# Lecture 35. Lidar Error Analysis: Further Discussion

Accuracy and precision in lidar measurements

- Error propagation and derivation
- Error estimation in lidar simulation
- Error estimation in lidar data processing

Summary

#### **Review Accuracy versus Precision**

□ The accuracy of an experiment is a measure of how close the result of the experiment is to the true value.

□ The precision is a measure of how well the result has been determined, without reference to its agreement with the true value. The precision is also a measure of the reproducibility of the result in a given experiment.

Accuracy concerns about bias, i.e., how far away is the measurement result from the true value? Precision concerns about uncertainty, i.e., how certain or how sure are we about the measurement result?



Accuracy is mainly determined by: (1) How much we understand the physical interactions and processes involved in the measurements or observations, e.g., atomic parameters and absorption cross-section, isotopes, branching ratio, Hanle effect, atomic layer saturation effect, transmission/extinction, interference absorption, etc. (2) How well we know the lidar system parameters, e.g., laser central frequency, laser linewidth and lineshape, photo detector/discriminator calibration, receiver filter function, overlapping function, chopper function, etc.

□ It happened in the history of physical experiments (e.g., quantum frequency standard) that when people understood more about the physical processes or interactions, the claimed experimental accuracy decreased. This is because some systematic errors (bias) caused by certain interactions were not included in earlier error analysis, as people were not aware of them.

□ This could also happen to lidar measurements, e.g., if we were not aware of the branching ratio issue in resonance fluorescence lidar so did not include it in our data reduction, it could bias the results towards one direction. Similar things apply to saturation and Hanle effects, isotopes, extinction, detector calibration.

□ In the lower atmosphere, Brillouin scattering causes pressure broadening to Rayleigh returns (otherwise, pure Doppler broadening). If not considered, the wind and temperature measurements would be biased.

□ In the DIAL, if some interference gases were unknown to people thus were not considered or compensated in data reduction, bias could be resulted.

□ In Rayleigh integration lidars, the major issues affecting accuracy would be the photo detector/discriminator calibration (saturation), overlapping, chopper, and filter functions, interference from aerosol scattering, and atmosphere constant change in the upper atmosphere when air is NOT well mixed.

□ In Raman lidars, how well we know the Raman scattering cross-section, filter function (determine how many Raman lidars are detected), aerosol interference, etc would affect the accuracy.

□ In high-spectral resolution lidar, how well we know the spectral analyzer and how stable the spectral analyzer is, will affect the accuracy and long-term stability.

□ If we do not know our lidar parameters well, bias could also be resulted, e.g., the chirp issue in Na, K, or Fe Doppler lidar due to pulsed amplification. If we were not aware of PMT and discriminator saturation issue, systematic bias could result from our ignorance. If we couldn't measure the narrowband filter function well for daytime observations, systematic errors would occur.

□ For horizontal wind measurements, how accurate we know the off-zenith angle and the azimuth angle would also affect our measurement accuracy.

□ For lidar researchers, one of our major tasks is to understand the physical processes as good as possible (e.g., measuring atomic parameters accurately from lab experiments, seeking and understanding all possible physical interactions involved in the scattering or absorption and fluorescence processes like saturation effects, understanding the details of laser and detection process) and improve our experimental conditions to either avoid or compensate for the systematic errors.

These usually demand experimenters to be highly knowledgeable of atomic, molecular, and laser physics and spectroscopy, measurement procedure, etc. That's why we emphasize the spectroscopy knowledge is more fundamental to lidar technology advancement, rather than optical/laser engineering.

Achieving high accuracy also requires experimenters to control and measure the lidar parameters very accurately and precisely. -- Easy to say but difficult to do. Calibrating your measurement tools is also very important.

□ On the lidar design aspect, it would be good to develop lidar systems that are stable and less subject to laser frequency drift or chirp, etc.

Also, sometimes it is necessary to take the trade-off between accuracy and precision, depending on the experimental purposes/goals.

Absolute temperature and wind values are the most difficult quantities to measure in lidar field, while relative perturbations are much easier to determine.

□ In lidar observations of atmosphere, the situation is more complicated as the atmosphere also experiences large geophysical variability. The geophysical variability can sometimes cover the accuracy problems of lidar measurements, and also makes the estimation of accuracy very difficult to perform.

□ Inter-instrument comparison (i.e., comparison between different lidars or between lidars and other instruments in common volume and simultaneous measurements) may be necessary in the assessment of lidar measurement accuracy. However, currently most people do not pay attention to the accuracy assessment, probably due to lack of knowledge or lack of funding and time.

□ For students taking this class, you should be at least aware of these issues and keep them in mind when you design and/or use a lidar system or lidar data.

Old words say "People with less knowledge are more confident" or "Compound ignorance". But I would rather you are less confident about the results with more knowledge and awareness of accuracy issues.

