Lecture 34. Lidar Error Analysis

- Introduction
- Accuracy versus Precision
- Classification of Measurement Error
- Sources of Errors
- Analytical Expression of Errors
- Example: Na Doppler Lidar
- Summary

Introduction

- For all physical experiments, errors and uncertainties exist that must be reduced by improved understanding of physical processes, improved experimental techniques, and repeated measurements. Those errors remaining must be estimated to establish the validity of our results.
- Error is defined as "the difference between an observed or calculated value and the true value".
- Usually we do not know the "true" value; otherwise there would be no reason for performing the experiment. We may know approximately what it should be, however, either from earlier experiments or from theoretical predictions. Such approximations can serve as a guide but we must always determine in a systematic way from the data and the experimental conditions themselves how much confidence we can have in our experimental results.
- Before going further, let us rule out one kind of errors – illegitimate errors that originate from mistakes in measurement or computation.
- A good reference book for general error analysis is "Data Reduction and Error Analysis for the Physical Sciences" by Philip R. Bevington and D. Keith Robinson (3rd edition, 2003).

Accuracy versus Precision

- It is important to distinguish between the terms accuracy and precision, because in error analysis, accuracy and precision are two different concepts, describing different aspects of a measurement.
- The accuracy of an experiment is a measure of how close the result of the experiment is to the true value.
- The precision is a measure of how well the result has been determined, without reference to its agreement with the true value. The precision is also a measure of the reproducibility of the result in a given experiment.
- Accuracy concerns about bias, i.e., how far away is the measurement result from the true value? Precision concerns about uncertainty, i.e., how certain or how sure are we about the measurement result?
- For any measurement, the results are commonly supposed to be a mean value with a confidence range: $x_i \pm \Delta x_i$

