Lecture 10. Temperature Lidar (1)

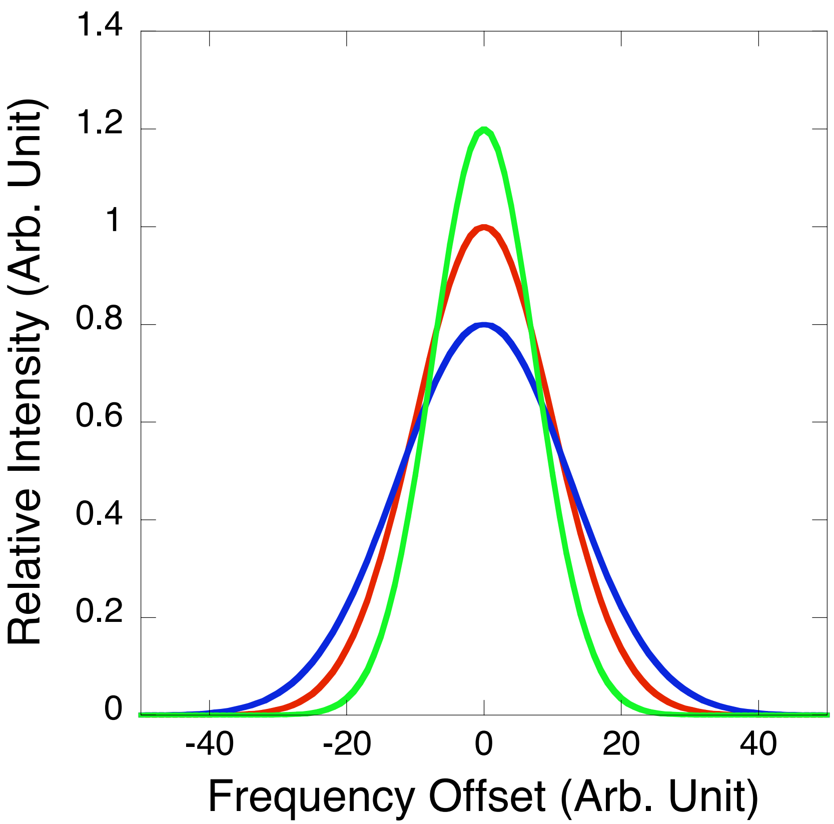
- How to measure temperature?
- Review of Techniques for Temperature Measurements
- Doppler Technique for Temperature and Wind Measurements
- Resonance Fluorescence Na Doppler Lidar
- Summary

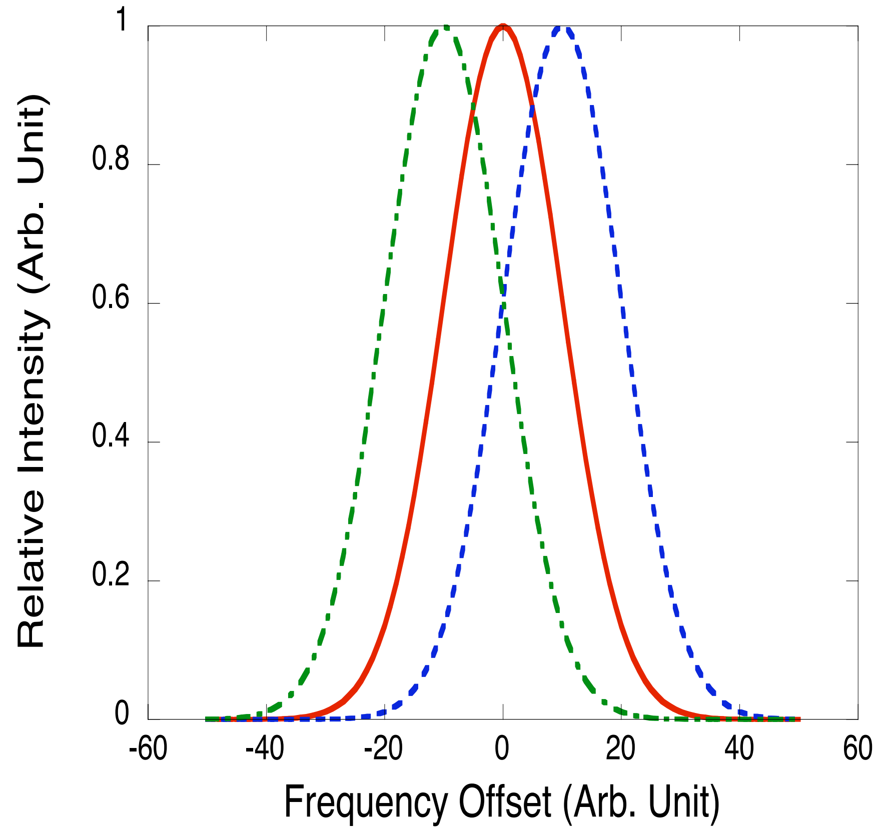
How to Measure Temperature ?

