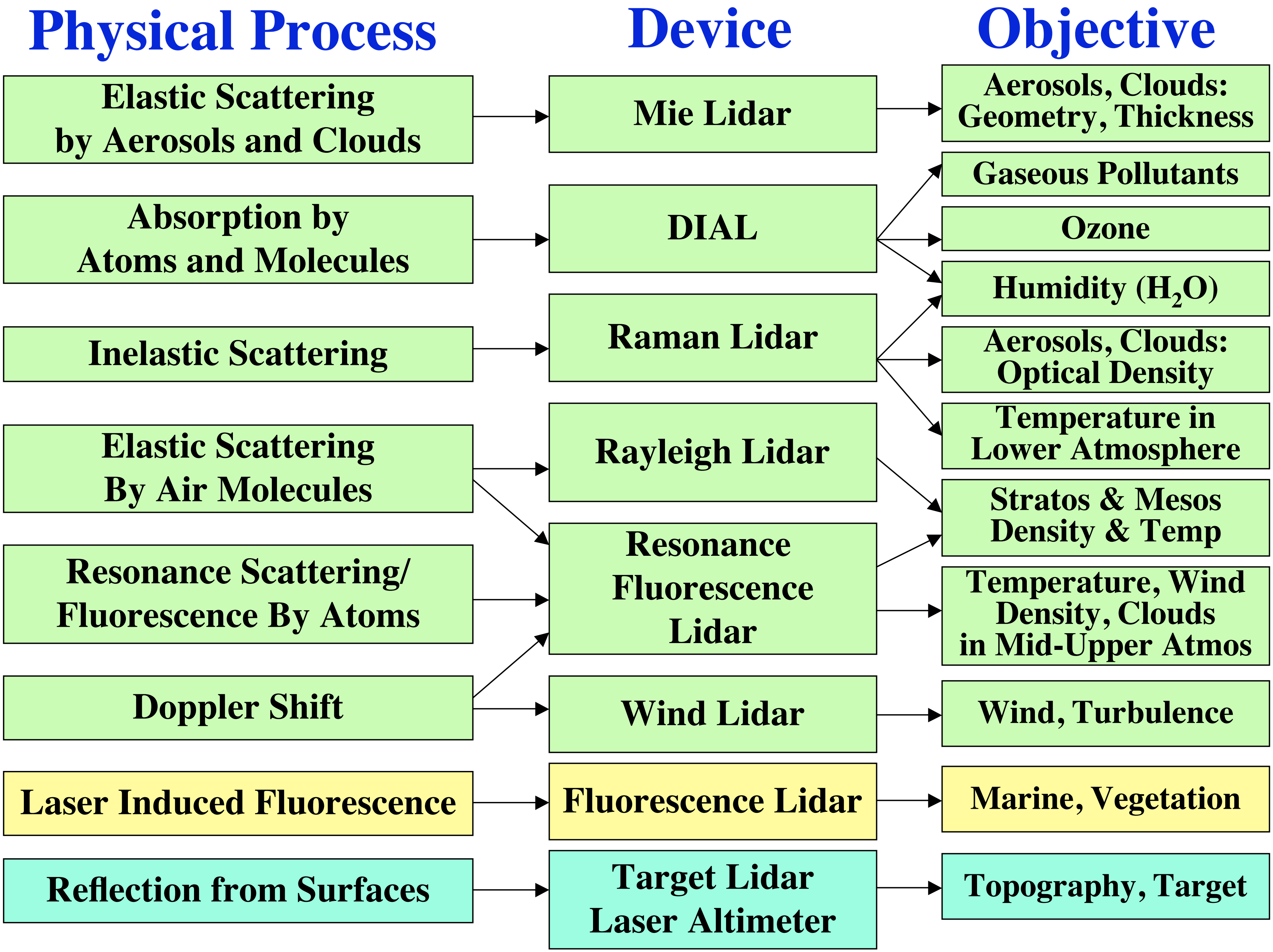
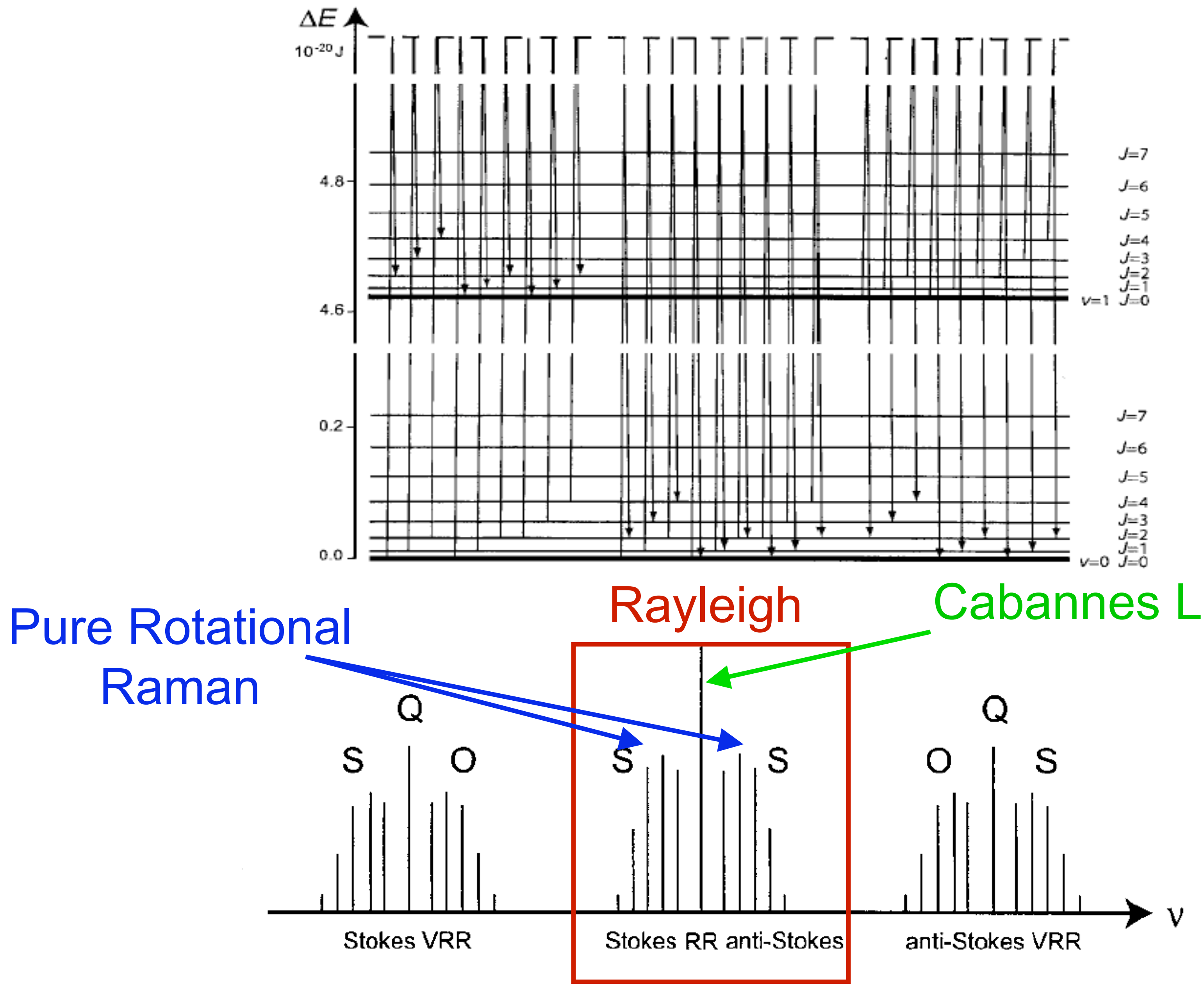
Lecture 06. Fundamentals of Lidar Remote Sensing (4)