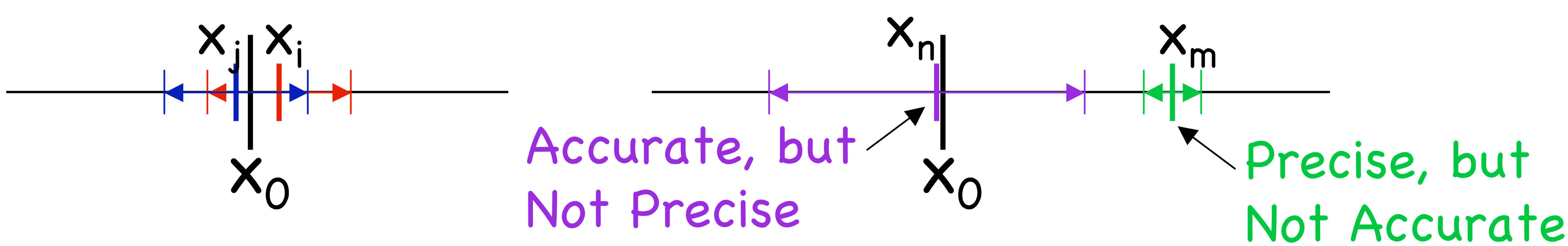


Illustration of Accuracy and Precision

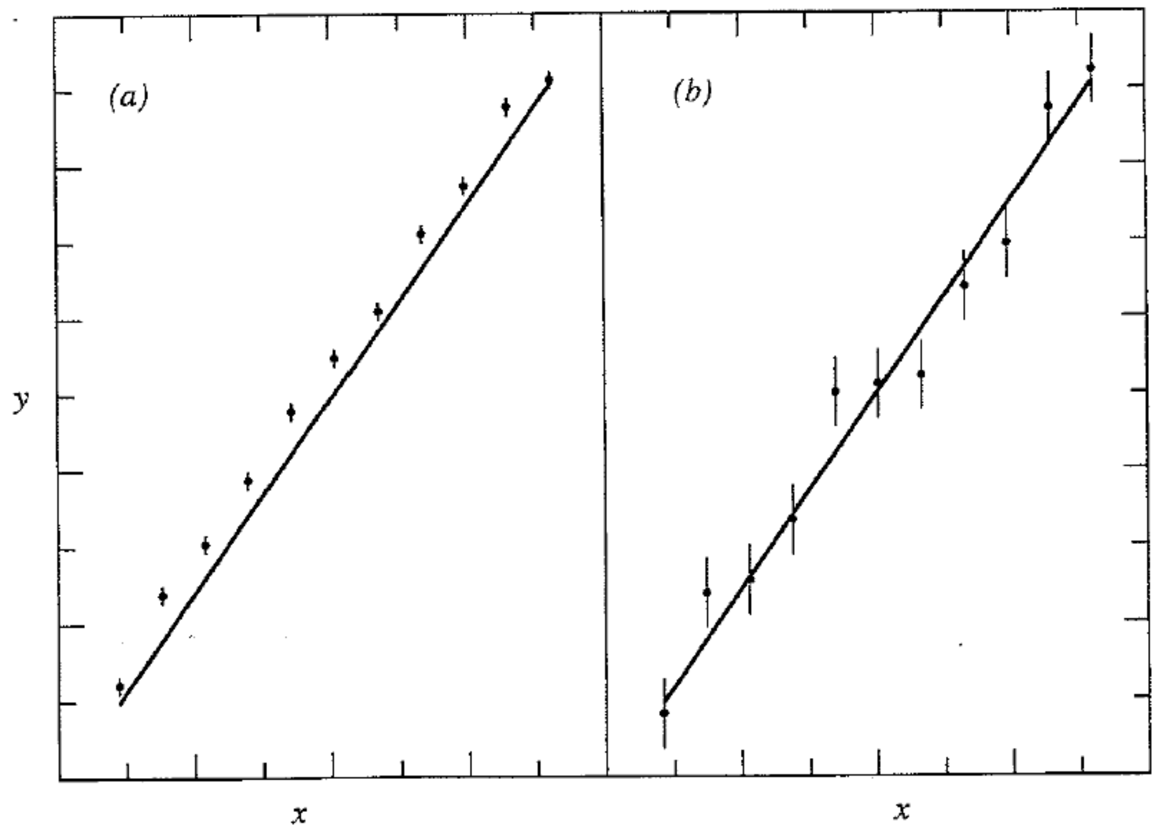


FIGURE 1.1
Illustration of the difference between precision and accuracy. (*a*) Precise but inaccurate data. (*b*) Accurate but imprecise data. True values are represented by the straight lines.

[Data Reduction and Error Analysis, Bevington and Robinson, 2003]

Classification of Measurement Error

- Measurement errors are classified into two major categories: Systematic errors and random errors.

- Systematic errors are errors that will make our results different from the "true" values with reproducible discrepancies. Errors of this type are note easy to detect and not easily studied by statistical analysis. They must be estimated from an analysis of the experimental conditions, techniques, and our understanding of physical interactions. A major part of the planning of an experiment should be devoted to understanding and reducing sources of systematic errors.

- Random errors are fluctuations in observations that yield different results each time the experiment is repeated, and thus require repeated experimentation to yield precise results.

- Another way to describe systematic and random errors are: Experimental uncertainties that can be revealed by repeating the measurements are called random errors; those that cannot be revealed in this way are called systematic errors.

