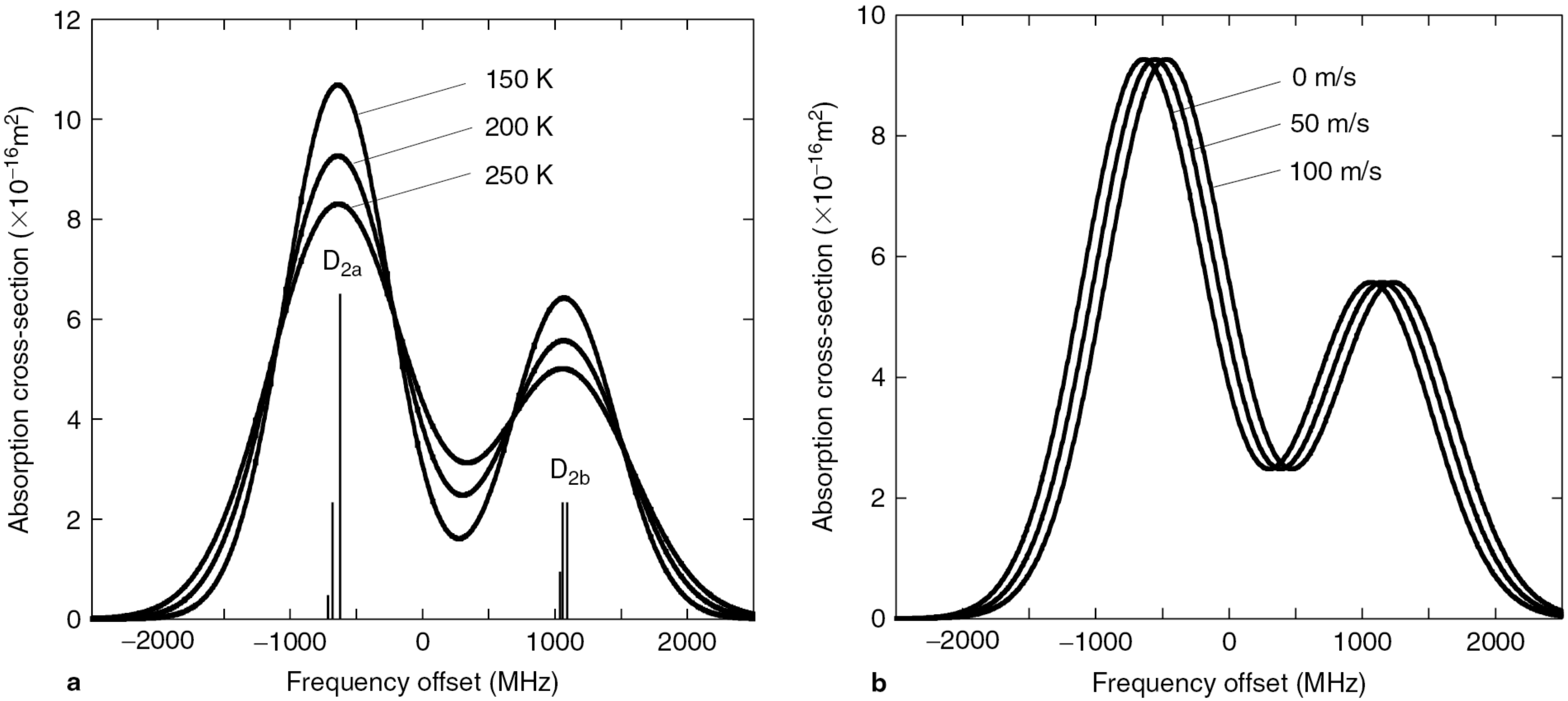
Lecture 11. Temperature Lidar (2)
Doppler Ratio Technique

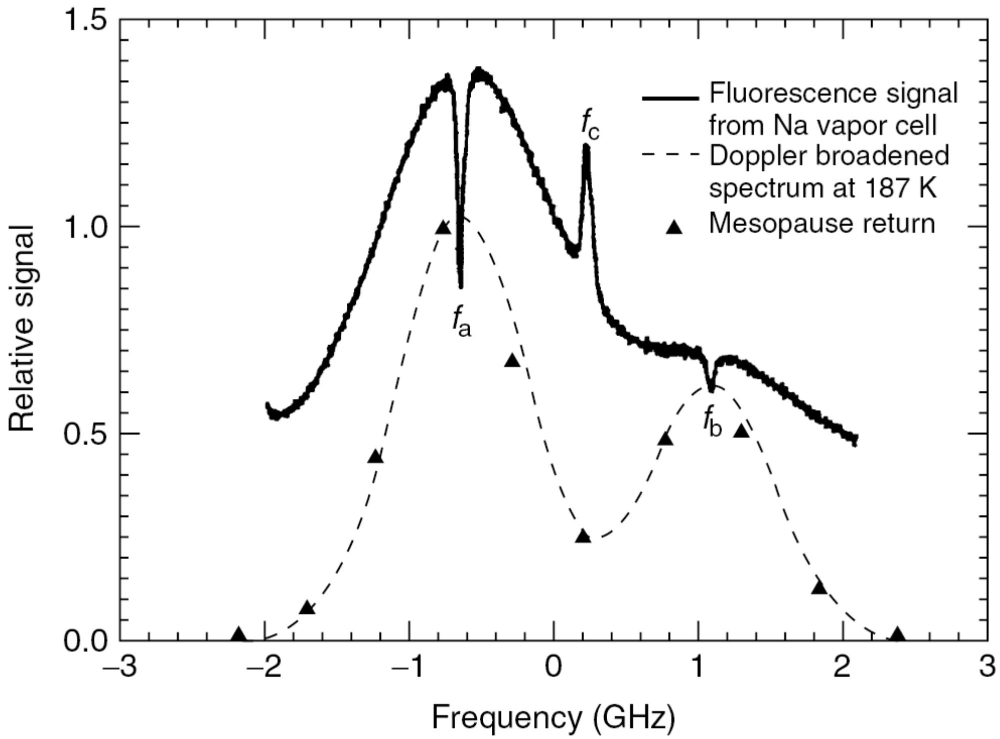
- HWK Projects #1 and #2
- Ratio versus scanning techniques
- Principle of Doppler ratio technique
- Comparison of calibration curves
- Summary