- Review physical processes in lidar equation
- Example calculation in physical processes
- Solution for scattering form lidar equation
- Solution for fluorescence form lidar equation
- Solution for differential absorption lidar equation
- Solution for resonance fluorescence lidar
- Solution for Rayleigh and Mie lidar
- Summary

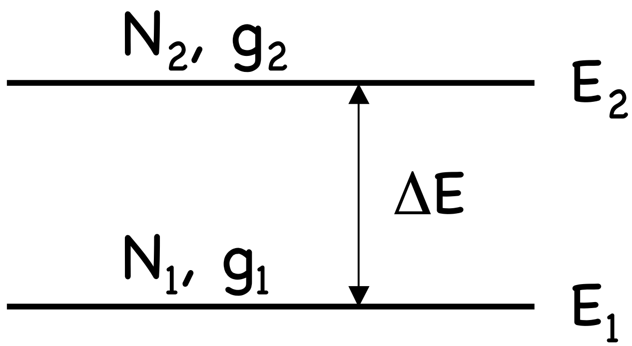

Physical Process
Device
Objective
Elastic Scattering by Aerosols and Clouds
Mie Lidar
Aerosols, Clouds: Geometry, Thickness
Absorption by Atoms and Molecules
DIAL
Gaseous Pollutants
Ozone
Humidity (H_2O)
Inelastic Scattering
Raman Lidar
Aerosols, Clouds: Optical Density
Temperature in Lower Atmosphere
Elastic Scattering By Air Molecules
Rayleigh Lidar
Stratos & Mesos Density & Temp
Resonance Scattering/ Fluorescence By Atoms
Resonance Fluorescence Lidar
Temperature, Wind Density, Clouds in Mid-Upper Atmos
Doppler Shift
Wind Lidar
Wind, Turbulence
Laser Induced Fluorescence
Fluorescence Lidar
Marine, Vegetation
Reflection from Surfaces
Target Lidar Laser Altimeter
Topography, Target

Scattering by Molecules in Atmosphere



Boltzmann Distribution

- Maxwell-Boltzmann distribution is the law of particle population distribution according to energy levels (under thermodynamic equilibrium)



$$\frac{N_2}{N_1} = \frac{g_2}{g_1}\exp\left\{-\left(E_2 - E_1\right)/k_B T\right\}$$