Use temperature-dependent effects or phenomena

- Doppler Technique - Doppler broadening (not only for Na, K, and Fe, but also for Rayleigh scattering, as long as Doppler broadening dominate and can be detected)
- Boltzmann Technique - population ratio (not only for Fe, but also for molecular spectroscopy in optical remote sensing and rotational Raman lidar)
- Integration Technique (Rayleigh or Raman) - integration lidar technique using ideal gas law and assuming hydrostatic equilibrium (not only for modern lidar, but also for cw searchlight and rocket falling sphere - some way to measure atmosphere number density)
- Rotational Raman Technique - temperature dependence of population ratio, similar to Boltzmann technique

Doppler Technique





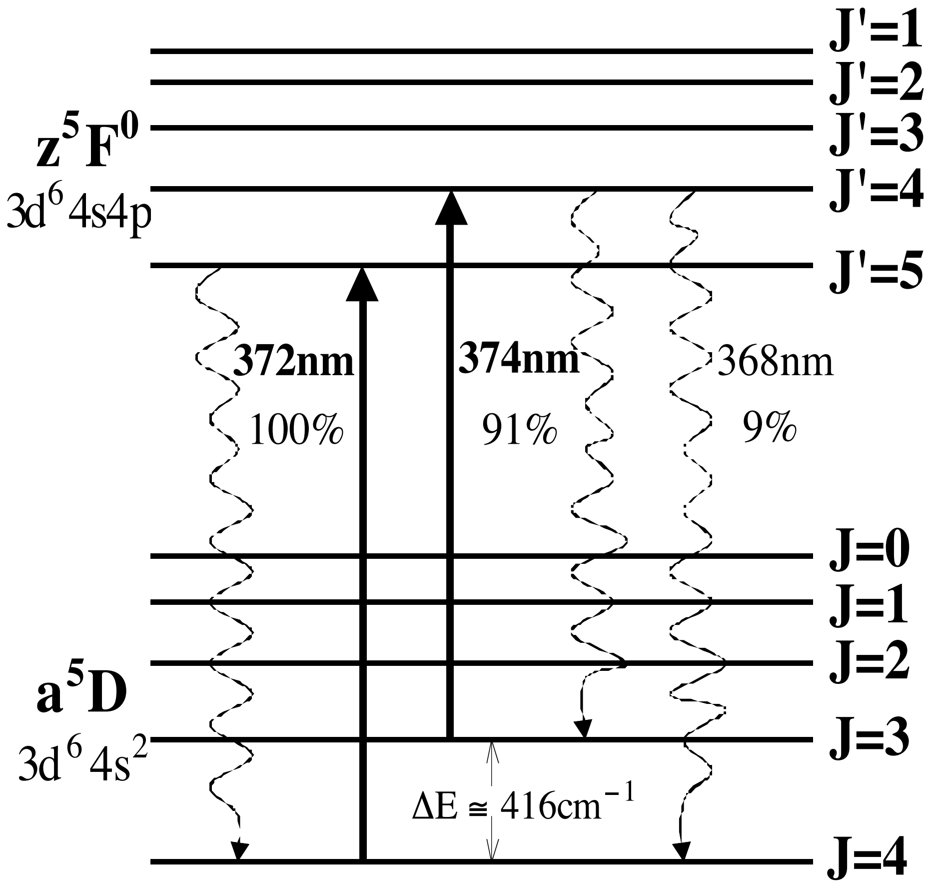
$$\sigma_{Drms} = \frac{\omega_0}{c}\sqrt{\frac{k_B T}{M}} = \frac{1}{\lambda_0}\sqrt{\frac{k_B T}{M}} = \sqrt{\frac{k_B T}{M\lambda_0^2}}$$