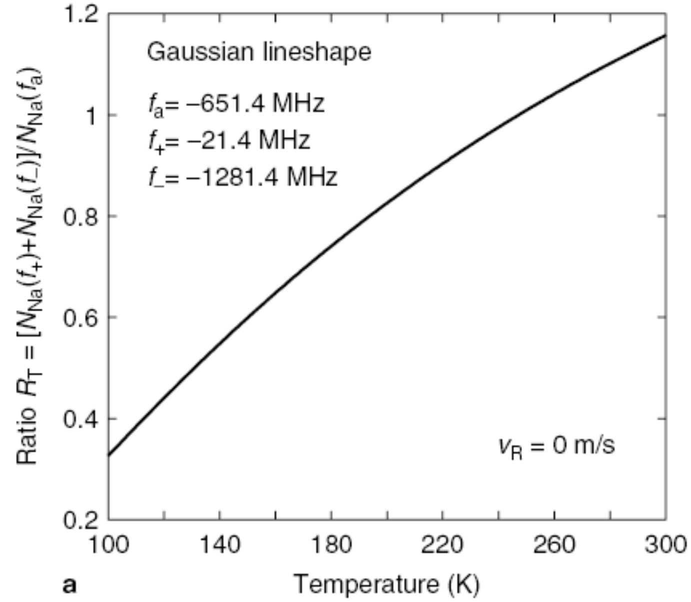
HWK Projects #1 and #2

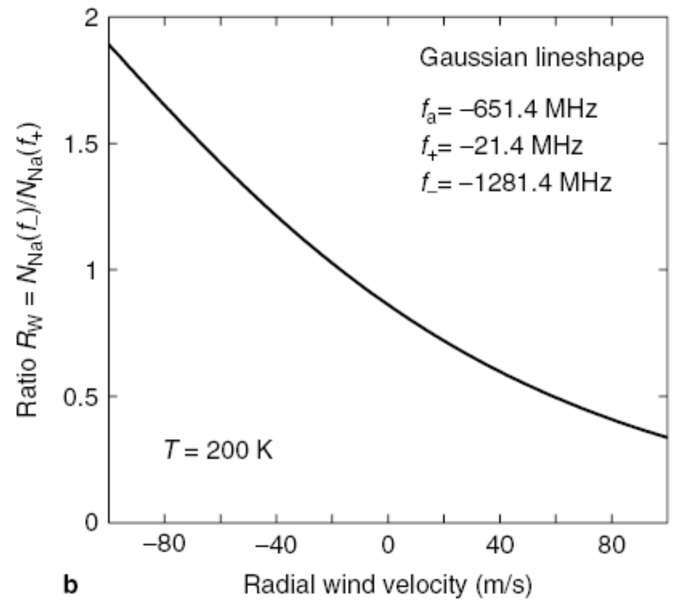


The effective cross section is a convolution of the atomic absorption cross section and the laser line shape.

Ratio versus Scanning Techniques







$$R_{\mathrm{T}}(z) = \frac{N_{\mathrm{norm}}(f_+, z, t_1) + N_{\mathrm{norm}}(f_-, z, t_2)}{N_{\mathrm{norm}}(f_{\mathrm{a}}, z, t_3)}$$

