Lecture 13. Lidar Data Inversion

- Review Doppler lidar architecture
- Daytime capability of Na Doppler lidar
- Introduction
- Common raw data format
- Basic ideas (clues) for data inversion
- Preprocess
- Main process
- Summary
Daytime Capability: Faraday Filter

- Faraday effect is the rotation of light polarization by some media under magnetic field.

\[ n = \sqrt{1 + \chi} \approx 1 + \frac{1}{2} \chi = 1 + \frac{1}{2} \chi' - i \frac{1}{2} \chi'' \quad (5.74) \]

\( \chi \) is the electric susceptibility of Na vapor
Faraday Filter

\[ \chi' = \frac{Ne^2f}{2m\omega\epsilon_0} \frac{\omega_0 - \omega}{(\omega_0 - \omega)^2 + (\gamma/2)^2} \]

\[ \chi'' = \frac{Ne^2f}{2m\omega\epsilon_0} \frac{\gamma/2}{(\omega_0 - \omega)^2 + (\gamma/2)^2} \]

Dispersion

Resonance absorption

![Diagram showing frequency response of a Faraday filter with peaks at \(v_0 - \Delta v\), \(v_0\), and \(v_0 + \Delta v\).]
Faraday Filter

- Phase shift between two circular polarizations

\[ \Delta \varphi = 2\pi \frac{l \Delta n}{\lambda} \]
Introduction: Lidar Data Inversion

- Lidar data inversion deals with the problems of how to derive meaningful physical parameters from raw photon counts.
- It is basically a reverse procedure to the development of lidar equation.
- It is necessary to understand the detailed physical procedure from light transmitting, to light propagation, to light interaction with objects, and to light detection, in order to conduct data inversion correctly.
- In this lecture we discuss the data inversion for Na Doppler lidar (K and Fe lidar would be similar).
From Raw Data to Physical Parameters

Raw Data
\( N_s(R) \)

\( T(R) \)
\( V_R(R) \)
\( n_c(R) \)
\( \beta(R) \)
\( \delta(R) \)
\( ... ... \)

Data Inversion & Error Analysis
Data Retrieval

Data Analysis & Interpretation
Science Study

 PMC, PSC, Aerosols
Constituent Density
Sporadic Layers
Meteors
Climatology
GWs, Tides, PWs
Momentum Flux
Heat Flux
Constituent Flux
Instability
\( ... ... ... \)
# Common Raw Data Format of Lidar

- Headers + One Column Photon Counts (ASCII or Binary)

<table>
<thead>
<tr>
<th>Total Bin #</th>
<th>Low Bin #</th>
<th># of Freq</th>
<th>Set No.</th>
<th>Profile No.</th>
<th># of Shots</th>
</tr>
</thead>
<tbody>
<tr>
<td>10240</td>
<td>1</td>
<td>3</td>
<td>10</td>
<td>1</td>
<td>1500</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>2</td>
<td>7.060833</td>
<td>1</td>
<td>3.05</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
<td>0</td>
<td>1</td>
<td>3.05</td>
<td>20.708</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-156.258</td>
</tr>
</tbody>
</table>

- Month: 12, 0
- Day: 1543, 0
- Year-2000: 0, 1574, 4694, ...
- Azimuth Angle: 12, 1543, 0, 0, 1574, 4694, ...
- Bin Resolution: 12, 1543, 0, 0, 1574, 4694, ...
- Off-Zenith Angle: 12, 1543, 0, 0, 1574, 4694, ...
- Base Altitude (km): 12, 1543, 0, 0, 1574, 4694, ...
- Lat: ...
- Longi: ...
Considerations in Data Inversion

- How to obtain associated information like date, time, location, base altitude, operation conditions?
  -- from data header and other info source
- How to obtain range or altitude information?
  -- from bin number, data header and other source

\[
R = n_{bin} \cdot t_{bin} \cdot \frac{c}{2} \quad \quad z = R \cdot \cos \theta + z_{base}
\]