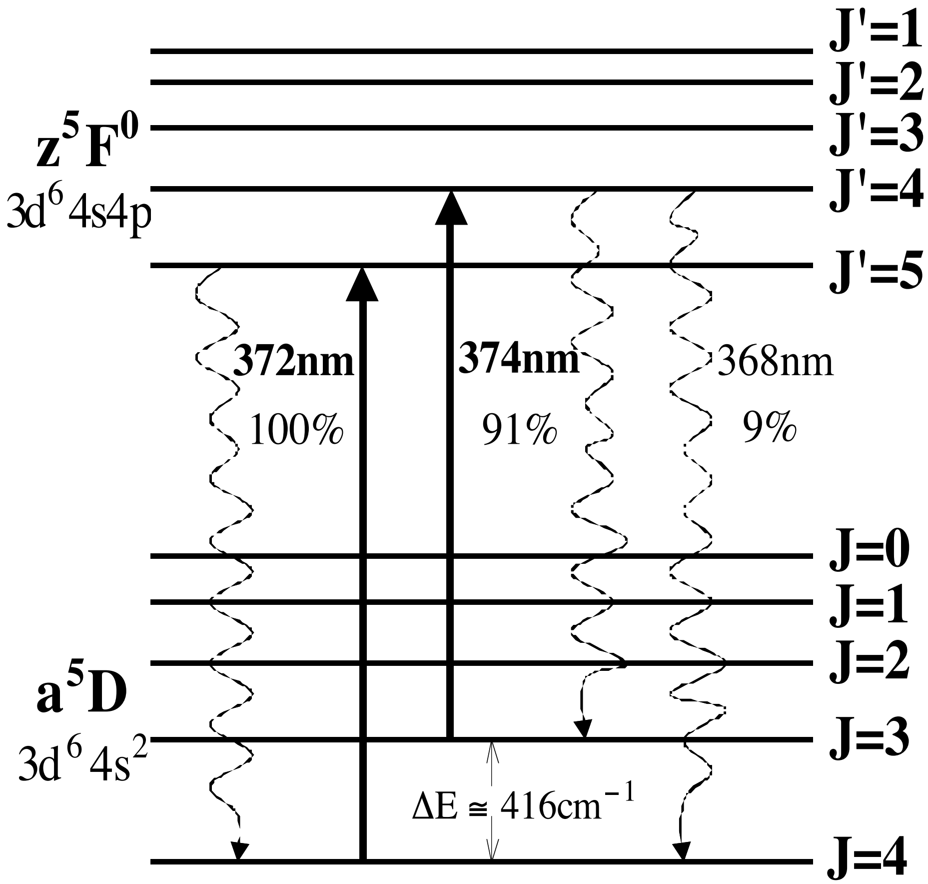
$$\frac{N_k}{N} = \frac{g_k \exp(-E_k/k_B T)}{\sum_i g_i \exp(-E_i/k_B T)}$$

$$T = \frac{\Delta E/k_B}{\ln\left(\frac{g_2}{g_1}\cdot\frac{N_1}{N_2}\right)}$$

N_1 and N_2 – particle populations on energy levels E_1 and E_2
g_1 and g_2 – degeneracy for energy levels E_1 and E_2, $\Delta E = E_2 - E_1$
k_B – Boltzmann constant, T – Temperature, N – total population

Population Ratio ⇨ Temperature

Boltzmann Technique



Atomic Fe Energy Level

[Gelbwachs, 1994; Chu et al., 2002]