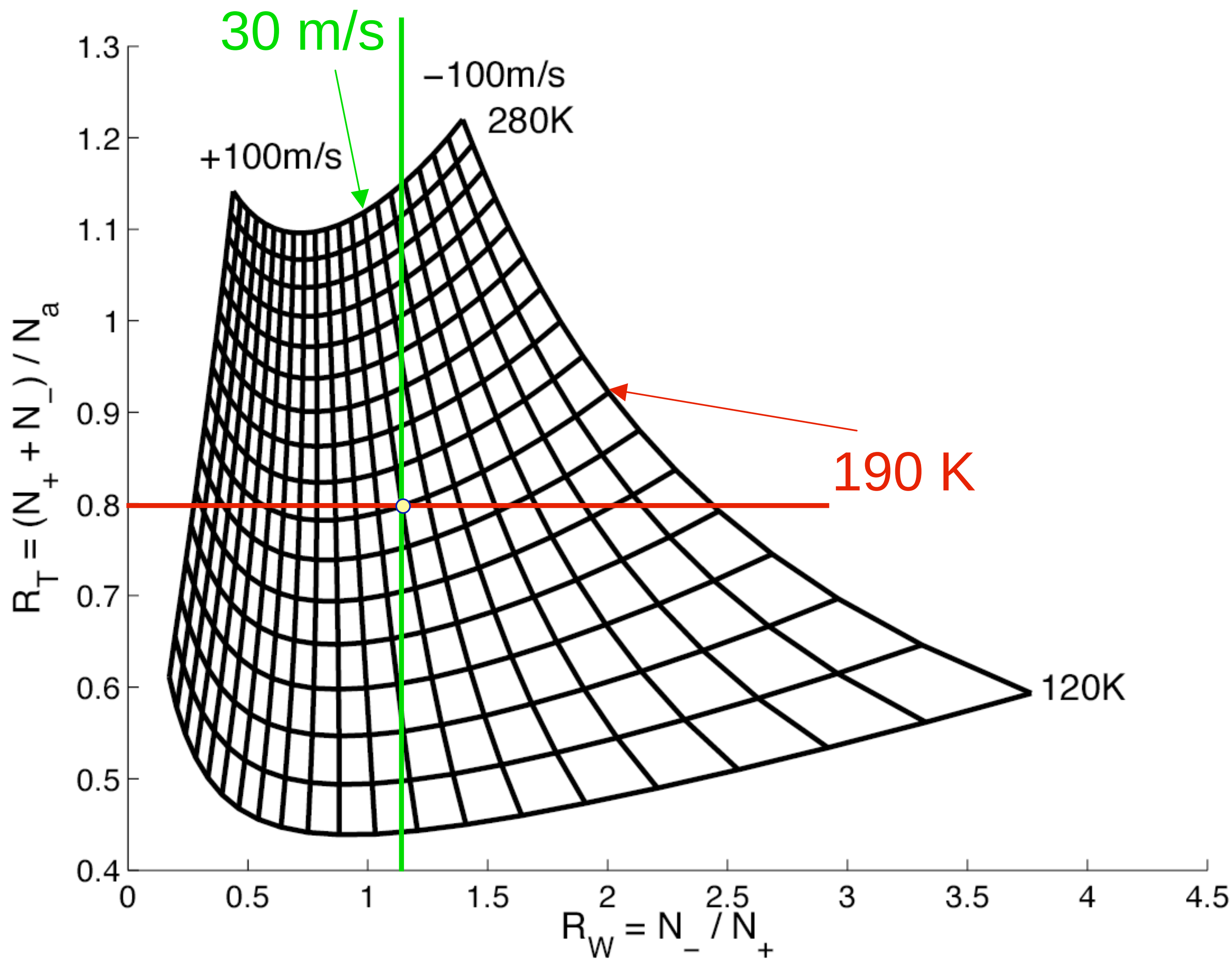
$$\approx \frac{\sigma_{\mathrm{eff}}(f_+, z) + \sigma_{\mathrm{eff}}(f_-, z)}{\sigma_{\mathrm{eff}}(f_{\mathrm{a}}, z)}$$

$$R_{\mathrm{W}}(z) = \frac{N_{\mathrm{norm}}(f_-, z, t_2)}{N_{\mathrm{norm}}(f_+, z, t_1)} \approx \frac{\sigma_{\mathrm{eff}}(f_-, z)}{\sigma_{\mathrm{eff}}(f_+, z)}$$

Main Ideas Behind Ratio Technique

- Three unknown parameters (temperature, radial wind, and Na number density) require 3 lidar equations at 3 frequencies as minimum ⇨ highest resolution.
- In the ratio technique, Na number density is cancelled out. So we have two ratios R_T and R_W that are independent of Na density but both dependent on T and W.
- The idea is to derive temperature and radial wind from these two ratios first, and then derive Na number density using computed temperature and wind at each altitude bin.
- However, because the Na extinction coefficient is involved, the upper bins are related to lower bins, and extinction coefficient is related to Na density and effective cross-section. The solution is to start from the bottom of the Na layer and then work bin by bin to the layer top.

Principle of Doppler Ratio Technique



Doppler Ratio Technique

- Lidar equation for resonance fluorescence (Na, K, or Fe)

$$N_S(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_{eff}(\lambda,z)n_c(z)R_B(\lambda) + \sigma_R(\pi,\lambda)n_R(z)\right]\Delta z\left(\frac{A}{4\pi z^2}\right) \times \left(T_a^2(\lambda)T_c^2(\lambda,z)\right)\left(\eta(\lambda)G(z)\right) + N_B$$

R_B = 1 for current Na Doppler lidar since return photons at all wavelengths are received by the broadband receiver, so no fluorescence is filtered off.

