Doppler shift and broadening, while Rayleigh scattering experiences 2 times of Doppler shift and broadening.

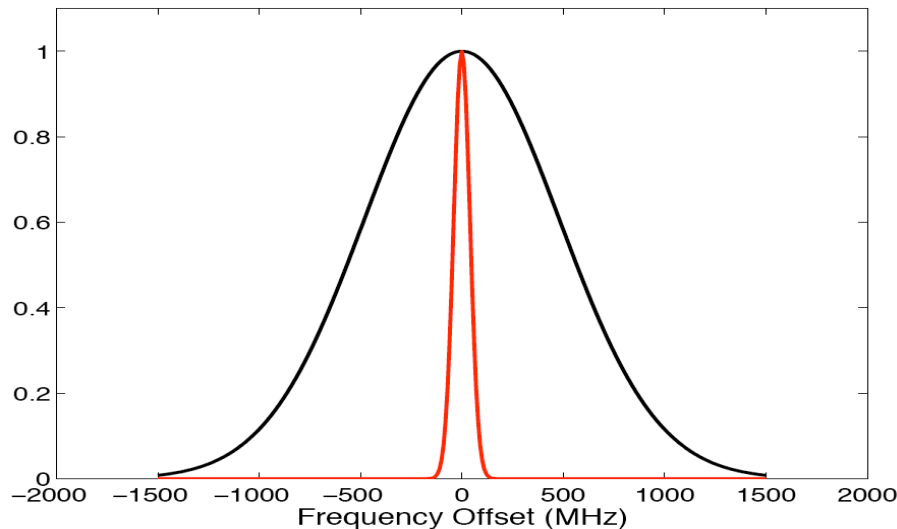
Review Doppler Technique

How will the lidar return signal strength (vs. laser frequency) change when the lidar receiver is broadband and we scan the narrowband laser frequency - in resonance fluorescence case and in Rayleigh scattering case?



Review Doppler Technique

- The effective cross section is a convolution of the atomic absorption cross section and the laser line shape.



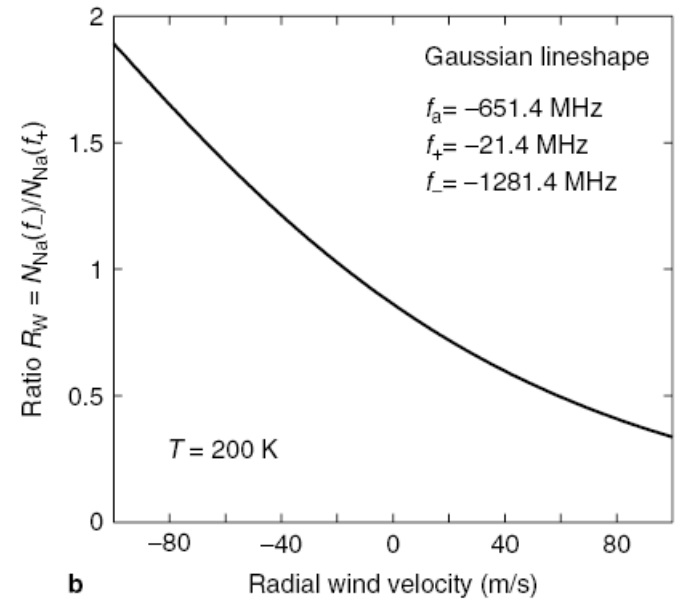
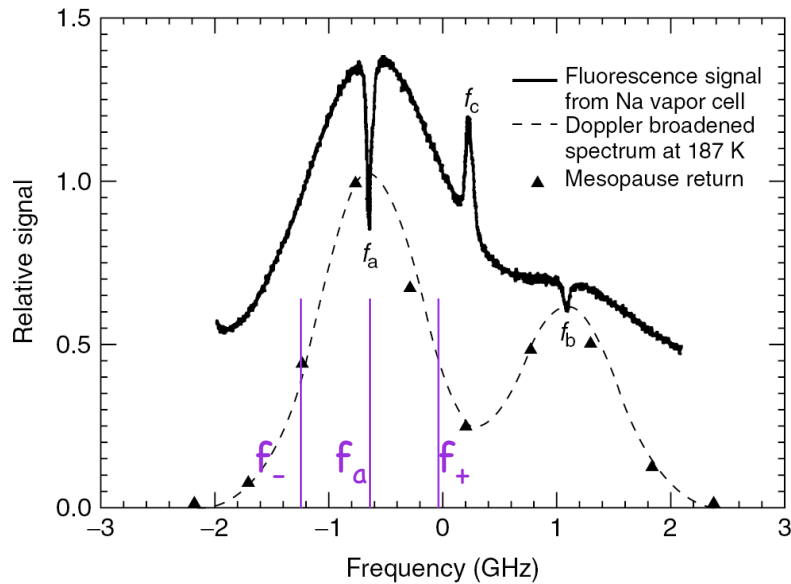
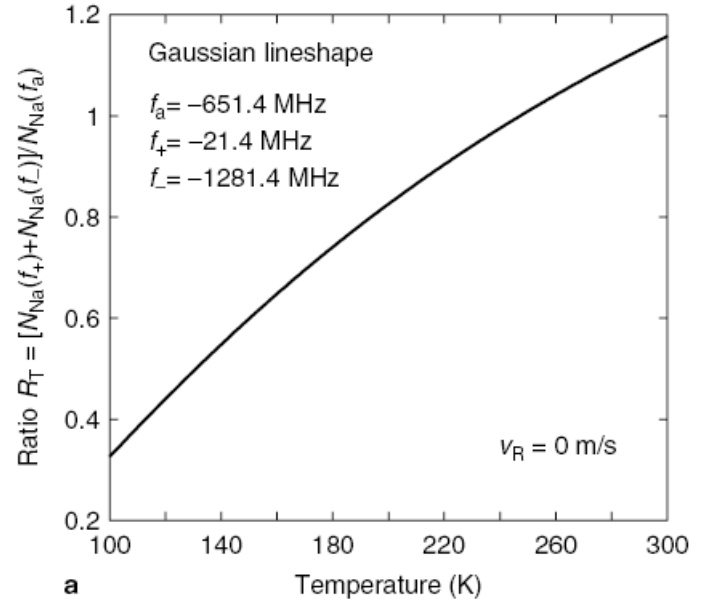
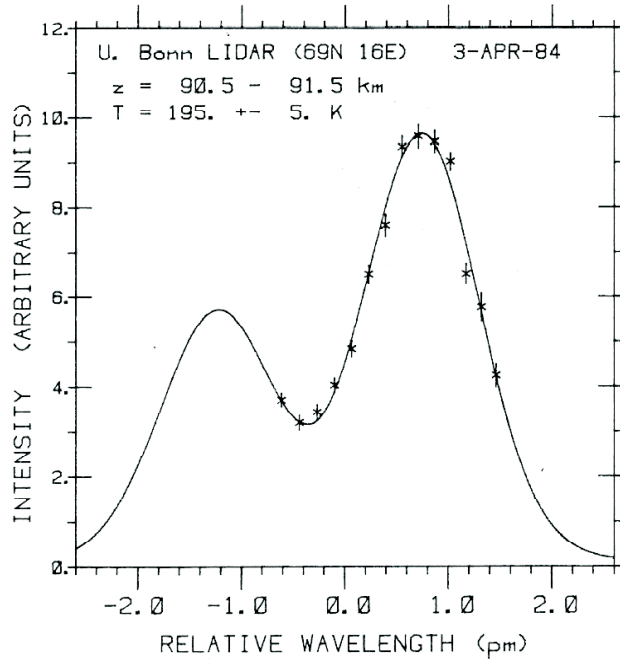
For Gaussian shapes of atomic absorption and laser lineshape:

$$\sigma_e = \sqrt{\sigma_D^2 + \sigma_L^2}$$

- How would you calculate the effective cross section when atoms have isotopes, e.g., K (potassium)?

$$\sigma_{eff}(overall) = \sum_{k=1}^N \left[\sigma_{eff,k}(isotope) \times Abdn_k \right]$$