Illustration of Accuracy and Precision

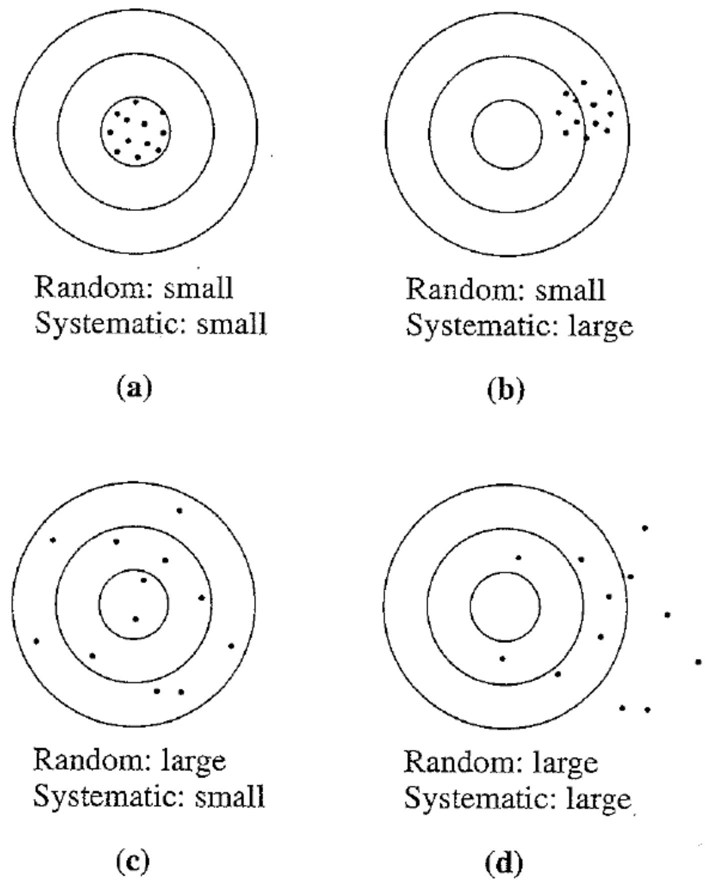


Figure 4.1. Random and systematic errors in target practice. **(a)** Because all shots arrived close to one another, we can tell the random errors are small. Because the distribution of shots is centered on the center of the target, the systematic errors are also small. **(b)** The random errors are still small, but the systematic ones are much larger—the shots are "systematically" off-center toward the right. **(c)** Here, the random errors are large, but the systematic ones are small—the shots are widely scattered but not systematically off-center. **(d)** Here, both random and systematic errors are large.

Illustration of Accuracy and Precision

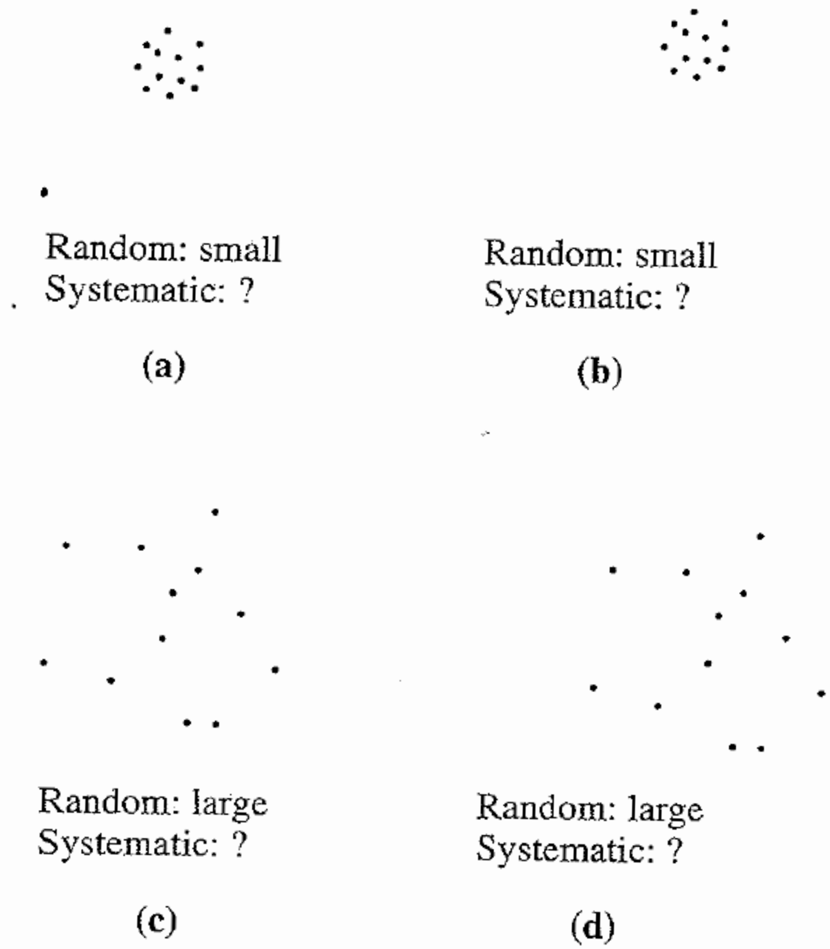


Figure 4.2. The same experiment as in Figure 4.1 redrawn without showing the position of the target. This situation corresponds closely to the one in most real experiments, in which we do not know the true value of the quantity being measured. Here, we can still assess the random errors easily but cannot tell anything about the systematic ones.