- Pure Na signal and pure Rayleigh signal in Na region are

$$N_{Na}(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_{eff}(\lambda,z)n_c(z)\right]\Delta z\left(\frac{A}{4\pi z^2}\right)\left(T_a^2(\lambda)T_c^2(\lambda,z)\right)\left(\eta(\lambda)G(z)\right)$$

$$N_R(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_R(\pi,\lambda)n_R(z)\right]\Delta z\left(\frac{A}{z^2}\right)\left(T_a^2(\lambda)T_c^2(\lambda,z)\right)\left(\eta(\lambda)G(z)\right)$$

- So we have

$$N_S(\lambda,z) = N_{Na}(\lambda,z) + N_R(\lambda,z) + N_B$$

Doppler Ratio Technique

- Lidar equation at pure molecular scattering region (35-55km)

$$N_S(\lambda, z_R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_R(\pi,\lambda)n_R(z_R)\right]\Delta z\left(\frac{A}{z_R^{\ 2}}\right)T_a^{\ 2}(\lambda, z_R)\left(\eta(\lambda)G(z_R)\right) + N_B$$

- Pure Rayleigh signal in molecular scattering region is

$$N_R(\lambda, z_R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_R(\pi,\lambda)n_R(z_R)\right]\Delta z\left(\frac{A}{z_R^{\ 2}}\right)T_a^{\ 2}(\lambda, z_R)\left(\eta(\lambda)G(z_R)\right)$$

- So we have

$$N_S(\lambda, z_R) = N_R(\lambda, z_R) + N_B$$

- The ratio between Rayleigh signals at z and z_R is given by

$$\frac{N_R(\lambda,z)}{N_R(\lambda,z_R)} = \frac{\left[\sigma_R(\pi,\lambda)n_R(z)\right]T_a^{\ 2}(\lambda,z)T_c^2(\lambda,z)G(z)}{\left[\sigma_R(\pi,\lambda)n_R(z_R)\right]T_a^{\ 2}(\lambda,z_R)G(z_R)}\frac{z_R^{\ 2}}{z^2} = \frac{n_R(z)}{n_R(z_R)}\frac{z_R^{\ 2}}{z^2}T_c^2(\lambda,z)$$

Where n_R is the (total) atmospheric number density, usually obtained from atmospheric models like MSIS00.

Doppler Ratio Technique

- From above equations, we obtain

$$N_{Na}(\lambda,z) = N_S(\lambda,z) - N_B - N_R(\lambda,z) \qquad N_R(\lambda,z_R) = N_S(\lambda,z_R) - N_B$$

- Normalized Na photon count is defined as

$$N_{Norm}(\lambda,z) = \frac{N_{Na}(\lambda,z)}{N_R(\lambda,z_R)T_c^2(\lambda,z)} \frac{z^2}{z_R^2}$$

- So from physics point of view, we have

$$N_{Norm}(\lambda,z) = \frac{N_{Na}(\lambda,z)}{N_R(\lambda,z_R)T_c^2(\lambda,z)} = \frac{\sigma_{eff}(\lambda,z)n_c(z)}{\sigma_R(\pi,\lambda)n_R(z_R)} \frac{1}{4\pi}$$

- From actual photon counts, we have

$$N_{Norm}(\lambda,z) = \frac{N_{Na}(\lambda,z)}{N_R(\lambda,z_R)T_c^2(\lambda,z)} \frac{z^2}{z_R^2} = \frac{N_S(\lambda,z) - N_B - N_R(\lambda,z)}{N_R(\lambda,z_R)T_c^2(\lambda,z)} \frac{z^2}{z_R^2}$$

$$= \frac{N_S(\lambda,z) - N_B}{N_S(\lambda,z_R) - N_B} \frac{z^2}{z_R^2} \frac{1}{T_c^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}$$

Doppler Ratio Technique

- From physics, the ratios of R_T and R_W are then given by

$$R_T = \frac{N_{Norm}(f_+,z)+N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)} = \frac{\dfrac{\sigma_{eff}(f_+,z)n_c(z)}{\sigma_R(\pi,f_+)n_R(z_R)}+\dfrac{\sigma_{eff}(f_-,z)n_c(z)}{\sigma_R(\pi,f_-)n_R(z_R)}}{\dfrac{\sigma_{eff}(f_a,z)n_c(z)}{\sigma_R(\pi,f_a)n_R(z_R)}} = \frac{\sigma_{eff}(f_+,z)+\sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}$$

$$R_W = \frac{N_{Norm}(f_+,z)-N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)} = \frac{\dfrac{\sigma_{eff}(f_+,z)n_c(z)}{\sigma_R(\pi,f_+)n_R(z_R)}-\dfrac{\sigma_{eff}(f_-,z)n_c(z)}{\sigma_R(\pi,f_-)n_R(z_R)}}{\dfrac{\sigma_{eff}(f_a,z)n_c(z)}{\sigma_R(\pi,f_a)n_R(z_R)}} = \frac{\sigma_{eff}(f_+,z)-\sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}$$

Here, Rayleigh backscatter cross-section is regarded as the same for three frequencies, since the frequency difference is so small. Na number density is also the same for three frequency channels, and so is the atmosphere number density at Rayleigh normalization altitude.