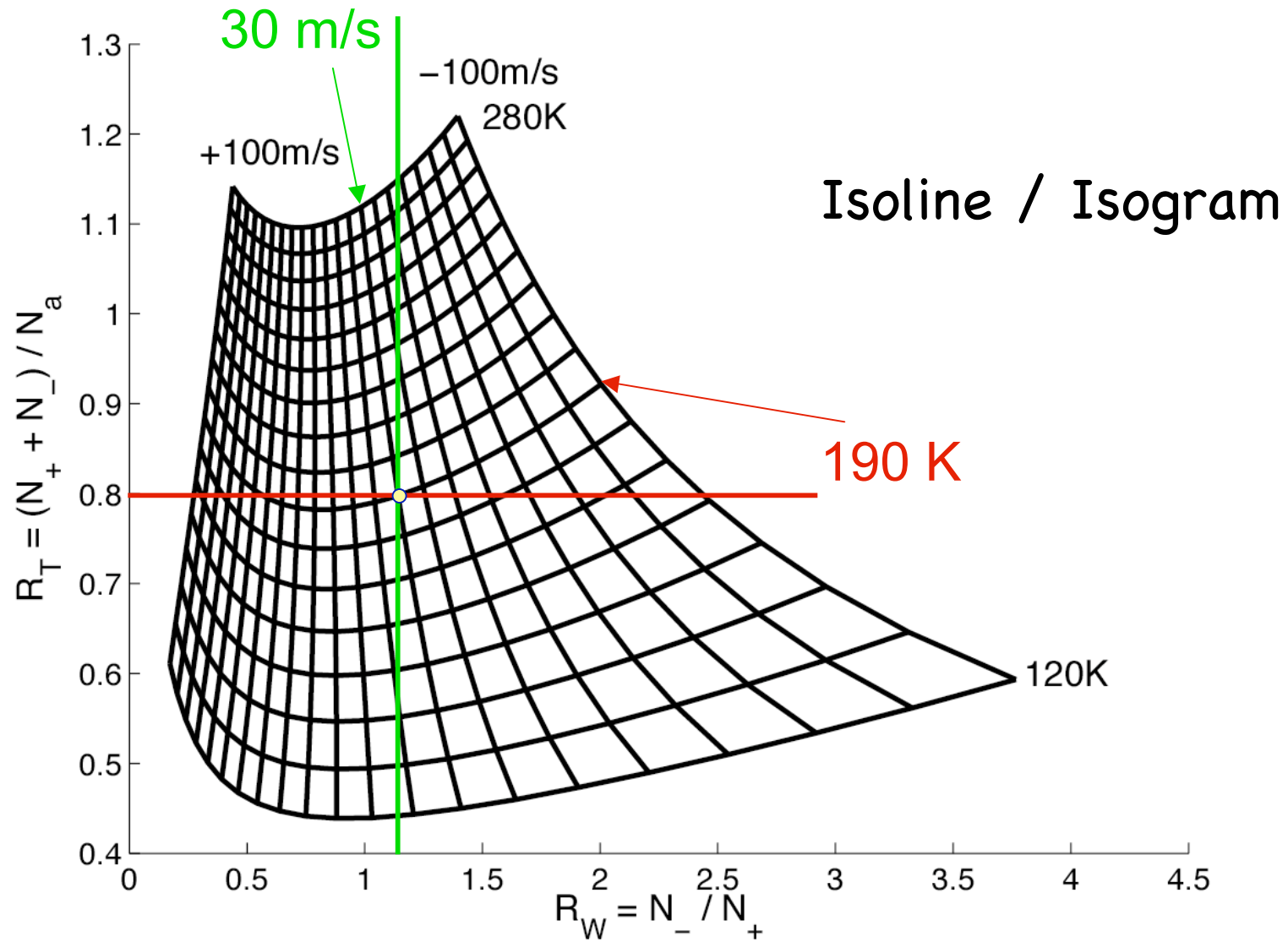
Scanning versus Ratio Techniques



Main Ideas Behind Ratio Technique

- ❑ Three unknown parameters (temperature, radial wind, and Na number density) require 3 lidar equations at 3 frequencies as minimum \Rightarrow highest resolution.
- ❑ In the ratio technique, Na number density is cancelled out. So we have two ratios R_T and R_W that are independent of Na density but both dependent on T and W.
- ❑ The idea is to derive temperature and radial wind from these two ratios first, and then derive Na number density using computed temperature and wind at each altitude bin.
- ❑ However, because the Na extinction coefficient is involved, the upper bins are related to lower bins, and extinction coefficient is related to Na density and effective cross-section. The solution is to start from the bottom of the Na layer and then work bin by bin to the layer top.

Principle of Doppler Ratio Technique



Doppler Ratio Technique

- Lidar equation for resonance fluorescence (Na, K, or Fe)

$$N_S(\lambda, z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) \left[\sigma_{eff}(\lambda, z)n_c(z)R_B(\lambda) + \sigma_R(\pi, \lambda)n_R(z) \right] \Delta z \left(\frac{A}{4\pi z^2} \right) \times \left(T_a^2(\lambda)T_c^2(\lambda, z) \right) (\eta(\lambda)G(z)) + N_B$$

$R_B = 1$ for current Na Doppler lidar since return photons at all wavelengths are received by the broadband receiver, so no fluorescence is filtered off.

- Pure Na signal and pure Rayleigh signal in Na region are

$$N_{Na}(\lambda, z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) \left[\sigma_{eff}(\lambda, z)n_c(z) \right] \Delta z \left(\frac{A}{4\pi z^2} \right) \left(T_a^2(\lambda)T_c^2(\lambda, z) \right) (\eta(\lambda)G(z))$$

$$N_R(\lambda, z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) \left[\sigma_R(\pi, \lambda)n_R(z) \right] \Delta z \left(\frac{A}{z^2} \right) \left(T_a^2(\lambda)T_c^2(\lambda, z) \right) (\eta(\lambda)G(z))$$

- So we have

$$N_S(\lambda, z) = N_{Na}(\lambda, z) + N_R(\lambda, z) + N_B$$

Doppler Ratio Technique

- Lidar equation at pure molecular scattering region (35-55km)

$$N_S(\lambda, z_R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) \left[\sigma_R(\pi, \lambda) n_R(z_R) \right] \Delta z \left(\frac{A}{z_R} \right)^2 T_a^2(\lambda, z_R) (\eta(\lambda) G(z_R)) + N_B$$

- Pure Rayleigh signal in molecular scattering region is

$$N_R(\lambda, z_R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) \left[\sigma_R(\pi, \lambda) n_R(z_R) \right] \Delta z \left(\frac{A}{z_R} \right)^2 T_a^2(\lambda, z_R) (\eta(\lambda) G(z_R))$$

- So we have

$$N_S(\lambda, z_R) = N_R(\lambda, z_R) + N_B$$

- The ratio between Rayleigh signals at z and z_R is given by

$$\frac{N_R(\lambda, z)}{N_R(\lambda, z_R)} = \frac{\left[\sigma_R(\pi, \lambda) n_R(z) \right] T_a^2(\lambda, z) T_c^2(\lambda, z) G(z) \frac{z_R^2}{z^2}}{\left[\sigma_R(\pi, \lambda) n_R(z_R) \right] T_a^2(\lambda, z_R) G(z_R) \frac{z_R^2}{z^2}} = \frac{n_R(z)}{n_R(z_R)} \frac{z_R^2}{z^2} T_c^2(\lambda, z)$$

Where n_R is the (total) atmospheric number density, usually obtained from atmospheric models like MSIS00.