Example: Fe Boltzmann

$$\frac{N(J=4)}{N(J=3)} = \frac{g_1}{g_2}\exp\{\Delta E / k_B T\}$$

$$g_1 = 2*4+1 = 9$$

$$g_2 = 2*3+1 = 7$$

$$\Delta E = 416(cm^{-1})$$

$$= hc \times 416 \times 100(J)$$

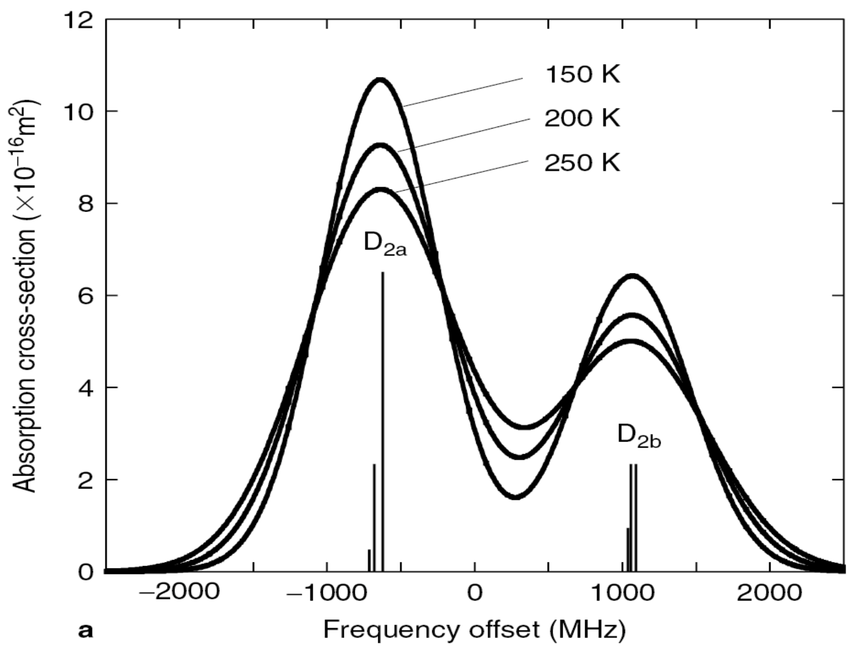
$$\Delta E / k_B = 598.43K$$

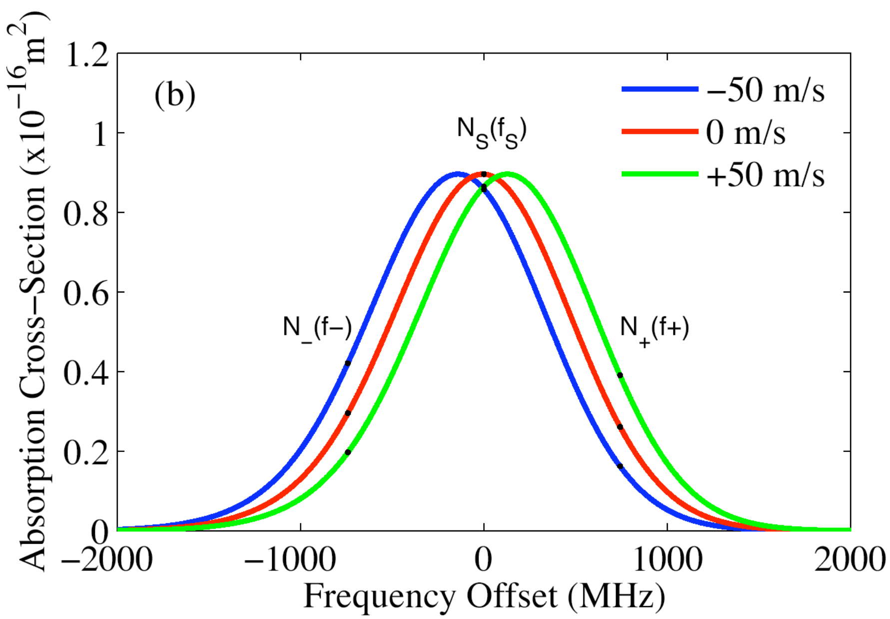
$For \quad T = 200,$

$$\frac{N(J=4)}{N(J=3)} = \frac{9}{7} e^{598.43/200} = 25.6$$

Doppler Shift and Broadening

❑ Doppler Technique - Doppler linewidth broadening and Doppler frequency shift are temperature-dependent and wind-dependent, respectively (applying to both Na, K, Fe resonance fluorescence and molecular scattering)