Doppler Ratio Technique

- From actual photon counts, we have

$$R_T = \frac{N_{Norm}(f_+,z)+N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}$$

$$= \frac{\left(\frac{N_S(f_+,z)-N_B}{N_S(f_+,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_+,z)}-\frac{n_R(z)}{n_R(z_R)}\right)+\left(\frac{N_S(f_-,z)-N_B}{N_S(f_-,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_-,z)}-\frac{n_R(z)}{n_R(z_R)}\right)}{\frac{N_S(f_a,z)-N_B}{N_S(f_a,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_a,z)}-\frac{n_R(z)}{n_R(z_R)}}$$

$$R_W = \frac{N_{Norm}(f_+,z)-N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}$$

$$= \frac{\left(\frac{N_S(f_+,z)-N_B}{N_S(f_+,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_+,z)}-\frac{n_R(z)}{n_R(z_R)}\right)-\left(\frac{N_S(f_-,z)-N_B}{N_S(f_-,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_-,z)}-\frac{n_R(z)}{n_R(z_R)}\right)}{\frac{N_S(f_a,z)-N_B}{N_S(f_a,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{T_c^2(f_a,z)}-\frac{n_R(z)}{n_R(z_R)}}$$

How Does Ratio Technique Work?

- From physics, we calculate the ratios of R_T and R_W as

$$R_T = \frac{\sigma_{eff}(f_+,z) + \sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}$$

$$R_W = \frac{\sigma_{eff}(f_+,z) - \sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}$$

- From actual photon counts, we calculate the ratios as

$$R_T = \frac{N_{Norm}(f_+,z) + N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}$$

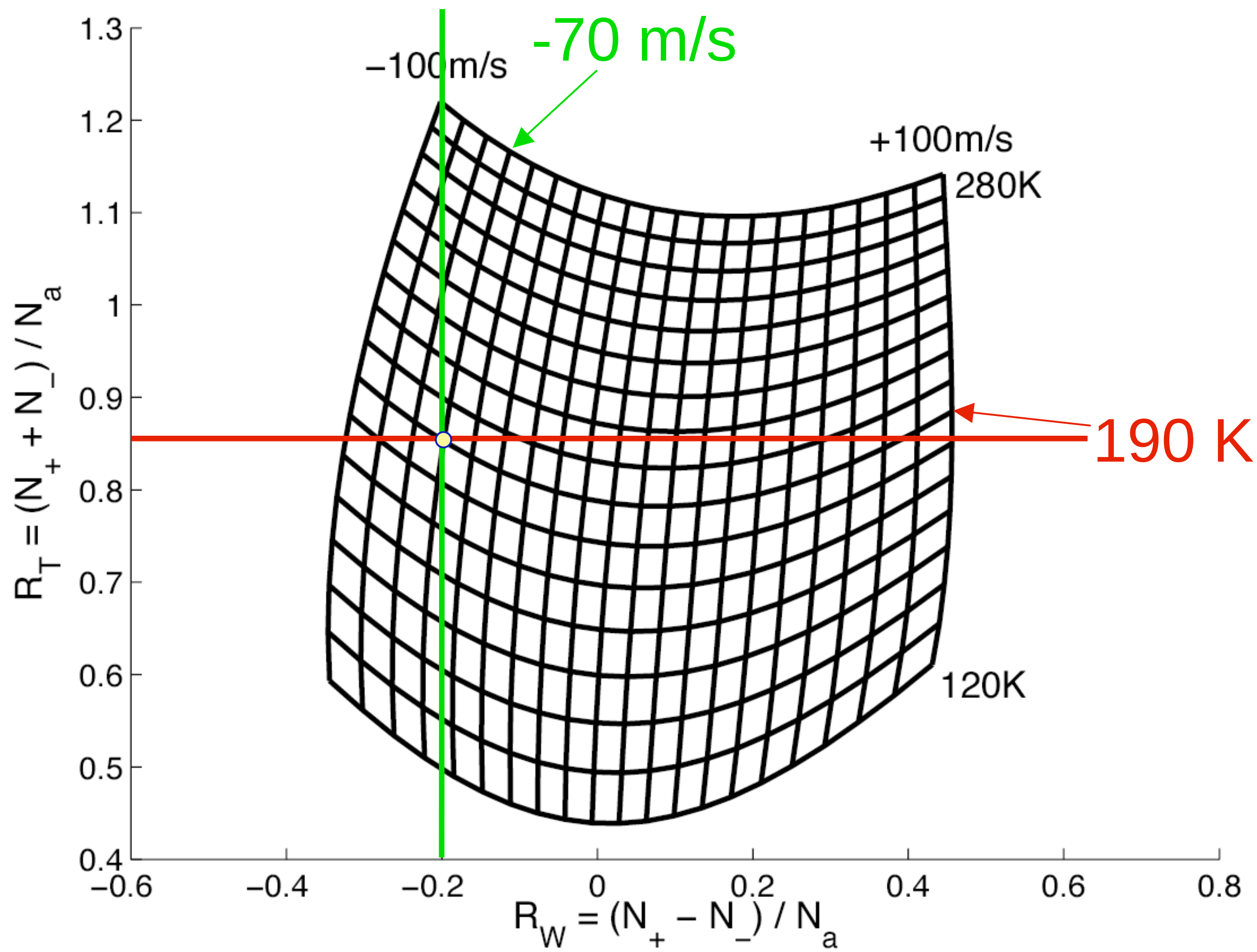
$$= \frac{\left(\frac{N_S(f_+,z) - N_B}{N_S(f_+,z_R) - N_B} \frac{z^2}{z_R{}^2} \frac{1}{T_c^2(f_+,z)} - \frac{n_R(z)}{n_R(z_R)}\right) + \left(\frac{N_S(f_-,z) - N_B}{N_S(f_-,z_R) - N_B} \frac{z^2}{z_R{}^2} \frac{1}{T_c^2(f_-,z)} - \frac{n_R(z)}{n_R(z_R)}\right)}{\frac{N_S(f_a,z) - N_B}{N_S(f_a,z_R) - N_B} \frac{z^2}{z_R{}^2} \frac{1}{T_c^2(f_a,z)} - \frac{n_R(z)}{n_R(z_R)}}$$