Errors vs. Accuracy & Precision

- The accuracy of an experiment is generally dependent on how well we can control or compensate for systematic errors.
- The precision of an experiment depends upon how well we can overcome random errors.
- A given accuracy implies an equivalent precision and, therefore, also depends on some extent on random errors.

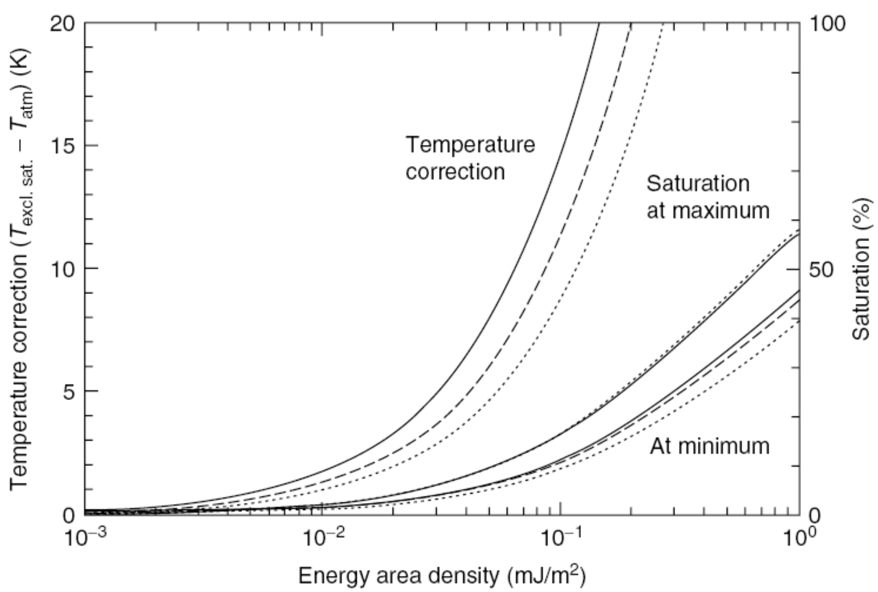
Accuracy (Bias)	Precision (Uncertainty)
↕	↕
Systematic Errors	Random Errors

Error Analysis: Accuracy

- Systematic errors determine the measurement accuracy.
- Possible sources: imprecise information of (1) atomic absorption cross-section, (2) laser absolute frequency calibration, (3) laser lineshape, (4) receiver filter function, (5) photo detector calibration, (6) geometric factor.
- Determination of $\sigma_{abs}(\nu)$: Hanle effect, Na layer saturation, and optical pumping effect.

Hanle effect modified A_n:
5, 5, 2, 14, 5, 1 $\rightarrow$
5, 5.48, 2, 15.64, 5, 0.98

- Absolute laser frequency calibration and laser lineshape.
- Receiver filter function and geometric factor.



Na Layer Saturation

Error Analysis: Precision

- Random errors determine the measurement precision.
- Possible sources: (1) random uncertainty associated with laser jitter and electronic jitter, (2) shot noise associated with photon-counting system. The latter ultimately limits the precision because of the statistic nature of photon-detection processes.
- In normal lidar photon counting, photon counts obey Poisson distribution. Therefore, for a given photon count N, the corresponding uncertainty is $\Delta N = \sqrt{N}$
- For three frequency technique, the relative errors are