Doppler Ratio Technique

From above equations, we obtain

$$N_{Na}(\lambda, z) = N_S(\lambda, z) - N_B - N_R(\lambda, z)$$

$$N_R(\lambda, z_R) = N_S(\lambda, z_R) - N_B$$

Normalized Na photon count is defined as

$$N_{Norm}(\lambda, z) = \frac{N_{Na}(\lambda, z)}{N_R(\lambda, z_R) T_c^2(\lambda, z)} \frac{z^2}{z_R^2}$$

So from physics point of view, we have

$$N_{Norm}(\lambda, z) = \frac{N_{Na}(\lambda, z)}{N_R(\lambda, z_R) T_c^2(\lambda, z)} = \frac{\sigma_{eff}(\lambda, z) n_c(z)}{\sigma_R(\pi, \lambda) n_R(z_R)} \frac{1}{4\pi}$$

From actual photon counts, we have

$$\begin{aligned} N_{Norm}(\lambda, z) &= \frac{N_{Na}(\lambda, z)}{N_R(\lambda, z_R) T_c^2(\lambda, z)} \frac{z^2}{z_R^2} = \frac{N_S(\lambda, z) - N_B - N_R(\lambda, z)}{N_R(\lambda, z_R) T_c^2(\lambda, z)} \frac{z^2}{z_R^2} \\ &= \frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda, z)} \frac{n_R(z)}{n_R(z_R)} \end{aligned}$$

Doppler Ratio Technique

From physics, the ratios of R_T and R_W are then given by

$$R_T = \frac{N_{Norm}(f_+, z) + N_{Norm}(f_-, z)}{N_{Norm}(f_a, z)} = \frac{\frac{\sigma_{eff}(f_+, z)n_c(z)}{\sigma_R(\pi, f_+)n_R(z_R)} + \frac{\sigma_{eff}(f_-, z)n_c(z)}{\sigma_R(\pi, f_-)n_R(z_R)}}{\frac{\sigma_{eff}(f_a, z)n_c(z)}{\sigma_R(\pi, f_a)n_R(z_R)}} = \frac{\sigma_{eff}(f_+, z) + \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

$$R_W = \frac{N_{Norm}(f_+, z) - N_{Norm}(f_-, z)}{N_{Norm}(f_a, z)} = \frac{\frac{\sigma_{eff}(f_+, z)n_c(z)}{\sigma_R(\pi, f_+)n_R(z_R)} - \frac{\sigma_{eff}(f_-, z)n_c(z)}{\sigma_R(\pi, f_-)n_R(z_R)}}{\frac{\sigma_{eff}(f_a, z)n_c(z)}{\sigma_R(\pi, f_a)n_R(z_R)}} = \frac{\sigma_{eff}(f_+, z) - \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

Here, Rayleigh backscatter cross-section is regarded as the same for three frequencies, since the frequency difference is so small. N_a number density is also the same for three frequency channels, and so is the atmosphere number density at Rayleigh normalization altitude.

Doppler Ratio Technique

□ From actual photon counts, we have

$$R_T = \frac{N_{Norm}(f_+, z) + N_{Norm}(f_-, z)}{N_{Norm}(f_a, z)}$$

$$= \frac{\left(\frac{N_S(f_+, z) - N_B}{N_S(f_+, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_+, z)} - \frac{n_R(z)}{n_R(z_R)} \right) + \left(\frac{N_S(f_-, z) - N_B}{N_S(f_-, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_-, z)} - \frac{n_R(z)}{n_R(z_R)} \right)}{\frac{N_S(f_a, z) - N_B}{N_S(f_a, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_a, z)} - \frac{n_R(z)}{n_R(z_R)}}$$

$$R_W = \frac{N_{Norm}(f_+, z) - N_{Norm}(f_-, z)}{N_{Norm}(f_a, z)}$$

$$= \frac{\left(\frac{N_S(f_+, z) - N_B}{N_S(f_+, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_+, z)} - \frac{n_R(z)}{n_R(z_R)} \right) - \left(\frac{N_S(f_-, z) - N_B}{N_S(f_-, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_-, z)} - \frac{n_R(z)}{n_R(z_R)} \right)}{\frac{N_S(f_a, z) - N_B}{N_S(f_a, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_a, z)} - \frac{n_R(z)}{n_R(z_R)}}$$

How Does Ratio Technique Work?

- From physics, we calculate the ratios of R_T and R_W as

$$R_T = \frac{\sigma_{eff}(f_+, z) + \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

$$R_W = \frac{\sigma_{eff}(f_+, z) - \sigma_{eff}(f_-, z)}{\sigma_{eff}(f_a, z)}$$

- From actual photon counts, we calculate the ratios as

$$R_T = \frac{N_{Norm}(f_+, z) + N_{Norm}(f_-, z)}{N_{Norm}(f_a, z)}$$

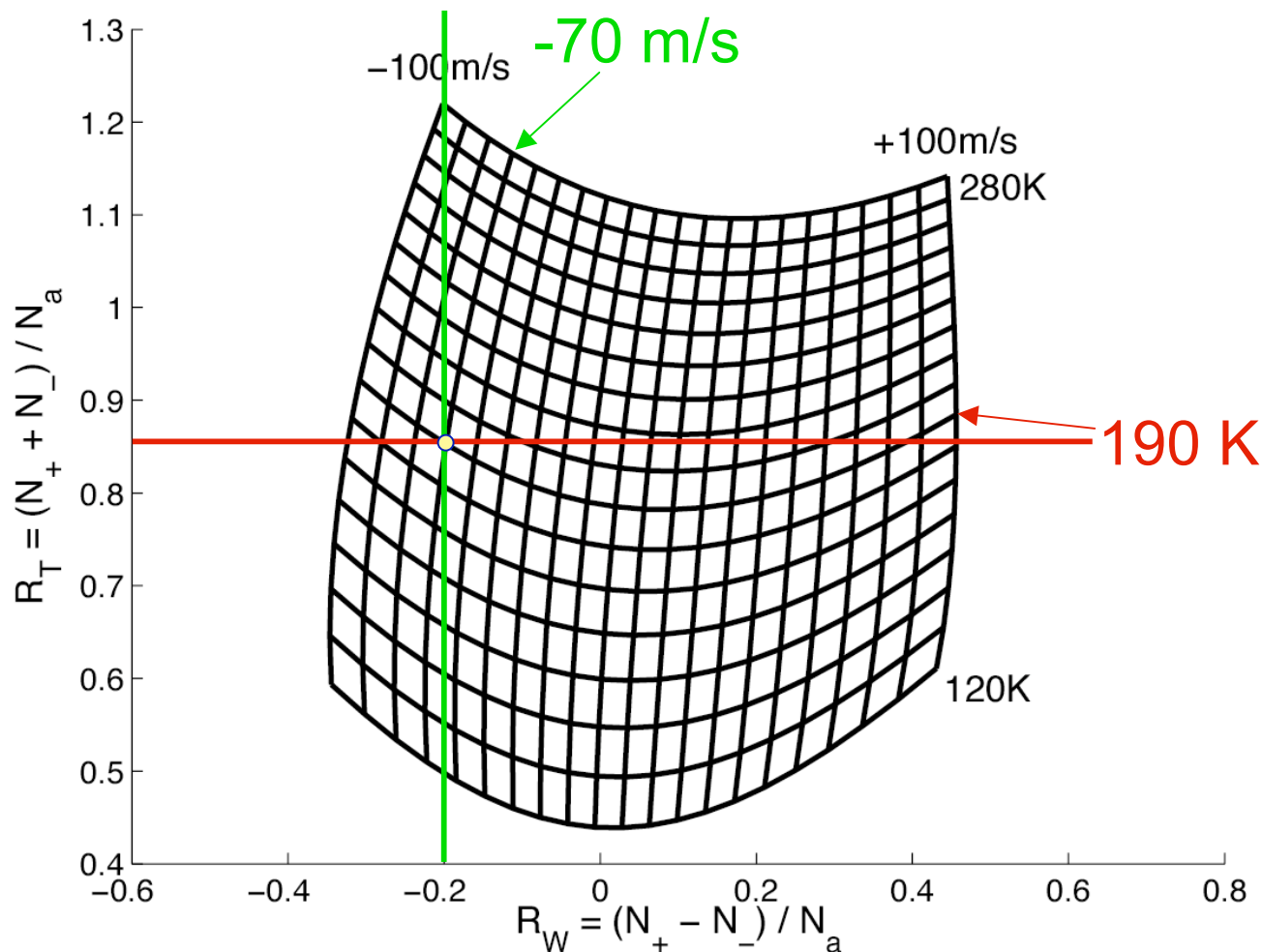
$$= \frac{\left(\frac{N_S(f_+, z) - N_B}{N_S(f_+, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_+, z)} - \frac{n_R(z)}{n_R(z_R)} \right) + \left(\frac{N_S(f_-, z) - N_B}{N_S(f_-, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_-, z)} - \frac{n_R(z)}{n_R(z_R)} \right)}{\frac{N_S(f_a, z) - N_B}{N_S(f_a, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_a, z)} - \frac{n_R(z)}{n_R(z_R)}}$$

$$R_W = \frac{N_{Norm}(f_+, z) - N_{Norm}(f_-, z)}{N_{Norm}(f_a, z)}$$

$$= \frac{\left(\frac{N_S(f_+, z) - N_B}{N_S(f_+, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_+, z)} - \frac{n_R(z)}{n_R(z_R)} \right) - \left(\frac{N_S(f_-, z) - N_B}{N_S(f_-, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_-, z)} - \frac{n_R(z)}{n_R(z_R)} \right)}{\frac{N_S(f_a, z) - N_B}{N_S(f_a, z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_a, z)} - \frac{n_R(z)}{n_R(z_R)}}$$