R is range, \(n_{bin}\) is bin number, \(t_{bin}\) is bin width in time, \(c\) is light speed, \(z\) is absolute altitude, \(\theta\) is off-zenith angle, and \(z_{base}\) is the base altitude relative to sea-level.
## Another Example of Lidar Raw Data

| Total Bin # | Low Bin # | # of Freq | Set No. | Profile No. | # of Shots | Month | Day | Year-2000 | Time (UT) | Azimuth Angle | Bin Resolution | Off-Zenith Angle | Base Altitude (km) | Lat | Longi |
|-------------|-----------|-----------|---------|------------|------------|-------|-----|-----------|-----------|---------------|----------------|-----------------|------------------|------------------|-----|-------|
| 10240       | 1         | 3         | 11      | 2          | 1500       |       |     |           |           |               |                |                 |                  |       |       |
| 4           | 11        | 2         |         |            |            |       |     |           |           |               |                |                 |                  |       |       |
| 180         | 7         | 30        |         |            |            |       |     |           |           |               |                |                 |                  |       |       |

**Photon Counts**

- 12
- 1532
- 0
- 0
- 2400
- 3771
- ...
- ...
- ...

**Data Elements**

- Total Bin #
- Low Bin #
- # of Freq
- Set No.
- Profile No.
- # of Shots
- Month
- Day
- Year-2000
- Time (UT)
- Azimuth Angle
- Bin Resolution
- Off-Zenith Angle
- Base Altitude (km)
- Lat
- Longi
Basic Clue: Lidar Equation & Solution

From lidar equation and its solution to derive preprocess procedure of lidar data inversion

\[ N_S(\lambda, z) = \left( \frac{P_L(\lambda) \Delta t}{hc/\lambda} \right) [\sigma_{\text{eff}}(\lambda, z)n_c(z)R_B(\lambda) + \sigma_R(\pi, \lambda)n_R(z)] \Delta z \left( \frac{A}{4\pi z^2} \right) \]

\[ \times \left( T_a^2(\lambda)T_c^2(\lambda, z) \right)(\eta(\lambda)G(z)) + N_B \]

\[ N_S(\lambda, z_R) = \left( \frac{P_L(\lambda) \Delta t}{hc/\lambda} \right) [\sigma_R(\pi, \lambda)n_R(z_R)] \Delta z \left( \frac{A}{z_R^2} \right) T_a^2(\lambda, z_R)(\eta(\lambda)G(z_R)) + N_B \]

\[ N_{\text{Norm}}(\lambda, z) = \frac{N_{Na}(\lambda, z)}{N_R(\lambda, z_R)T_c^2(\lambda, z)} \frac{z^2}{z_R^2} = \frac{N_S(\lambda, z) - N_B - N_R(\lambda, z)}{N_S(\lambda, z_R) - N_B} \frac{z^2}{T_c^2(\lambda, z) z_R^2} \]

\[ = \left[ \frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} \frac{z^2}{z_R^2} - \frac{n_R(z)}{n_R(z_R)} \right] \frac{1}{T_c^2(\lambda, z)} \]
Basic Clue: Ratio Computation

- From physics, we calculate the ratios of $R_T$ and $R_W$ as

$$R_T = \frac{\sigma_{\text{eff}}(f_+,z) + \sigma_{\text{eff}}(f_-,z)}{\sigma_{\text{eff}}(f_a,z)}$$

$$R_W = \frac{\sigma_{\text{eff}}(f_+,z) - \sigma_{\text{eff}}(f_-,z)}{\sigma_{\text{eff}}(f_a,z)}$$

- From actual photon counts, we calculate the ratios as

$$R_T = \frac{N_{\text{Norm}}(f_+,z) + N_{\text{Norm}}(f_-,z)}{N_{\text{Norm}}(f_a,z)}$$

$$= \left( \frac{N_S(f_+,z) - N_B}{N_S(f_+,z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_+,z)} - \frac{n_R(z)}{n_R(z_R)} \right) + \left( \frac{N_S(f_-,z) - N_B}{N_S(f_-,z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_-,z)} - \frac{n_R(z)}{n_R(z_R)} \right)$$