□ Of course, the ultimate goal is to enhance our knowledge to improve accuracy or compensate systematic errors so that we are both very knowledgeable and confident in our measurement results.

#### **Precision in Lidar Measurements**

□ Random errors determine the measurement precision.

Possible sources: (1) random uncertainty associated with laser jitter and electronic jitter, (2) shot noise associated with photoncounting system. The latter ultimately limits the precision because of the statistic nature of photon-detection processes.

This differentiation of metric ratio method applies to both systematic and random errors, depending on the error sources: are they systematic bias or random jitter?
For example, the error in f<sub>a</sub> can be systematic bias and random jitter, which will lead to systematic error and uncertainty, respectively.

#### **Error Propagation**

□ Systematic and random errors will propagate to the measurement errors of temperature and wind. T and W errors can be derived by the use of differentials of the corresponding ratios  $R_T$  and  $R_W$ .

□ For 2-frequency technique,

$$R_T(f_a, f_c, T, \mathbf{v}_{\mathbf{R}}, \sigma_L) = \frac{\sigma_{eff}(f_c, T, \mathbf{v}_{\mathbf{R}}, \sigma_L)}{\sigma_{eff}(f_a, T, \mathbf{v}_{\mathbf{R}}, \sigma_L)}$$

Temperature errors are given by the derivatives

$$\Delta T = \frac{\partial T}{\partial R_T} \Delta R_T + \frac{\partial T}{\partial f_a} \Delta f_a + \frac{\partial T}{\partial f_c} \Delta f_c + \frac{\partial T}{\partial \sigma_L} \Delta \sigma_L + \frac{\partial T}{\partial v_R} \Delta v_R$$

Using implicit differentiation, we have

$$\begin{split} \Delta T &= \Delta R_T \left( \frac{\partial R_T}{\partial R_T} \middle/ \frac{\partial R_T}{\partial T} \right) + \Delta f_a \left( \frac{\partial R_T}{\partial f_a} \middle/ \frac{\partial R_T}{\partial T} \right) + \Delta f_c \left( \frac{\partial R_T}{\partial f_c} \middle/ \frac{\partial R_T}{\partial T} \right) \\ &+ \Delta \sigma_L \left( \frac{\partial R_T}{\partial \sigma_L} \middle/ \frac{\partial R_T}{\partial T} \right) + \Delta v_R \left( \frac{\partial R_T}{\partial v_R} \middle/ \frac{\partial R_T}{\partial T} \right) \end{split}$$

## **Error Propagation**

 $\square$  The derivatives of  $R_{T}$  to each system parameters are

$$\frac{\partial R_T}{\partial x} = R_T \left[ \frac{\partial \sigma_{eff}(f_c) / \partial x}{\sigma_{eff}(f_c)} - \frac{\partial \sigma_{eff}(f_a) / \partial x}{\sigma_{eff}(f_a)} \right]$$

□ For example, the uncertainty in  $R_T$  caused by photon noise results in the temperature error:  $\Delta R_T \begin{bmatrix} \partial \sigma_{eff}(f_c) / \partial T & \partial \sigma_{eff}(f_a) / \partial T \end{bmatrix}^{-1}$ 

$$\Delta I = \frac{1}{\partial R_T} \Delta R_T = \frac{1}{R_T} \left[ \frac{1}{\sigma_{eff}(f_c)} - \frac{1}{\sigma_{eff}(f_a)} \right]$$

Where  $\Delta R_T/R_T$  is determined photon counts of both signals and background, and the bracket gives the coefficient of  $\Delta T$  to  $\Delta R_T/R_T$ .

$$\frac{\Delta R_T}{R_T} = \frac{\left(1 + \frac{1}{R_T}\right)^{1/2}}{\left(N_{f_a}\right)^{1/2}} \left[1 + \frac{B}{N_{f_a}} \frac{\left(1 + \frac{1}{R_T}\right)}{\left(1 + \frac{1}{R_T}\right)}\right]^{1/2}$$

## Error Analysis in Lidar Simulation

Add this part to your code: usually we only deal with the uncertainties caused by photon noise.

 $\square$   $\partial R_T / \partial T$  can be calculated numerically for different operating points.

□ Derive the  $\Delta R_{\tau}/R_{\tau}$  terms by yourself, considering background, Rayleigh normalization, etc.

## Error Analysis in Data Analysis

Add this part to your code: keep one set of photon counts for error analysis. This set of photon counts should not have PMT, chopper, and range corrections.

□ In principle, we should use different operating point for each temperature/wind condition. But for general purpose of error analysis, we can just use a nominal point, e.g., T = 200 K and V = 0 m/s.



□ Error analysis is an important part for lidar research. Many confusing ideas are the in field, especially on the accuracy versus precision issues.

Adding error analysis part to your lidar simulation and data processing code would be an important task for the rest of the semester.