$$R_W = \frac{N_{Norm}(f_+,z) - N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}$$

$$= \frac{\left(\frac{N_S(f_+,z) - N_B}{N_S(f_+,z_R) - N_B} \frac{z^2}{z_R{}^2} \frac{1}{T_c^2(f_+,z)} - \frac{n_R(z)}{n_R(z_R)}\right) - \left(\frac{N_S(f_-,z) - N_B}{N_S(f_-,z_R) - N_B} \frac{z^2}{z_R{}^2} \frac{1}{T_c^2(f_-,z)} - \frac{n_R(z)}{n_R(z_R)}\right)}{\frac{N_S(f_a,z) - N_B}{N_S(f_a,z_R) - N_B} \frac{z^2}{z_R{}^2} \frac{1}{T_c^2(f_a,z)} - \frac{n_R(z)}{n_R(z_R)}}$$

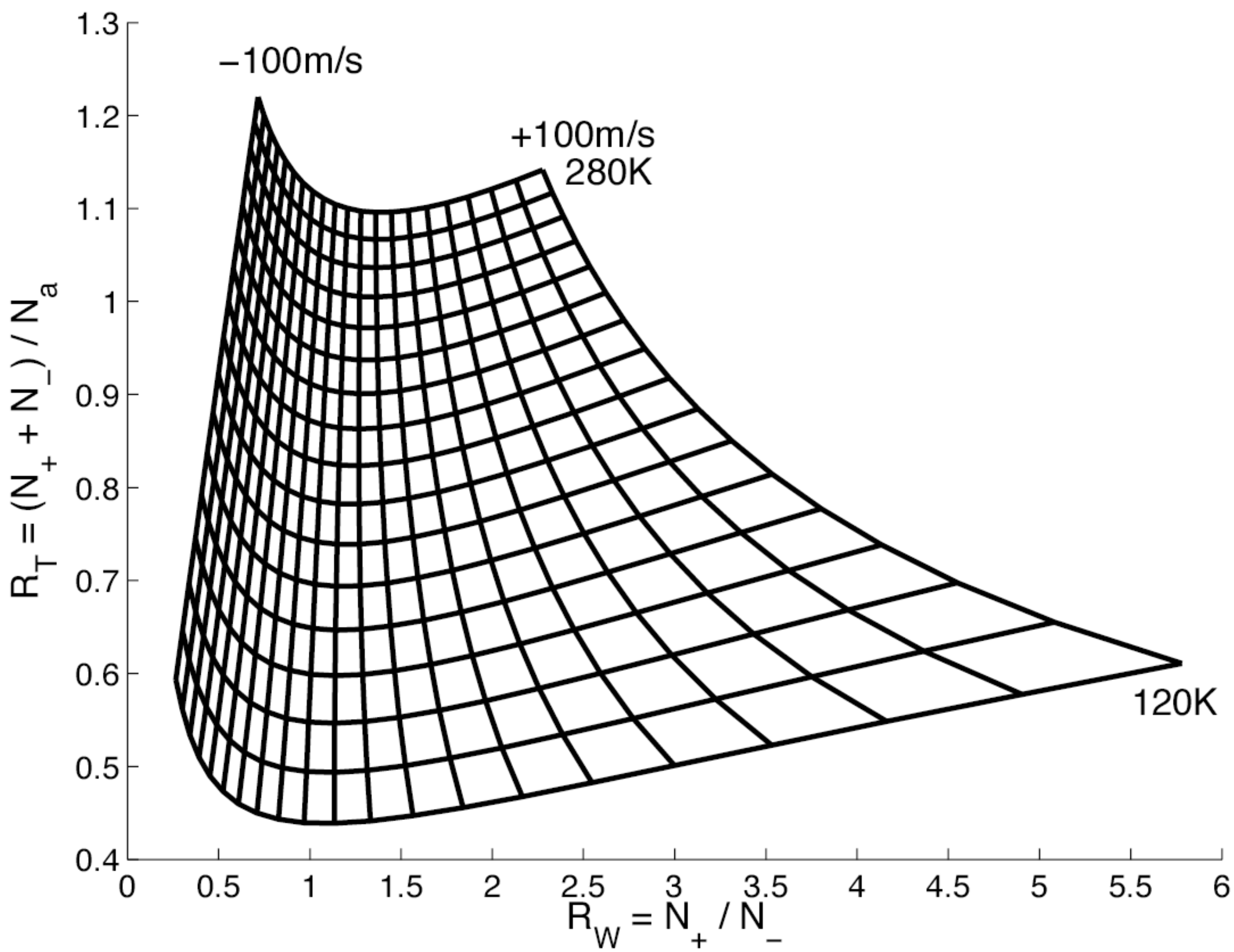
How Does Ratio Technique Work?