$$= \frac{N_S(f_a,z) - N_B}{N_S(f_a,z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_a,z)} - \frac{n_R(z)}{n_R(z_R)}$$

$$R_W = \frac{N_{\text{Norm}}(f_+,z) - N_{\text{Norm}}(f_-,z)}{N_{\text{Norm}}(f_a,z)}$$

$$= \left( \frac{N_S(f_+,z) - N_B}{N_S(f_+,z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_+,z)} - \frac{n_R(z)}{n_R(z_R)} \right) - \left( \frac{N_S(f_-,z) - N_B}{N_S(f_-,z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_-,z)} - \frac{n_R(z)}{n_R(z_R)} \right)$$

$$= \frac{N_S(f_a,z) - N_B}{N_S(f_a,z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(f_a,z)} - \frac{n_R(z)}{n_R(z_R)}$$
Main Process Procedure

- Compute Doppler calibration curves from physics
- Look up these two ratios on the calibration curves to infer the corresponding Temperature and Wind from isoline/isogram.

\[ R_T = \frac{N_+ + N_-}{N_a} \]

\[ R_W = \frac{N_+ - N_-}{N_a} \]
Na Density Derivation

- The Na density can be inferred from the peak freq signal

\[
n_{Na}(z) = \frac{N_{norm}(f_a,z)}{\sigma_a} 4\pi n_R(z_R) \sigma_R = \frac{N_{norm}(f_a,z)}{\sigma_a} 4\pi \times 2.938 \times 10^{-32} \frac{P(z_R)}{T(z_R)} \frac{1}{\lambda^{4.0117}}
\]

- The Na density can be inferred from a weighted average of all three frequency signals.

- The weighted effective cross-section is

\[
\sigma_{eff\_wgt} = \sigma_a + \alpha \sigma_+ + \beta \sigma_-
\]

where \(\alpha\) and \(\beta\) are chosen so that

\[
\frac{\partial \sigma_{eff\_wgt}}{\partial T} = 0; \quad \frac{\partial \sigma_{eff\_wgt}}{\partial N_R} = 0
\]

- The Na density is then calculated by

\[
n_{Na}(z) = 4\pi n_R(z_R) \sigma_R \frac{N_{norm}(f_a,z) + \alpha N_{norm}(f_+,z) + \beta N_{norm}(f_-,z)}{\sigma_a + \alpha \sigma_+ + \beta \sigma_-}
\]
Main Ideas to Derive Na T and W

- In the ratio technique, Na number density is cancelled out. So we have two ratios $R_T$ and $R_W$ that are independent of Na density but both dependent on T and W.

- The idea is to derive temperature and radial wind from these two ratios first, and then derive Na number density using computed temperature and wind at each altitude bin.

- To derive T and W from $R_T$ and $R_W$, the basic idea is to use look-up table or iteration methods to derive them: (1) compute $R_T$ and $R_W$ from physics point-of-view to generate the table or calibration curves, (2) compute $R_T$ and $R_W$ from actual photon counts, (3) check the table or calibration curves to find the corresponding T and W. (4) If $R_T$ and $R_W$ are out of range, then set to nominal T and W.

- However, because the Na extinction coefficient is involved, the upper bins are related to lower bins, and extinction coefficient is related to Na density and effective cross-section. The solution is to start from the bottom of the Na layer.
**Preprocess Procedure**

- Indicated from the lidar equation and its solution, the preprocess for Na Doppler lidar data is
  - Background estimation and subtraction \((- N_B\)
  - Range-dependence removal \((x z^2)\)
  - Rayleigh normalization \(1/(N_S(z_R)-N_B)\)
  - Rayleigh subtraction \([- n_R(z)/n_R(z_R)]\)

- More considerations on lidar hardware and detection - extra preprocess procedure
  - PMT and discriminator saturation correction
  - Chopper or electronic gain gain correction
PMT+Discriminator Saturation Correction

\[
\lambda_o = \frac{\lambda_S e^{-\lambda_S \tau_p}}{1 + \lambda_S \tau_d e^{-\lambda_S \tau_p}}
\]

\[
\lambda_S = \lambda_i \eta_{QE}
\]