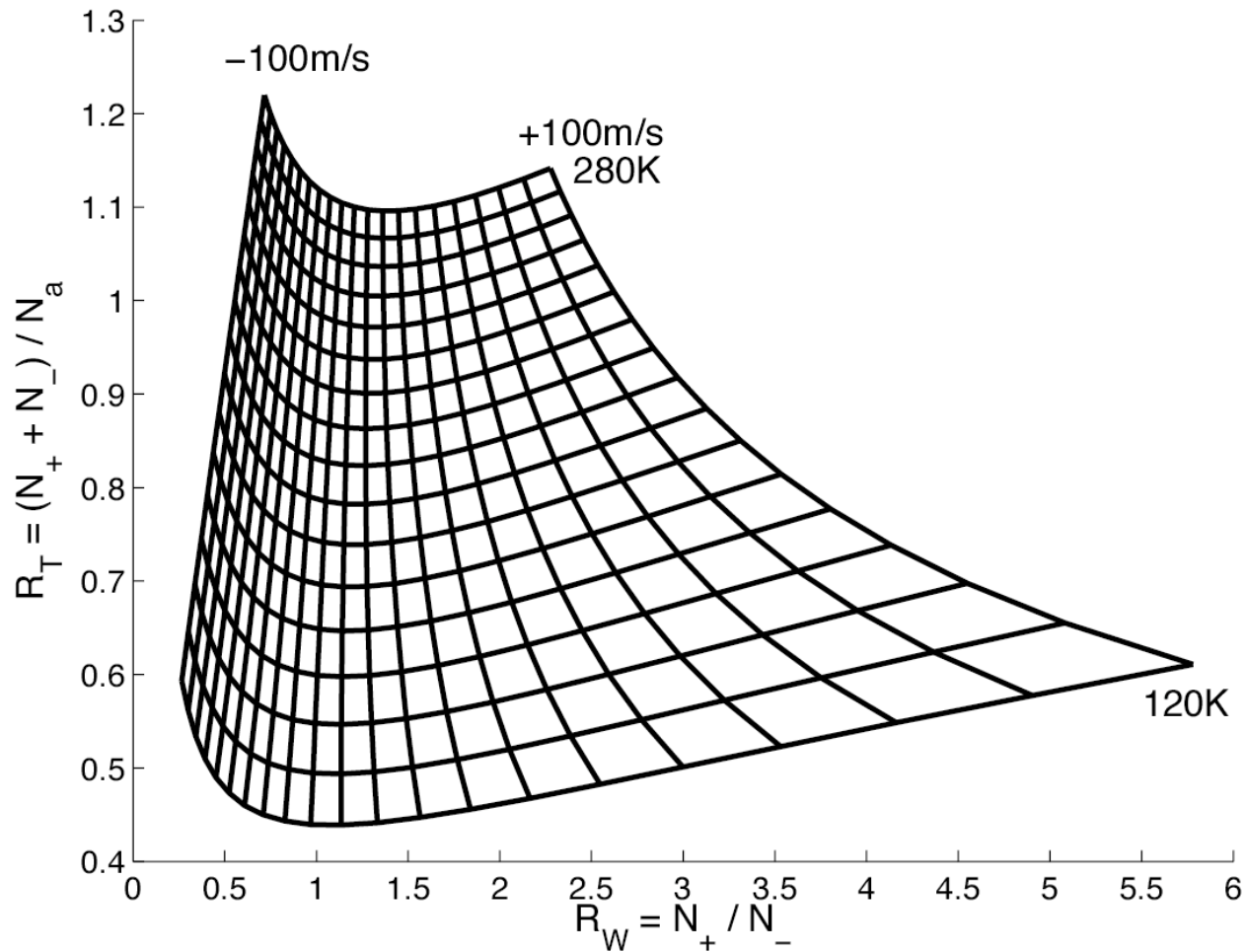
How Does Ratio Technique Work?

- ❑ Compute Doppler calibration curves from physics
- ❑ Look up these two ratios on the calibration curves to infer the corresponding Temperature and Wind from isoline/isogram.



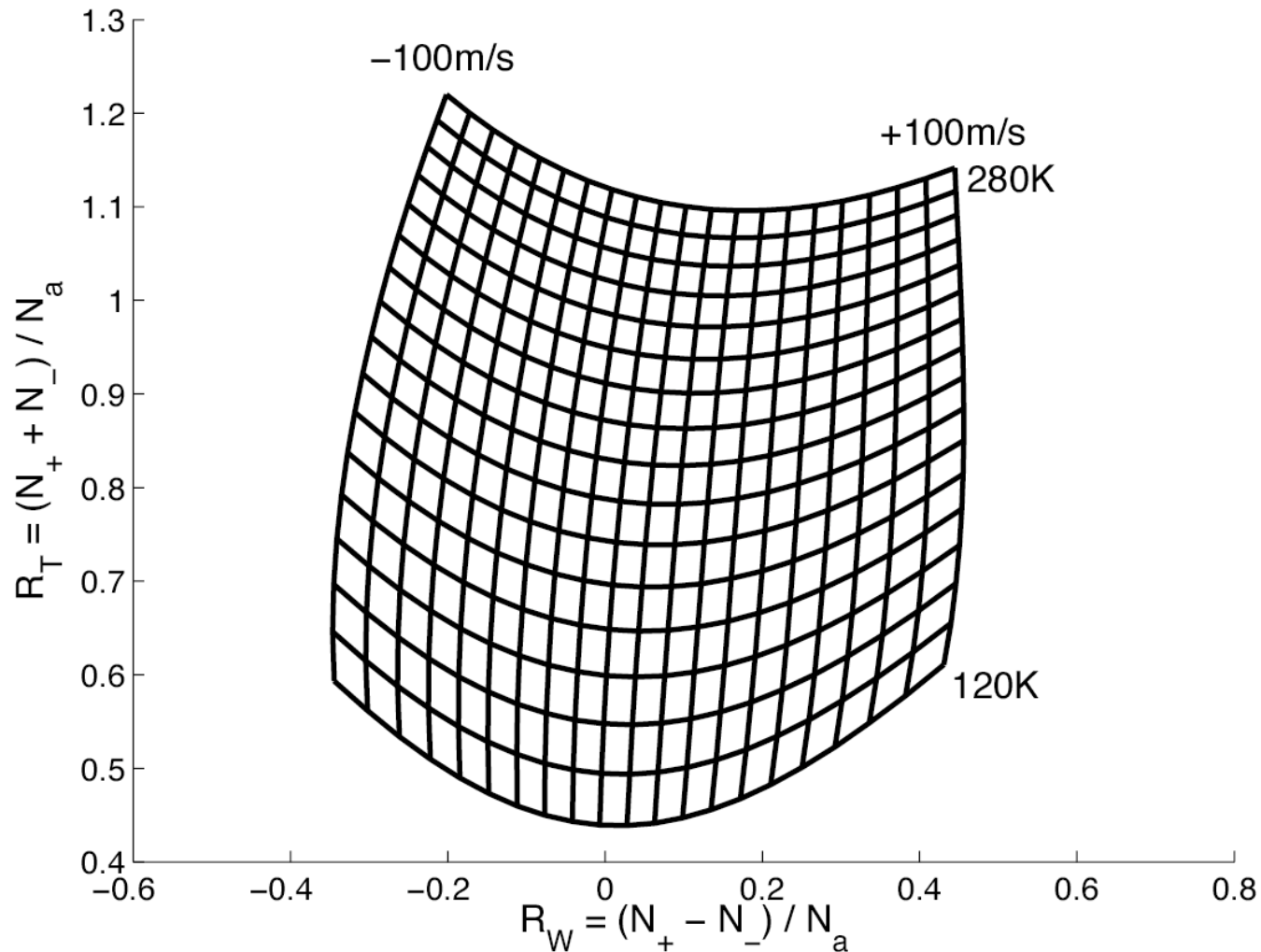
Comparison of Calibration Curves

- ❑ Different metrics of R_W result in different wind sensitivities
- ❑ The ratio $R_W = N_+ / N_-$ has inhomogeneous sensitivity