- Compute Doppler calibration curves from physics
- Look up these two ratios on the calibration curves to infer the corresponding Temperature and Wind.



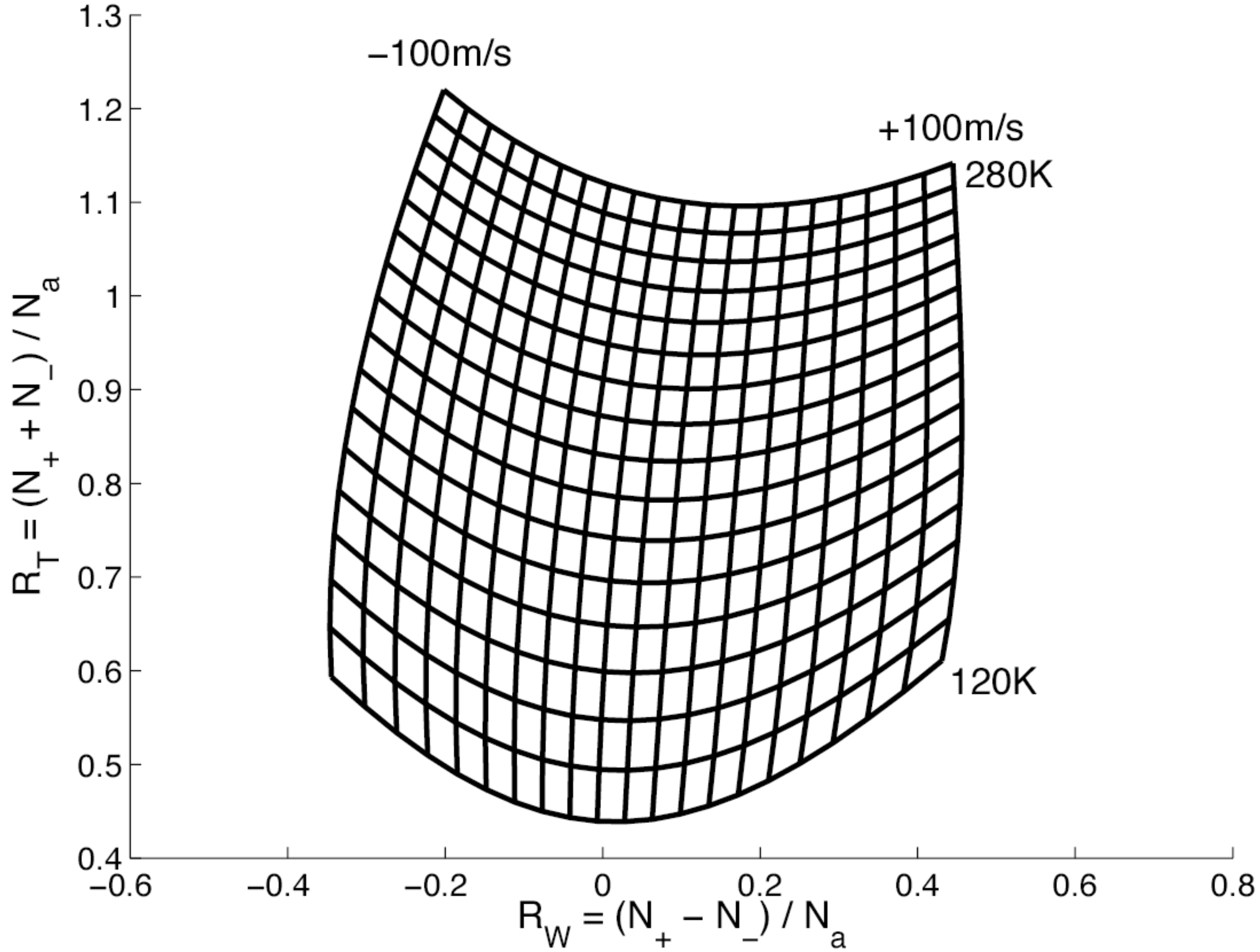
Comparison of Calibration Curves

- Different metrics of R_W result in different wind sensitivities
- The ratio $R_W = N_+/N_-$ has inhomogeneous sensitivity