Na lidar PMT + discriminator

\(\tau_p = 3.2 \text{ ns}\)

\(\tau_d = 10 \text{ ns}\)
Chopper Correction

- Chopper function is measured and then used to do chopper correction for lower atmosphere signals.
Background Estimate

- Background is estimated from high altitude signal

![Raw Data Profiles for 3-Frequency Na Doppler Lidar](image)

- **fa Channel**
- **f+ Channel**
- **f- Channel**

Background Estimate
Estimate of Rayleigh Normalization
Signal - Rayleigh Fit or Sum
Preprocess Procedure for Na Doppler Lidar

- Read data: for each set, and calculate $T$, $W$, and $n$ for each set
- PMT/Discriminator saturation correction
- Chopper correction
- Background estimate and subtraction
- Range-dependence removal (not altitude)
- Base altitude adjustment
- Take Rayleigh signal @ $z_R$ (Rayleigh fit or Rayleigh sum)
- Rayleigh normalization

\[
N_N(\lambda, z) = \frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B z_R} z^2
\]
**Main Process**

Create look-up table or calibration curves
From physics

\[
R_T = \frac{\sigma_{\text{eff}}(f_+, z) + \sigma_{\text{eff}}(f_-, z)}{\sigma_{\text{eff}}(f_a, z)}
\]

\[
R_W = \frac{\sigma_{\text{eff}}(f_+, z) - \sigma_{\text{eff}}(f_-, z)}{\sigma_{\text{eff}}(f_a, z)}
\]

- Load Atmosphere \( n_R, T_R, P_R \)
  Profiles from MSIS00

- Start from Na layer bottom
  \( E(z=z_b) = 1 \)
  Calculate \( N_{\text{norm}}(z=z_b) \) from photon counts and MSIS number density for each freq

\[
N_{\text{Norm}}(\lambda, z) = \left( \frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z) - N_B} \frac{z^2 - n_R(z)}{n_R(z)} \right) \frac{1}{T_c^2(\lambda, z)}
\]

- Calculate \( R_T \) and \( R_W \) from \( N_{\text{norm}} \)

- Are ratios reasonable?
  - Yes
    - Find \( T \) and \( W \) from the Table
  - No
    - Set to nominal values
      \( T = 200 \, K, W = 0 \, m/s \)

Calculate Na density \( n_c(z) \)
Main Process

1. Calculate $N_{\text{norm}}(z+\Delta z)$ from photon counts and MSIS number density for each freq

2. Calculate $R_T(z+\Delta z)$ & $R_W(z+\Delta z)$ from $N_{\text{Norm}}$

3. Are ratios reasonable?
   - Yes: Find $T$ and $W$ from the Table
   - No: Set to nominal values $T = 200 \, \text{K}, \ W = 0 \, \text{m/s}$

4. Calculate Na density $n_c(z)$

5. Reach Layer Top?
   - Yes: Save $T$, $W$, $n_c$ with altitude
   - No: Go back to step 3
Summary

- Lidar data inversion is to convert raw photon counts to meaningful physical parameters like temperature, wind, number density, and volume backscatter coefficient. It is a key step in the process of using lidar to study science.

- The basic procedure of data inversion originates from solutions of lidar equations, in combination with detailed considerations of hardware properties and limitations as well as detailed considerations of light propagation and interaction processes.

- The data inversion procedure consists of three main processes: (1) preprocess, (2) process of $T$ and $V_R$, (3) process of $n_c$ and $\beta$, etc.
Summary

- The preprocess is to convert the raw photon counts to corrected and normalized photon counts in consideration of hardware properties and limitations.
- The process of $T$ and $V_R$ is to convert the normalized photon counts to $T$ and $V_R$ through integration, iteration or looking-up table methods.
- The process of $n_c$ is to convert the normalized photon counts to meaningful number density, in combination with prior acquired knowledge or model knowledge of certain atmosphere information or atomic/molecular spectroscopy.
- These processes sometimes involve considerable binning, smoothing, or temporal integration in order to improve the signal-to-noise ratio (SNR) to result in meaningful results.