$$\frac{\Delta R_T}{R_T} = \frac{\left(1 + \frac{1}{R_T}\right)^{1/2}}{\left(N_{f_a}\right)^{1/2}} \left[1 + \frac{B}{N_{f_a}} \frac{\left(1 + \frac{2}{R_T{}^2}\right)}{\left(1 + \frac{1}{R_T}\right)}\right]^{1/2}$$

$$\frac{\Delta R_W}{R_W} = \frac{\left(1 + \frac{1}{R_W}\right)^{1/2}}{\left(N_{f_+}\right)^{1/2}} \left[1 + \frac{B}{N_{f_+}} \frac{\left(1 + \frac{1}{R_W{}^2}\right)}{\left(1 + \frac{1}{R_W}\right)}\right]^{1/2}$$

Calculation of Errors: Error Propagation

- Systematic and random errors will propagate to the measurement errors of temperature and wind. T and W errors can be derived by the use of differentials of the corresponding ratios R_T and R_W.

- For 2-frequency technique,

$$R_T(f_a, f_c, T, v_R, \sigma_L) = \frac{\sigma_{eff}(f_c, T, v_R, \sigma_L)}{\sigma_{eff}(f_a, T, v_R, \sigma_L)}$$

- Temperature errors are given by the derivatives

$$\Delta T = \frac{\partial T}{\partial R_T}\Delta R_T + \frac{\partial T}{\partial f_a}\Delta f_a + \frac{\partial T}{\partial f_c}\Delta f_c + \frac{\partial T}{\partial \sigma_L}\Delta \sigma_L + \frac{\partial T}{\partial v_R}\Delta v_R$$

- Using implicit differentiation, we have

$$\Delta T = \Delta R_T\left(\frac{\partial R_T}{\partial R_T}\Big/\frac{\partial R_T}{\partial T}\right) + \Delta f_a\left(\frac{\partial R_T}{\partial f_a}\Big/\frac{\partial R_T}{\partial T}\right) + \Delta f_c\left(\frac{\partial R_T}{\partial f_c}\Big/\frac{\partial R_T}{\partial T}\right)$$
$$+ \Delta \sigma_L\left(\frac{\partial R_T}{\partial \sigma_L}\Big/\frac{\partial R_T}{\partial T}\right) + \Delta v_R\left(\frac{\partial R_T}{\partial v_R}\Big/\frac{\partial R_T}{\partial T}\right)$$

Calculation of Errors: Error Propagation

- The derivatives of R_T to each system parameters are

$$\frac{\partial R_T}{\partial x} = R_T \left[\frac{\partial \sigma_{eff}(f_c)/\partial x}{\sigma_{eff}(f_c)} - \frac{\partial \sigma_{eff}(f_a)/\partial x}{\sigma_{eff}(f_a)} \right]$$

- For example, the uncertainty in R_T caused by photon noise results in the temperature error:

$$\Delta T = \frac{\partial T}{\partial R_T} \Delta R_T = \frac{\Delta R_T}{R_T} \left[\frac{\partial \sigma_{eff}(f_c)/\partial T}{\sigma_{eff}(f_c)} - \frac{\partial \sigma_{eff}(f_a)/\partial T}{\sigma_{eff}(f_a)} \right]^{-1}$$

Where $\Delta R_T/R_T$ is determined photon counts of both signals and background, and the bracket gives the coefficient of ΔT to $\Delta R_T/R_T$.

- This differentiation of metric ratio method applies to both systematic and random errors, depending on the error sources: are they systematic bias or random jitter?
- For example, the error in f_a can be systematic bias and random jitter, which will lead to systematic error and uncertainty, respectively.

Summary

- Accuracy and precision are two different concepts for lidar error analysis. Accuracy concerns about bias, usually determined by systematic errors. Precision concerns about uncertainty, mainly determined by random errors, and in lidar photon counting case, mainly by photon noise.
- Calculation of errors for ratio technique utilizes the differentiation of the metric ratios as described in text. It works for both systematic and random errors. Certainly, extra work is needed to identify the systematic errors and their sources. Photon noise obeys Poisson distribution.
- Reference our textbook: section 5.2.2.5.2