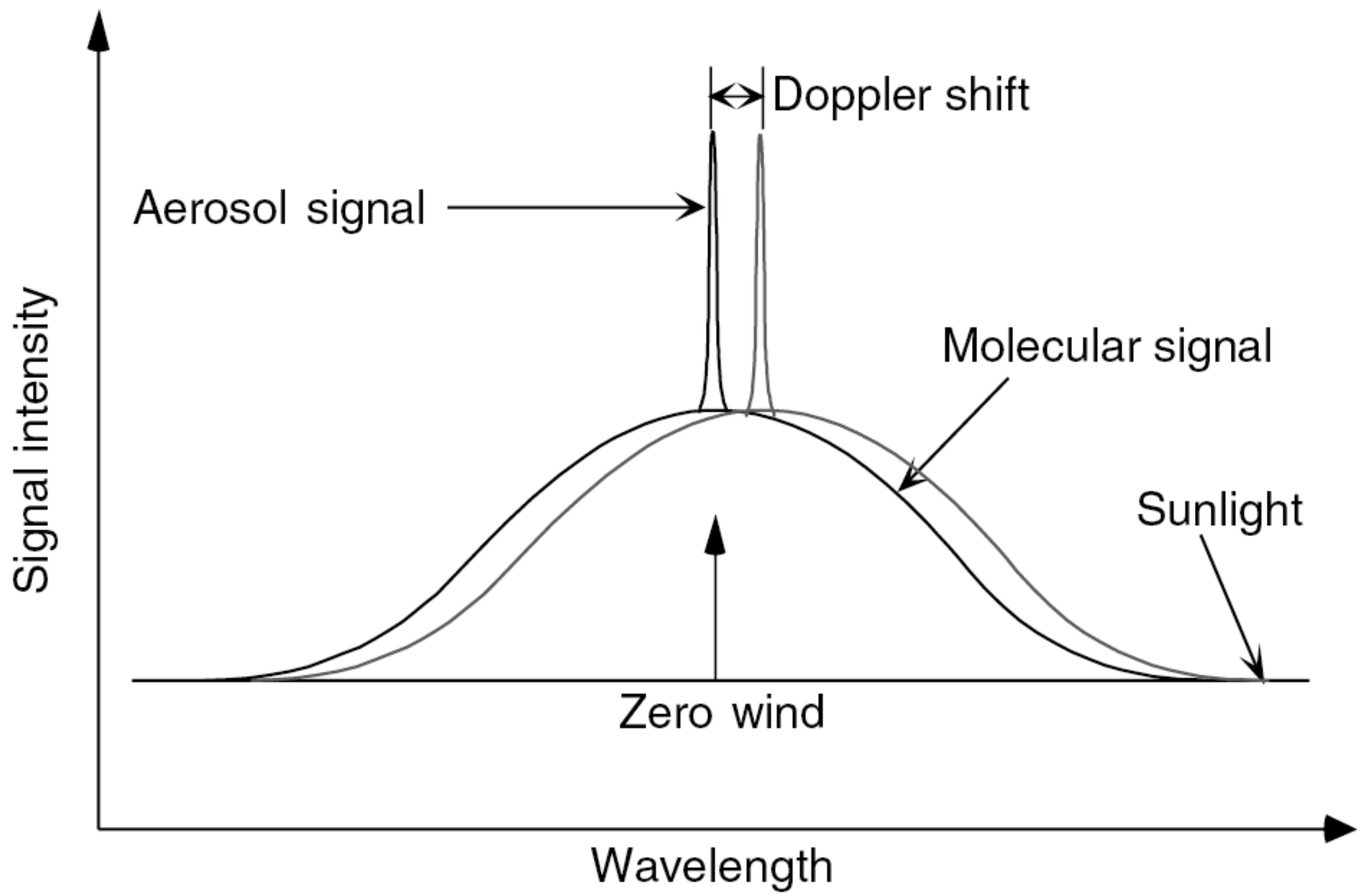




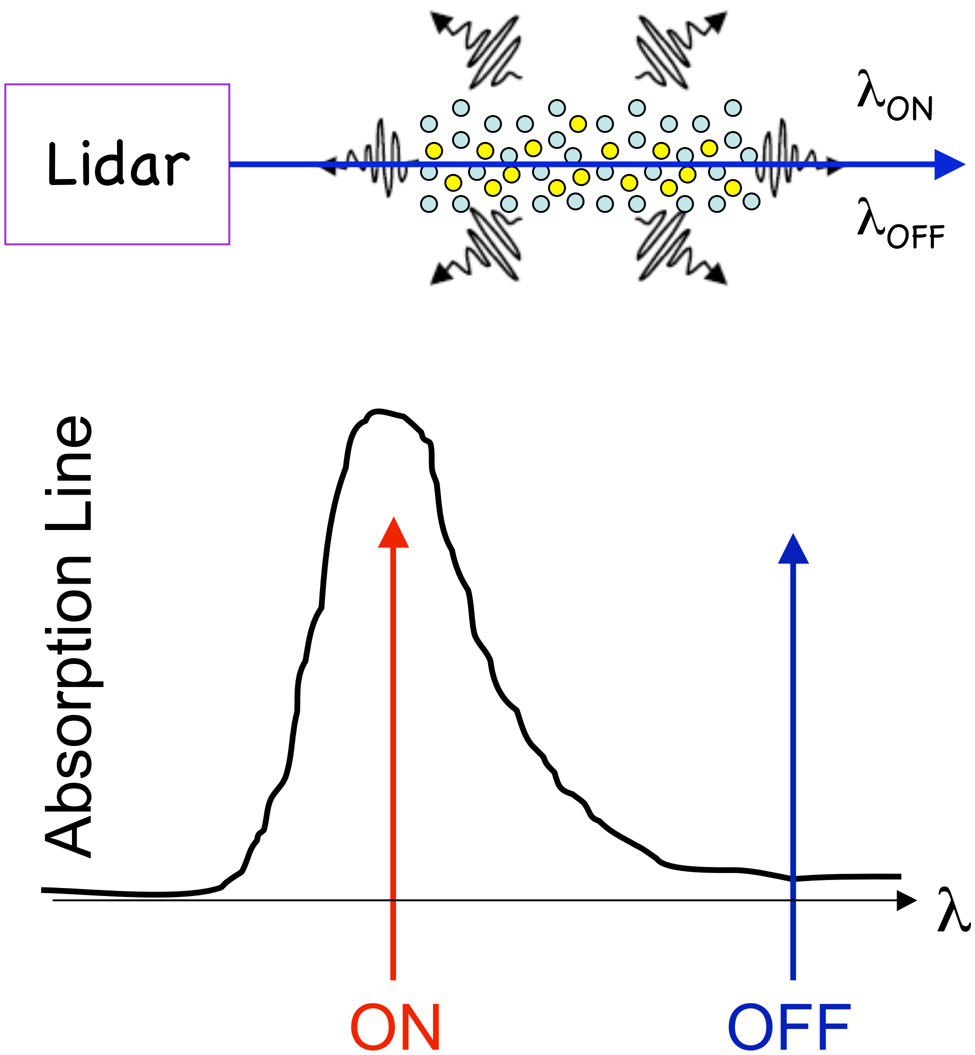
$$\sigma_{rms} = \frac{\omega_0}{c}\sqrt{\frac{k_B T}{M}} = \frac{1}{\lambda_0}\sqrt{\frac{k_B T}{M}}$$

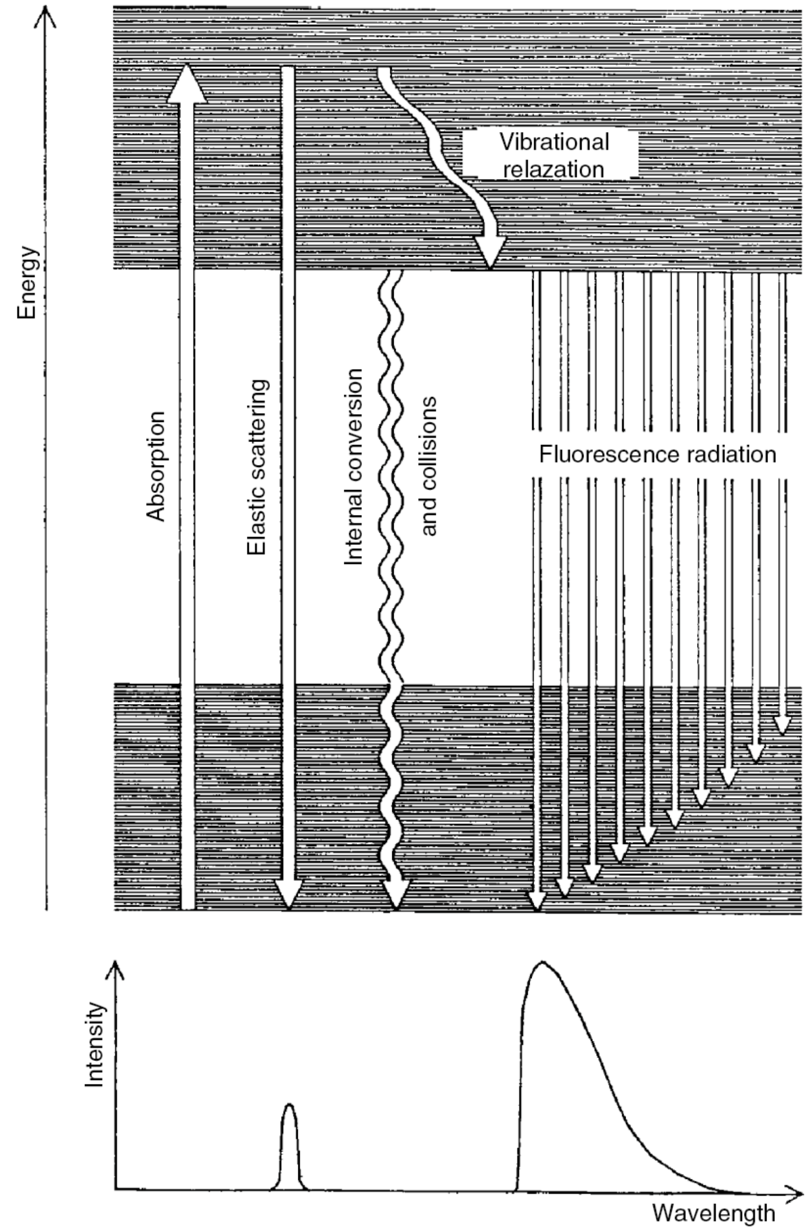
$$\Delta\omega = \omega - \omega_0 = -\vec{k}\cdot\vec{v} = -\omega_0 \frac{v\cos\theta}{c}$$