$$\Delta\omega = \omega - \omega_0 = -\vec{k}\cdot\vec{v} = -\omega_0 \frac{v\cos\theta}{c}$$

$$\nu' = \nu\left(1 - \frac{v_R}{c}\right) = \nu - \frac{v_R}{\lambda}$$

Doppler Spectrum (Width and Shift) ⇨ Temperature and Radial Wind

Boltzmann Technique



Atomic Fe Energy Level

[Gelbwachs, 1994; Chu et al., 2002]

Maxwell-Boltzmann Distribution in Thermal-dynamic Equilibrium

$$\frac{P_2(J=3)}{P_1(J=4)} = \frac{\rho_{Fe(374)}}{\rho_{Fe(372)}} = \frac{g_2}{g_1} exp(-\Delta E / k_B T)$$

⇩

$$T = \frac{\Delta E / k_B}{ln\left(\frac{g_2}{g_1} \cdot \frac{P_1}{P_2}\right)}$$

P_1, P_2 -- Fe populations
g_1, g_2 -- Degeneracy
k_B -- Boltzmann constant
T -- Temperature

Population Ratio ⇨ Temperature

Rayleigh Integration Technique

Hydrostatic Equation

$$dP = -\rho g dz$$

+

Ideal Gas Law

$$P = \rho R T$$

$$T(z) = T(z_o)\frac{\rho(z_o)}{\rho(z)} + \frac{1}{R}\int_{z}^{z_o} g(r)dr\frac{\rho(r)}{\rho(z)}$$