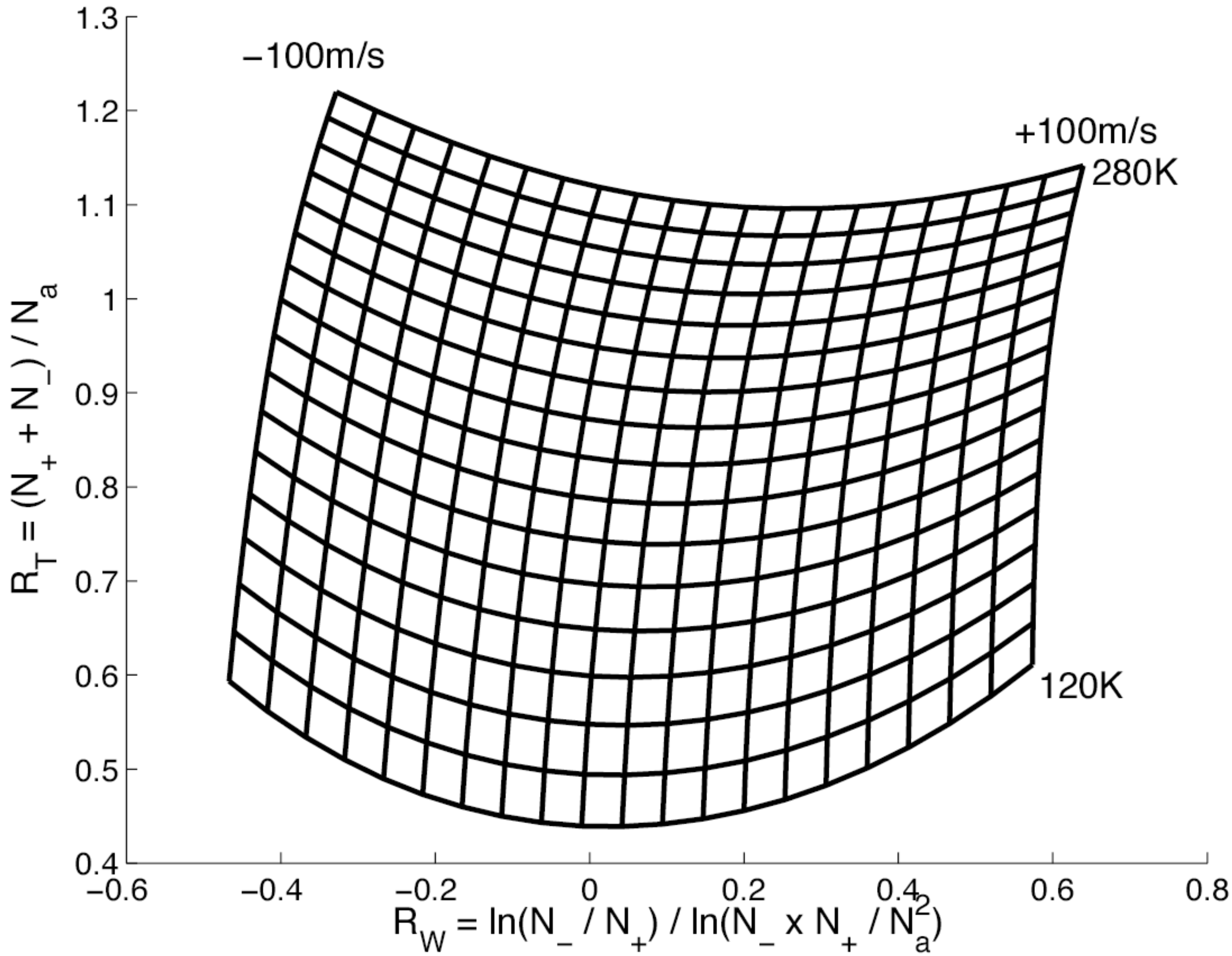
Comparison of Calibration Curves

- The ratio $R_W = (N_+ - N_-)/N_a$ has much better uniformity than the simplest ratio



Comparison of Calibration Curves

- The ratio $R_W = \ln(N_- / N_+) / \ln(N_- \times N_+ / N_a^2)$ has good uniformity



Summary

- Doppler ratio technique takes advantage of the high temporal resolution feature by limiting the lidar detection to 3 preset frequencies (usually one peak and two wing frequencies) for 3 unknown parameters (T, W, and density).
- By taking the ratios among signals at these three frequencies, R_T and R_W are sensitive functions of temperature and radial wind, respectively.
- We compute the ratios R_T and R_W from atomic physics first to form the lidar calibration curves, and then look up the two ratios calculated from actual photon counts on the calibration curves to infer the corresponding temperature T and radial wind W.
- Different metrics exhibit different inhomogeneity, resulting in different crosstalk between T and W errors.