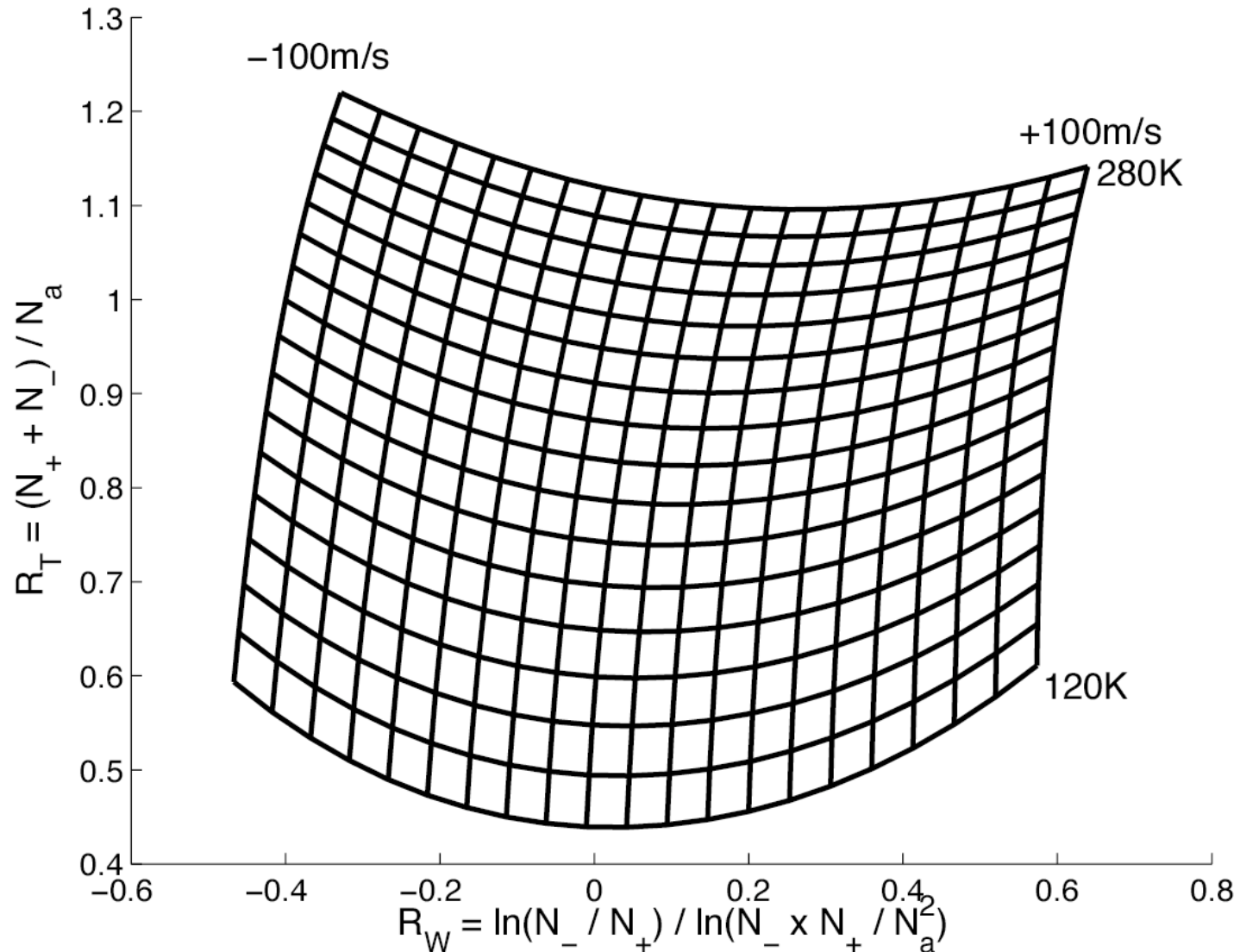
Comparison of Calibration Curves

- ❑ The ratio $R_W = (N_+ - N_-) / N_a$ has much better uniformity than the simplest ratio



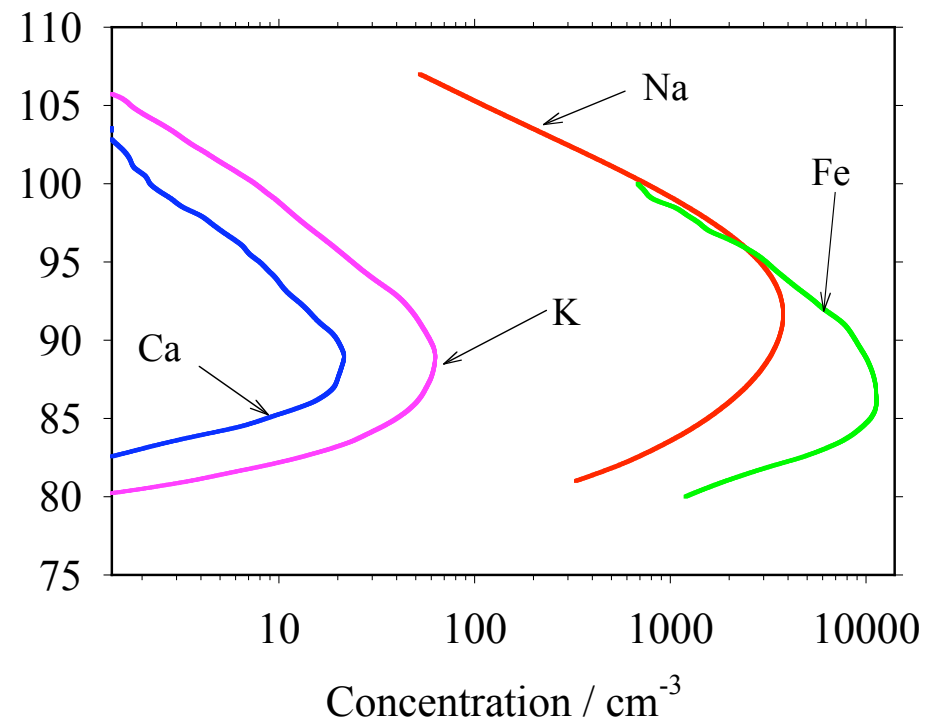
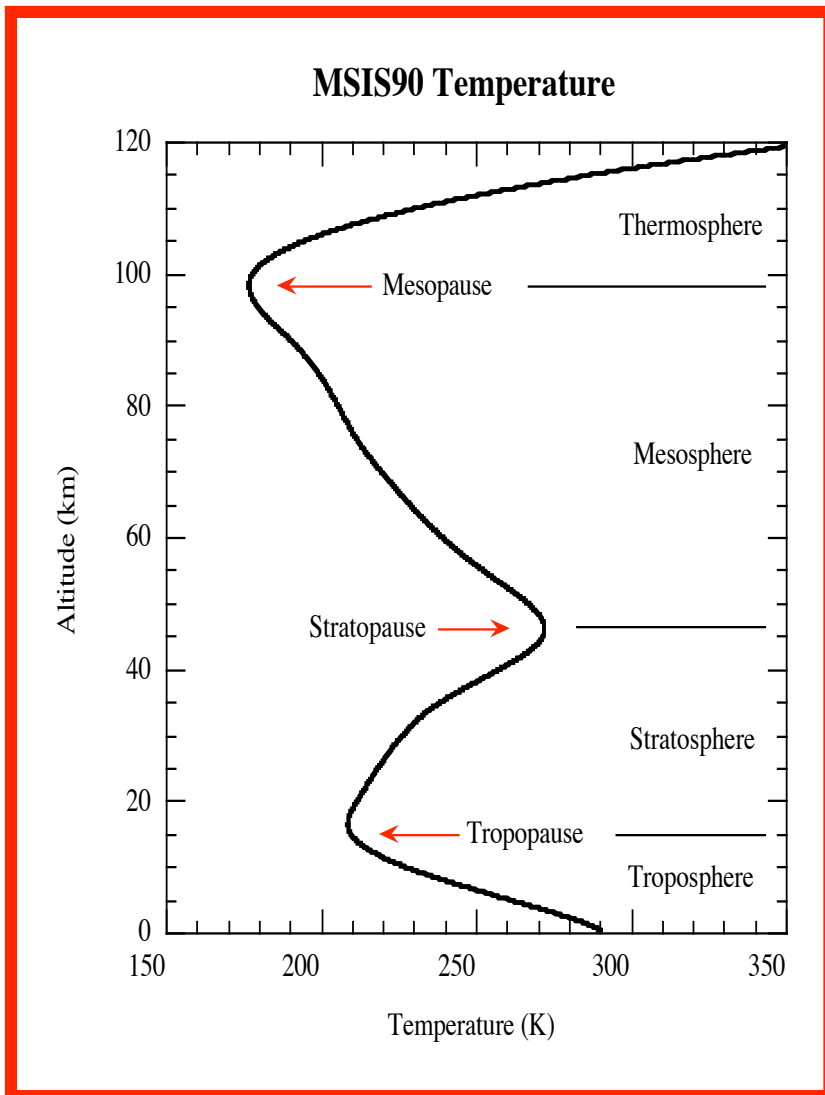
Comparison of Calibration Curves