Doppler Shift and Broadening



Absorption and Fluorescence





Backscatter Cross-Section Comparison

Physical Process	Backscatter Cross-Section	Mechanism
Mie (Aerosol) Scattering	10^{-8} - 10^{-10} cm^2sr^{-1}	Two-photon process Elastic scattering, instantaneous
Resonance Fluorescence	10^{-13} cm^2sr^{-1}	Two single-photon process (absorption and spontaneous emission) Delayed (radiative lifetime)
Molecular Absorption	10^{-19} cm^2sr^{-1}	Single-photon process
Fluorescence from molecule, liquid, solid	10^{-19} cm^2sr^{-1}	Two single-photon process Inelastic scattering, delayed (lifetime)
Rayleigh Scattering	10^{-27} cm^2sr^{-1}	Two-photon process Elastic scattering, instantaneous
Raman Scattering	10^{-30} cm^2sr^{-1}	Two-photon process Inelastic scattering, instantaneous

Rayleigh Backscatter Coefficient

$$\beta_{Rayleigh}(\lambda, z, \theta = \pi) = 2.938 \times 10^{-32} \frac{P(z)}{T(z)} \cdot \frac{1}{\lambda^{4.0117}} \left(m^{-1} sr^{-1}\right)$$

P in mbar and T in Kelvin at altitude z, λ in meter.

$$\beta(\theta) = \frac{\beta_T}{4\pi} P(\theta) = \frac{\beta_T}{4\pi} \times 0.7629 \times (1 + 0.9324 \cos^2\theta)$$

Rayleigh Backscatter Cross Section

$$\frac{d\sigma_m(\lambda)}{d\Omega} = 5.45 \cdot \left(\frac{550}{\lambda}\right)^4 \times 10^{-32} \left(m^2 sr^{-1}\right)$$

where λ is the wavelength in nm.

- For Rayleigh lidar, λ = 532 nm, ⇨ 6.22×10^{-32} $m^2 sr^{-1}$

Absorption Cross-Section for Atoms

Atomic absorption cross section σ_{abs}

for Na 589-nm D2 line is about 10^{-15} m^2

for Fe 372-nm line is about 10^{-16} m^2

How do you derive the backscatter cross section?

$$\frac{d\sigma_m}{d\Omega}(\theta = \pi) = \frac{\sigma_{abs}}{4\pi} = \frac{10^{-15} m^2}{4\pi}$$

$$= 7.95 \times 10^{-16} m^2 sr^{-1} = 7.95 \times 10^{-12} cm^2 sr^{-1}$$

Polarization in Scattering

- Depolarization can be resulted from

(1) Non-spherical particle shape (true for both aerosol/cloud and atmosphere molecules)

(2) Inhomogeneous refraction index

(3) Multiple scattering inside particle

- The range-resolved linear depolarization ratio is defined from lidar or optical observations as

$$\delta(R) = \frac{P_\perp}{P_{||}} = \frac{I_\perp}{I_{||}}$$

where P and I are the light power and intensity detected, respectively.

- According to Gary Gimmestad, the following definition is misleading:

$$\delta(R) = \left[\beta_\perp(R) / \beta_{||}(R)\right] \exp\left(T_{||} - T_\perp\right)$$

β and T are the backscattering coefficients and atmospheric transmittances.

Review Lidar Equation

- General lidar equation with angular scattering coefficient

$$N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

- General lidar equation with total scattering coefficient

$$N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta_T(\lambda,\lambda_L,R)\Delta R\right]\cdot\frac{A}{4\pi R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

- General lidar equation in angular scattering coefficient β and extinction coefficient α form

$$N_S(\lambda,R) = \left[\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right]\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\left(\frac{A}{R^2}\right)$$
$$\cdot\exp\left[-\int_0^R \alpha(\lambda_L,r')dr'\right]\exp\left[-\int_0^R \alpha(\lambda,r')dr'\right]\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

Specific Lidar Equations

- Lidar equation for Rayleigh lidar

$$N_S(\lambda,R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\beta(\lambda,R)\Delta R\right)\left(\frac{A}{R^2}\right)T^2(\lambda,R)\left(\eta(\lambda)G(R)\right) + N_B$$