Seeding Temperature

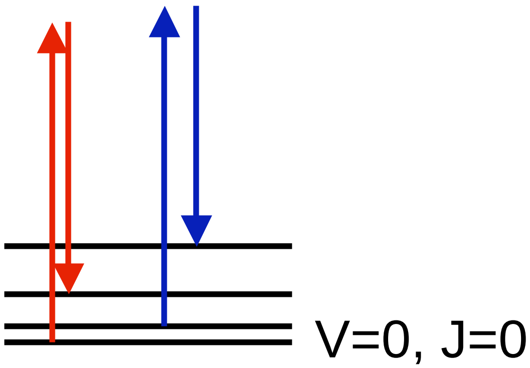
Relative Density

$T(z_0)$ - Seeding Temperature; **ρ - number density**

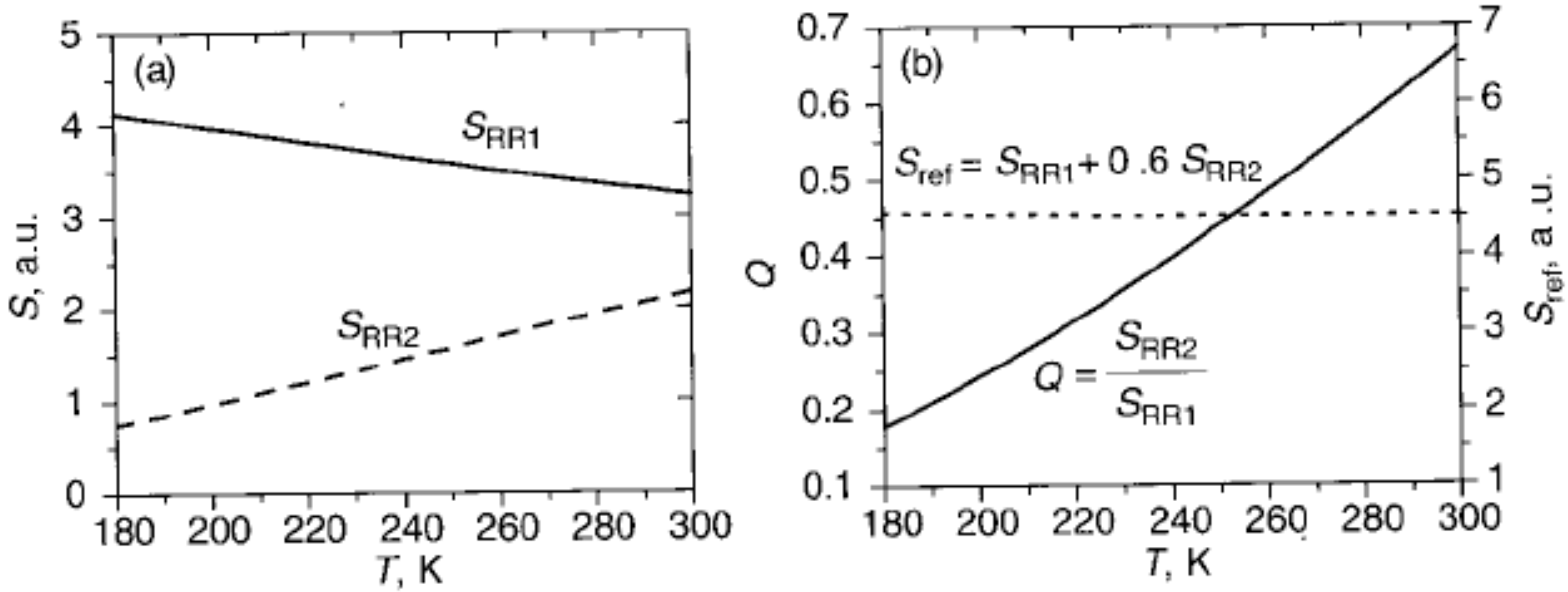
R - gas constant for dry air; **g - gravitational acceleration**

Lidar Backscatter Ratio ⇨ Relative Density ⇨ Temperature
(at different altitudes) **(Rayleigh)**