- The ratio $R_W = \ln(N_- / N_+) / \ln(N_- \times N_+ / N_a^2)$ has good uniformity



More Resonance Fluorescence Doppler Lidars

□ Besides Na, there are more metal species (K, Fe, Ca, Ca⁺, Mg, Li, ...) from meteor ablation. They can be used as tracers for Doppler lidar measurements in MLT region.

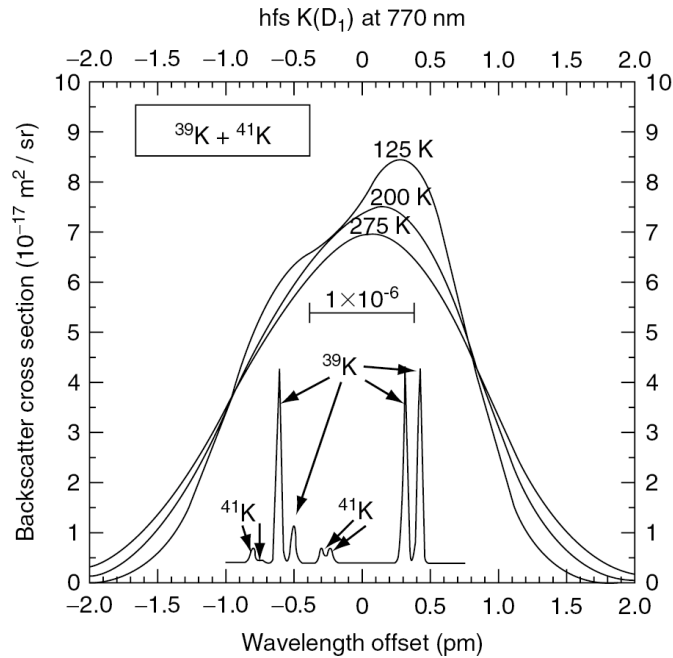
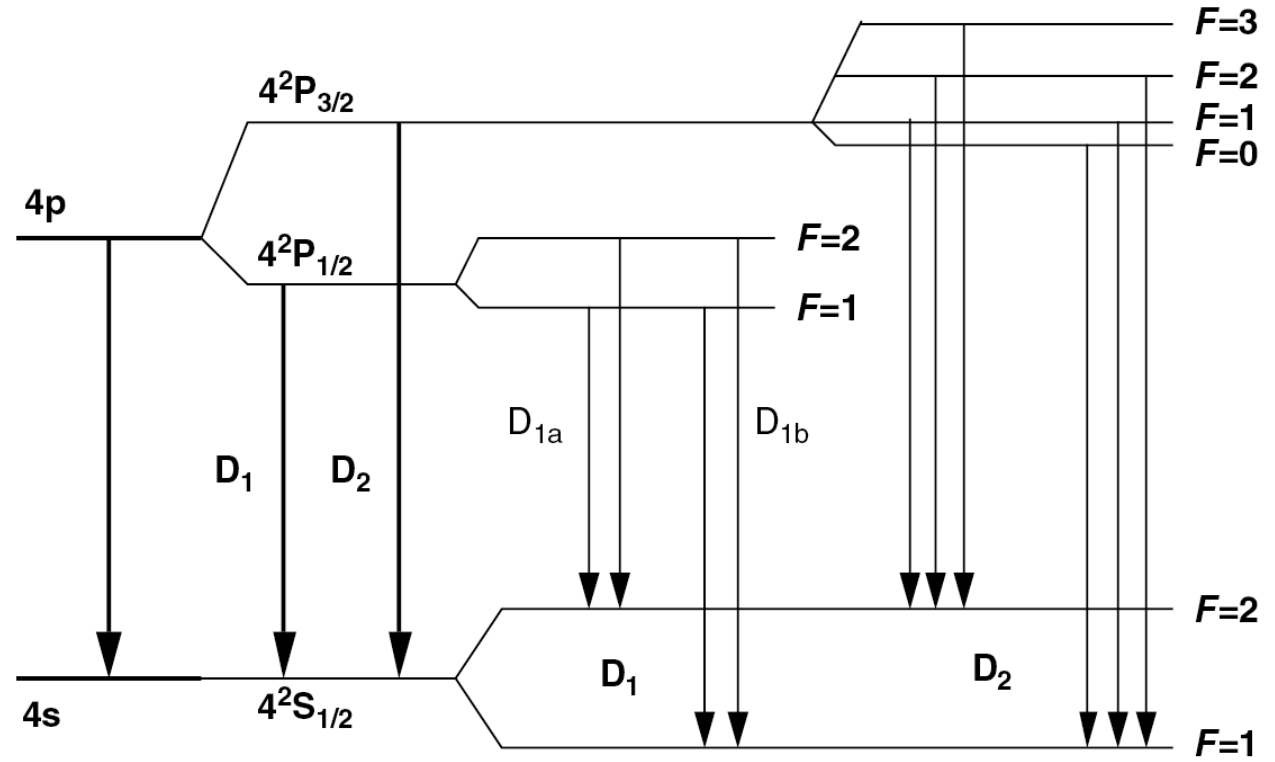


Metal Species in MLT Region

Species	Central wavelength (nm)	A_{ki} ($\times 10^8 \text{ s}^{-1}$)	Degeneracy g_k / g_i	Atomic Weight	Isotopes	Doppler rms Width (MHz)	σ_0 ($\times 10^{-12} \text{ cm}^2$)	Abundance ($\times 10^9 \text{ cm}^{-2}$)	Centroid Altitude (km)	Layer rms Width (km)
Na (D ₂)	589.1583	0.616	4 / 2	22.98977	23	456.54	14.87	4.0	91.5	4.6
Fe	372.0995	0.163	11 / 9	55.845	54, 56, 57, 58	463.79	0.944	10.2	88.3	4.5
K (D ₁)	770.1088	0.382	2 / 2	39.0983	39, 40, 41	267.90	13.42	4.5×10^{-2}	91.0	4.7
K (D ₂)	766.702	0.387	4 / 2	39.0983	39, 40, 41	267.90	26.92	4.5×10^{-2}	91.0	4.7
Ca	422.793	2.18	3 / 1	40.078	40, 42, 43, 44, 46, 48	481.96	38.48	3.4×10^{-2}	90.5	3.5
Ca ⁺	393.777	1.47	4 / 2	40.078	Same as Ca	517.87	13.94	7.2×10^{-2}	95.0	3.6

- ❑ In principle, all these species can be used as trace atoms for resonance fluorescence Doppler lidar measurements.
- ❑ Whether a Doppler lidar can be developed and used mainly depends on the availability and readiness of laser and electro-optic technologies. In addition, the constituent abundance and absorption cross-section are naturally determined.