- Lidar equation for resonance fluorescence lidar

$$N_S(\lambda,R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda,R)n_c(z)R_B(\lambda)\Delta R\right)\left(\frac{A}{4\pi R^2}\right)\left(T_a^2(\lambda,R)T_c^2(\lambda,R)\right)\left(\eta(\lambda)G(R)\right) + N_B$$

- Lidar equation for differential absorption lidar

$$N_S(\lambda_{on}^{off},R) = N_L(\lambda_{on}^{off})\left[\beta_{sca}(\lambda_{on}^{off},R)\Delta R\right]\left(\frac{A}{R^2}\right)\exp\left[-2\int_0^z \overline{\alpha}(\lambda_{on}^{off},r')dr'\right]$$
$$\times\exp\left[-2\int_0^z \sigma_{abs}(\lambda_{on}^{off},r')n_c(r')dr'\right]\left[\eta(\lambda_{on}^{off})G(R)\right] + N_B$$

General Lidar Equation

Assumptions: independent and single scattering

$$N_S(\lambda,R) = N_L(\lambda_L)\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

- N_S - expected photon counts detected at λ and distance R;
- 1st term - number of transmitted laser photons;
- 2nd term - probability that a transmitted photon is scattered by the scatters into a unit solid angle at angle θ;
- 3rd term - probability that a scatter photon is collected by the receiving telescope;
- 4th term - light transmission during light propagation from laser source to distance R and from distance R to receiver;
- 5th term - overall system efficiency;
- 6th term - background and detector noise.

More in General Lidar Equation

$$N_S(\lambda,R) = \left[\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right]\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

N_S (R) - expected received photon number from a distance R

P_L - transmitted laser power, λ_L - laser wavelength

Δt - integration time,

h - Planck's constant, c - light speed

β(R) - volume scatter coefficient at distance R for angle θ,

ΔR - thickness of the range bin

A - area of receiver,

T(R) - one way transmission of the light from laser source to distance R or from distance R to the receiver,

η - system optical efficiency,

G(R) - geometrical factor of the system,

N_B - background and detector noise photon counts.

Solution for Scattering Form Lidar Equation

- Scattering form lidar equation

$$N_S(\lambda,R)=\left[\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right]\cdot\left[\beta(\lambda,\lambda_L,R)\Delta R\right]\cdot\left[\frac{A}{R^2}\right]\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

- Solution for scattering form lidar equation

$$\beta(\lambda,\lambda_L,R)=\frac{N_S(\lambda,R)-N_B}{\left[\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right]\Delta R\left(\frac{A}{R^2}\right)\left[T(\lambda_L,R)T(\lambda,R)\right]\left[\eta(\lambda,\lambda_L)G(R)\right]}$$

Solution for
Fluorescence Form Lidar Equation

- Fluorescence form lidar equation

$$N_S(\lambda,R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda,R)n_c(R)R_B(\lambda)\Delta R\right)\left(\frac{A}{4\pi R^2}\right)\left(T_a^2(\lambda,R)T_c^2(\lambda,R)\right)\left(\eta(\lambda)G(R)\right) + N_B$$

- Solution for fluorescence form lidar equation

$$n_c(R) = \frac{N_S(\lambda,R) - N_B}{\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda)R_B(\lambda)\Delta R\right)\left(\frac{A}{4\pi R^2}\right)\left(\eta(\lambda)T_a{}^2(\lambda,R)T_c^2(\lambda,R)G(R)\right)}$$

Differential Absorption/Scattering Form

- For the laser with wavelength λ_{on} on the molecular absorption line

$$N_S(\lambda_{on},R) = N_L(\lambda_{on})\left[\beta_{sca}(\lambda_{on},R)\Delta R\right]\left(\frac{A}{R^2}\right)\exp\left[-2\int_0^R \overline{\alpha}(\lambda_{on},r')dr'\right]$$
$$\times\exp\left[-2\int_0^R \sigma_{abs}(\lambda_{on},r')n_c(r')dr'\right]\left[\eta(\lambda_{on})G(R)\right] + N_B$$

- For the laser with wavelength λ_{off} off the molecular absorption line

$$N_S(\lambda_{off},R) = N_L(\lambda_{off})\left[\beta_{sca}(\lambda_{off},R)\Delta R\right]\left(\frac{A}{R^2}\right)\exp\left[-2\int_0^R \overline{\alpha}(\lambda_{off},r')dr'\right]$$
$$\times\exp\left[-2\int_0^R \sigma_{abs}(\lambda_{off},r')n_c(r')dr'\right]\left[\eta(\lambda_{off})G(R)\right] + N_B$$

Differential Absorption/Scattering Form

- The ratio of photon counts from these two channels is a function of the differential absorption and scattering:

$$\frac{N_S(\lambda_{on},R)-N_B}{N_S(\lambda_{off},R)-N_B}=\frac{N_L(\lambda_{on})\beta_{sca}(\lambda_{on},R)}{N_L(\lambda_{off})\beta_{sca}(\lambda_{off},R)}\frac{\eta(\lambda_{on})}{\eta(\lambda_{off})}$$
$$\times\exp\left\{-2\int_0^R\left[\overline{\alpha}(\lambda_{on},r')-\overline{\alpha}(\lambda_{off},r')\right]dr'\right\}$$
$$\times\exp\left\{-2\int_0^R\left[\sigma_{abs}(\lambda_{on},r')-\sigma_{abs}(\lambda_{off},r')\right]n_c(r')dr'\right\}$$