K Atomic Energy Levels



K fine structure

K hyperfine structure

Transition	K(D_1)	K(D_2)
Wavelength air [nm]	769.8974	766.4911
Wavelength vacuum [nm]	770.1093	766.7021
Rel. intensity	24	25
A_{ik} [10^8 s^{-1}]	0.382 ($\pm 10\%$)	0.387 ($\pm 10\%$)
f -value	0.340	0.682
Terms $2S+1L_J$	$^2S_{1/2} - ^2P_{1/2}^o$	$^2S_{1/2} - ^2P_{3/2}^o$
$g_i - g_k$	2-2	2-4

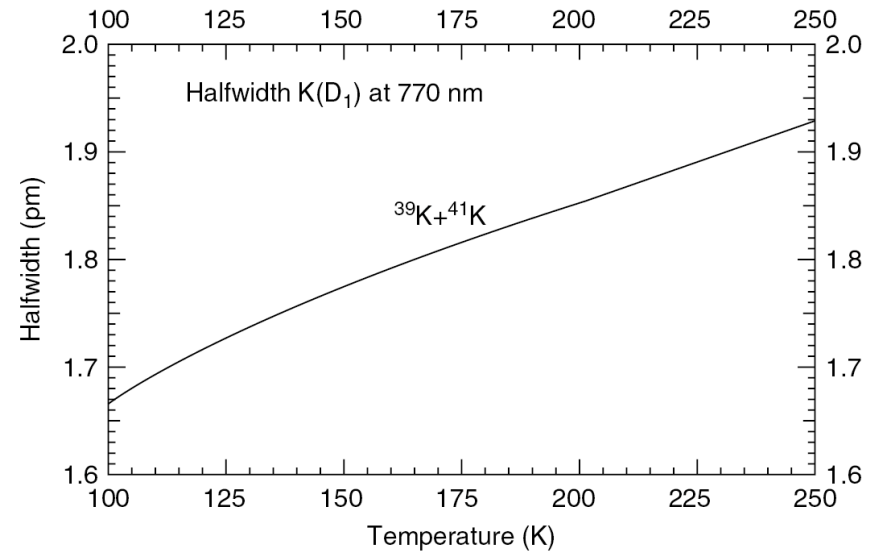
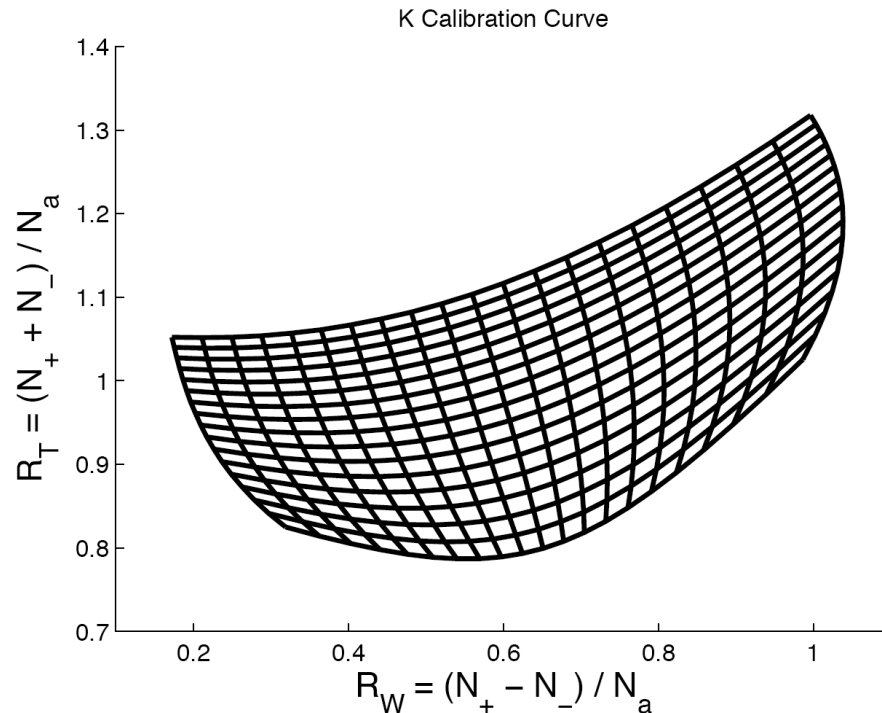
K Atomic Parameters

Isotope	Atomic mass	Abundance	Nuclear spin	K(D ₁) line shift
39	38.963 706 9(3)	0.932 581(44)	$I = 3/2$	0
40	39.963 998 67(29)	0.000 117(1)	$I = 4$	125.58 MHz
41	40.961 825 97(28)	0.067 302(44)	$I = 3/2$	235.28 MHz

Table 5.8 Quantum Numbers, Frequency Offsets, and Relative Line Strength for K (D₁) Hyperfine Structure Lines

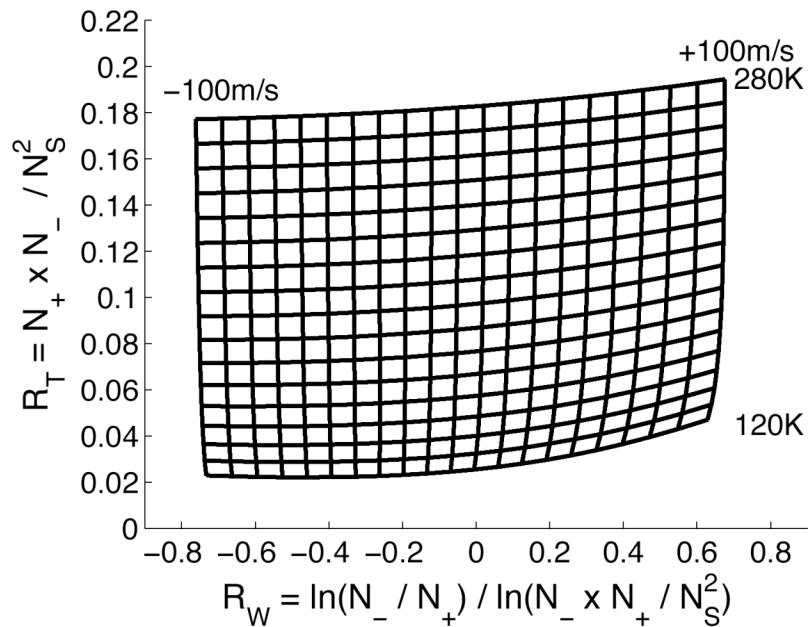
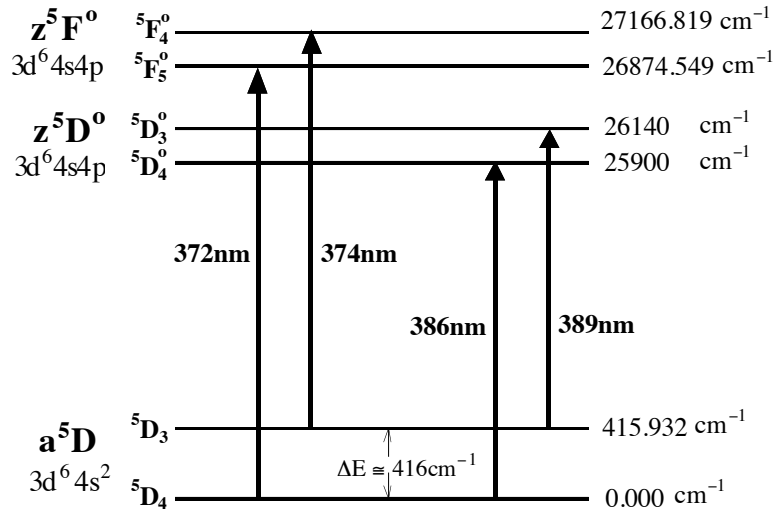
² S _{1/2}	² P _{1/2}	³⁹ K (MHz)	⁴¹ K (MHz)	Relative Line Strength
$F = 1$	$F = 2$	310	405	5/16
	$F = 1$	254	375	1/16
$F = 2$	$F = 2$	-152	151	5/16
	$F = 1$	-208	121	5/16