$$\Delta\sigma=\sigma_{abs}(\lambda_{on})-\sigma_{abs}(\lambda_{off})$$

Solution for
Differential Absorption Lidar Equation

- Solution for differential absorption lidar equation

$$n_c(R) = \frac{1}{2\Delta\sigma}\frac{d}{dR}\left\{\begin{array}{l} \ln\left[\dfrac{N_L(\lambda_{on})\beta_{sca}(\lambda_{on},R)}{N_L(\lambda_{off})\beta_{sca}(\lambda_{off},R)}\dfrac{\eta(\lambda_{on})}{\eta(\lambda_{off})}\right] \\ \qquad -\ln\left[\dfrac{N_S(\lambda_{on},R)-N_B}{N_S(\lambda_{off},R)-N_B}\right] \\ \qquad -2\int_0^R\left[\overline{\alpha}(\lambda_{on},r')-\overline{\alpha}(\lambda_{off},r')\right]dr' \end{array}\right\}$$

$$\Delta\sigma = \sigma_{abs}(\lambda_{on}) - \sigma_{abs}(\lambda_{off})$$

Solution for Resonance Fluorescence Lidar Equation

- Resonance fluorescence and Rayleigh lidar equations

$$N_S(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda,z)n_c(z)R_B(\lambda)\Delta z\right)\left(\frac{A}{4\pi z^2}\right)\left(T_a^{\ 2}(\lambda)T_c^2(\lambda,z)\right)\left(\eta(\lambda)G(z)\right)+N_B$$

$$N_R(\lambda,z_R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_R(\pi,\lambda)n_R(z_R)\Delta z\right)\left(\frac{A}{z_R^{\ 2}}\right)T_a^{\ 2}(\lambda,z_R)\left(\eta(\lambda)G(z_R)\right)+N_B$$

- Rayleigh normalization

$$\frac{n_c(z)}{n_R(z_R)} = \frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^{\ 2}}\cdot\frac{4\pi\sigma_R(\pi,\lambda)}{\sigma_{eff}(\lambda,z)R_B(\lambda)}\cdot\frac{T_a^{\ 2}(\lambda,z_R)G(z_R)}{T_a^{\ 2}(\lambda,z)T_c^2(\lambda,z)G(z)}$$

- Solution for resonance fluorescence

$$n_c(z) = n_R(z_R)\frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^{\ 2}}\cdot\frac{4\pi\sigma_R(\pi,\lambda)}{\sigma_{eff}(\lambda,z)R_B(\lambda)}\cdot\frac{1}{T_c^2(\lambda,z)}$$

Solution for Rayleigh and Mie Lidars

- Rayleigh and Mie (middle atmos) lidar equations

$$N_S(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\beta_R(z) + \beta_{aerosol}(z)\right)\Delta z\left(\frac{A}{z^2}\right)T_a^{\ 2}(\lambda,z)\left(\eta(\lambda)G(z)\right) + N_B$$

$$N_R(\lambda,z_R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\beta_R(z_R)\Delta z\right)\left(\frac{A}{z_R^{\ 2}}\right)T_a^{\ 2}(\lambda,z_R)\left(\eta(\lambda)G(z_R)\right) + N_B$$

- Rayleigh normalization

$$\frac{\beta_R(z) + \beta_{aerosol}(z)}{\beta_R(z_R)} = \frac{N_S(\lambda,z) - N_B}{N_R(\lambda,z_R) - N_B} \cdot \frac{z^2}{z_R^{\ 2}} \cdot \frac{\cancel{T_a^{\ 2}(\lambda,z_R)}\cancel{G(z_R)}}{\cancel{T_a^{\ 2}(\lambda,z)}\cancel{G(z)}}$$

- For Rayleigh scattering at z and z_R

$$\frac{\beta_R(z)}{\beta_R(z_R)} = \frac{\sigma_R(z)n_{atm}(z)}{\sigma_R(z_R)n_{atm}(z_R)} = \frac{n_{atm}(z)}{n_{atm}(z_R)}$$

Solution (Continued)

- Solution for Mie scattering in middle atmosphere

$$\beta_{aerosol}(z)=\beta_R(z_R)\left[\frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^2}-\frac{n_{atm}(z)}{n_{atm}(z_R)}\right]$$

$$\beta_R(\lambda,z_R,\pi)=2.938\times10^{-32}\frac{P(z_R)}{T(z_R)}\cdot\frac{1}{\lambda^{4.0117}}\left(m^{-1}sr^{-1}\right)$$

- Rayleigh normalization when aerosols not present

$$\frac{\beta_R(z)}{\beta_R(z_R)}=\frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^2}\cdot\frac{\cancel{T_a^2(\lambda,z_R)}\cancel{G(z_R)}}{\cancel{T_a^2(\lambda,z)}\cancel{G(z)}}$$

- Solution for relative number density in Rayleigh lidar

$$RND(z)=\frac{n_{atm}(z)}{n_{atm}(z_R)}=\frac{\beta_R(z)}{\beta_R(z_R)}=\frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^2}$$

Summary

- Solutions of lidar equation can be obtained by solving the lidar equation directly if all the lidar parameters and atmosphere conditions are well known.
- Solutions for three forms of lidar equations are shown: scattering form, fluorescence form, and differential absorption form.
- However, system parameters and atmosphere conditions may vary frequently and are NOT well known to experimenters.
- A good solution is to perform Rayleigh normalization to cancel out most of the system and atmosphere parameters so that the essential and known parts can be solved.