K Doppler Lidar Principle & Metrics

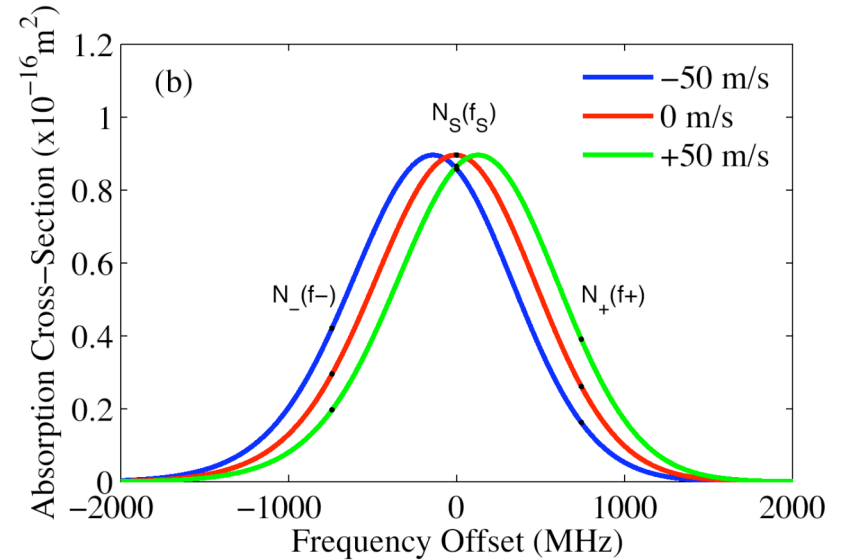
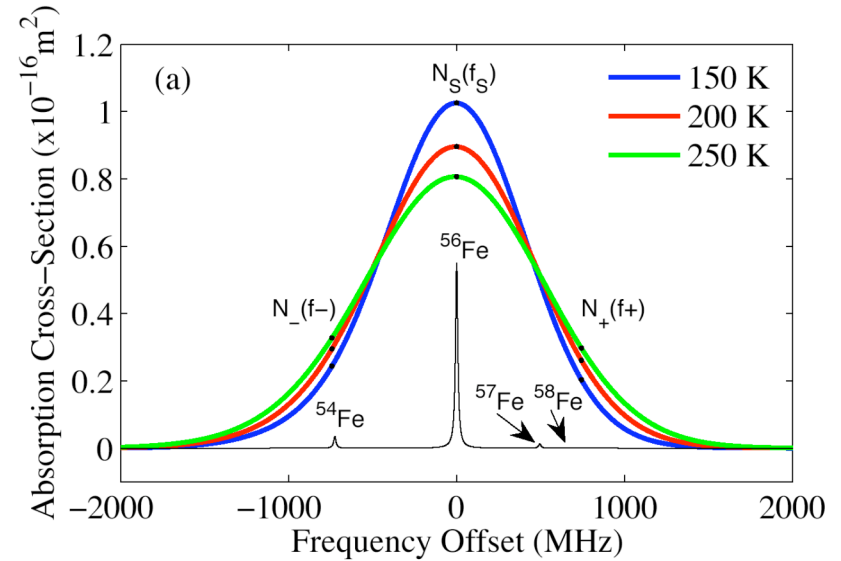


- ❑ Ratio technique versus scanning technique
- ❑ Scanning technique actually has its advantages on several aspects, depending on the laser system used - whether there is pedestal, background problems, etc.
- ❑ Ratio technique usually gives higher resolution.

Fe Doppler Lidar Principles



[Chu et al., ILRC, 2008]



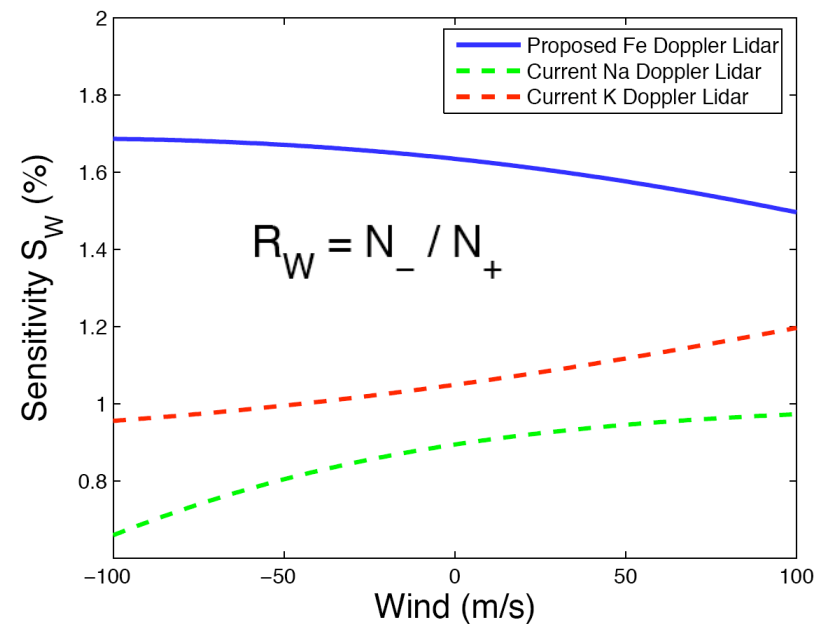
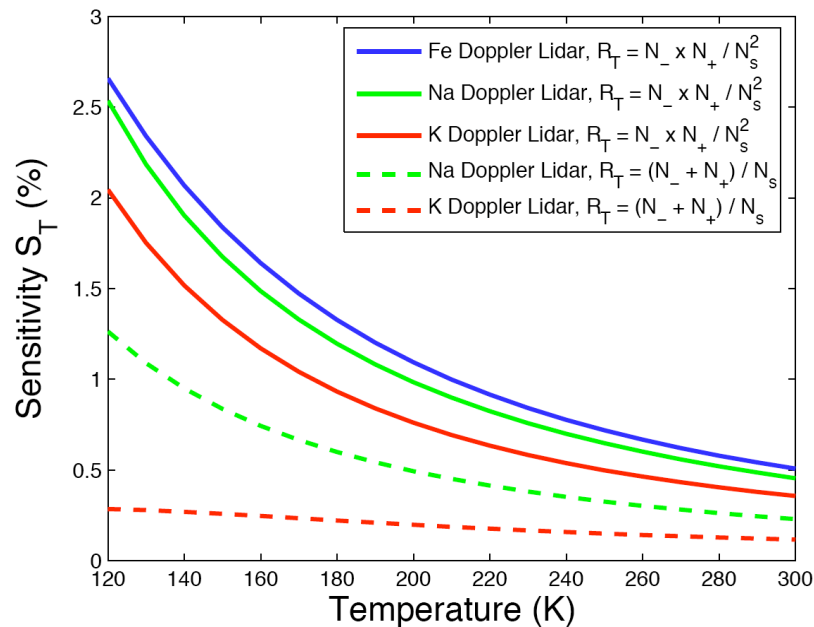
Fe (iron) 372-nm line

Sensitivity Comparison of Na, K, Fe Resonance Doppler Lidars

□ The temperature and wind sensitivities are defined as

$$S_T = \frac{\partial R_T / \partial T}{R_T} = \frac{\Delta R_T / R_T}{\Delta T}$$

$$S_W = \frac{\partial R_W / \partial W}{R_W} = \frac{\Delta R_W / R_W}{\Delta W}$$



□ Errors and signal-to-noise ratio (SNR) depend on atomic properties, geophysical parameters, and lidar parameters.

□ [Gardner et al., 2004] would be a good reference (with cautions).

Summary (1)

- ❑ Doppler ratio technique takes advantage of the high temporal resolution feature by limiting the lidar detection to 3 preset frequencies (usually one peak and two wing frequencies) for 3 unknown parameters (T , W , and density).
- ❑ By taking the ratios among signals at these three frequencies, R_T and R_W are sensitive functions of temperature and radial wind, respectively.
- ❑ We compute the ratios R_T and R_W from atomic physics first to form the lidar calibration curves, and then look up the two ratios calculated from actual photon counts on the calibration curves to infer the corresponding temperature T and radial wind W .
- ❑ Different metrics exhibit different inhomogeneity, resulting in different crosstalk between T and W errors.

Summary (2)

- ❑ There are several different atomic species originating from meteor ablation in the mesosphere and lower thermosphere (MLT) region. They all have the potentials to be tracers for resonance fluorescence Doppler lidars to measure the temperature and wind in MLT region.
- ❑ Na and K Doppler lidars are currently near mature status and making great contributions to MLT science.
- ❑ Fe Doppler lidar has very high future potential due to the high Fe abundance, advanced alexandrite laser technology, Doppler-free Fe spectroscopy, and bias-free measurements, etc.
- ❑ Solid-state Doppler lidars are demanded for science advancement, although dye-laser-based Na Doppler lidar is still the golden standard for now.