Rotation Raman Technique

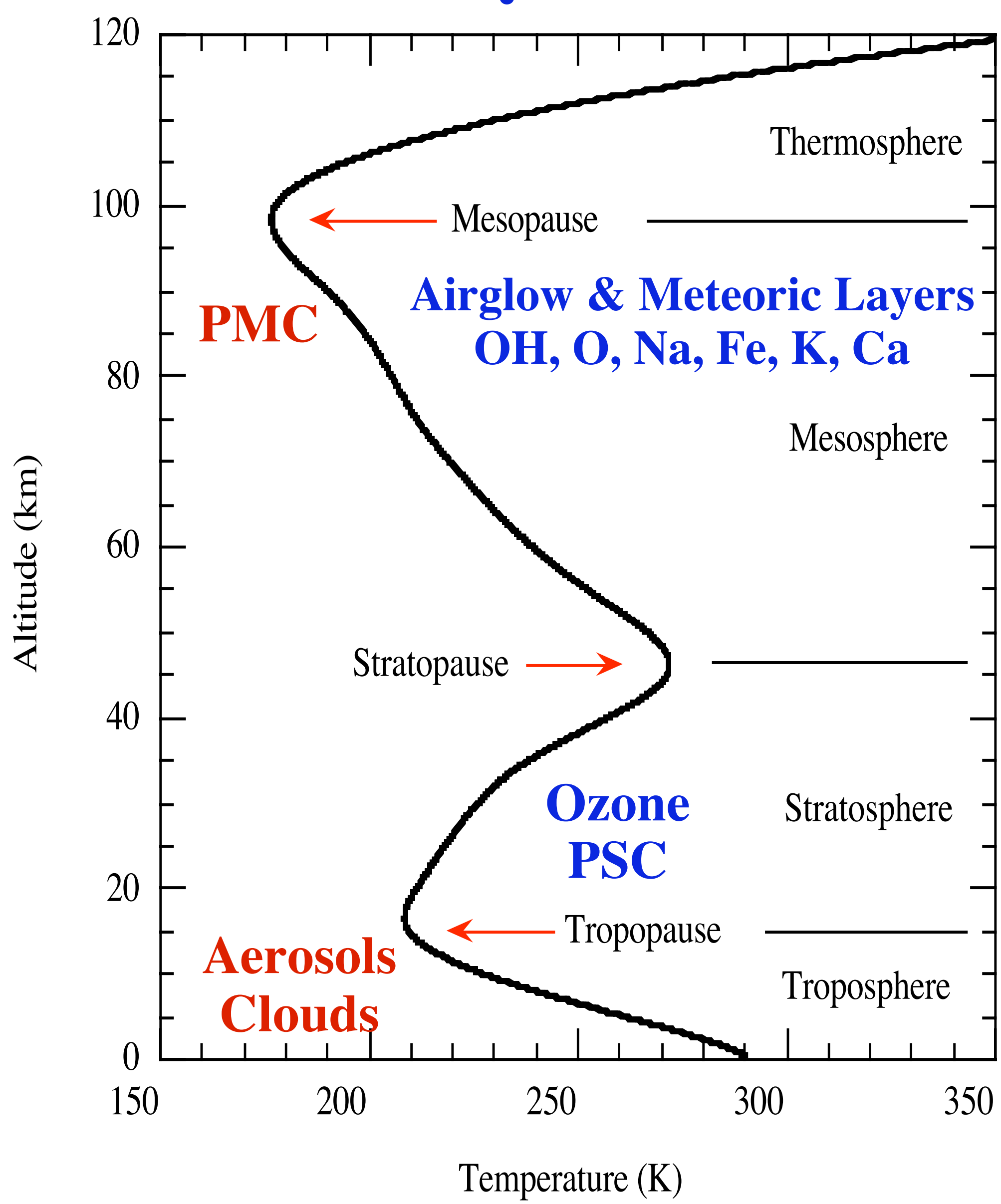


$$Q(T) = \frac{\sum_{i=O_2,N_2} \sum_{J_i} \tau_{RR2}(J_i)\eta_i \left(\frac{d\sigma}{d\Omega}\right)_{\pi}^{RR,i}(J_i)}{\sum_{i=O_2,N_2} \sum_{J_i} \tau_{RR1}(J_i)\eta_i \left(\frac{d\sigma}{d\Omega}\right)_{\pi}^{RR,i}(J_i)}$$



- Temperature can be derived from the ratio of two pure Rotational Raman line intensities. This is essentially the same principle as Boltzmann temperature technique!

Temperature Techniques



- 75-120km: resonance fluorescence Doppler technique (Na, K, Fe) & Boltzmann technique (Fe, OH, O_2)
- 30-90km: Rayleigh integration technique & Rayleigh Doppler technique
- Below 30 km: scattering Doppler technique and Raman (Boltzmann and integration) technique
- Boundary layer: DIAL, HSRL, Rotational Raman

Doppler Technique to Measure Temperature and Wind

- Doppler effect is commonly experienced by moving particles, such as atoms, molecules, and aerosols. It is the apparent frequency change of radiation or wave that is perceived by the particles moving relative to the source of the radiation or wave. This is called Doppler shift.
- Doppler frequency shift is proportional to the radial velocity along the line of sight (LOS) of the radiation –

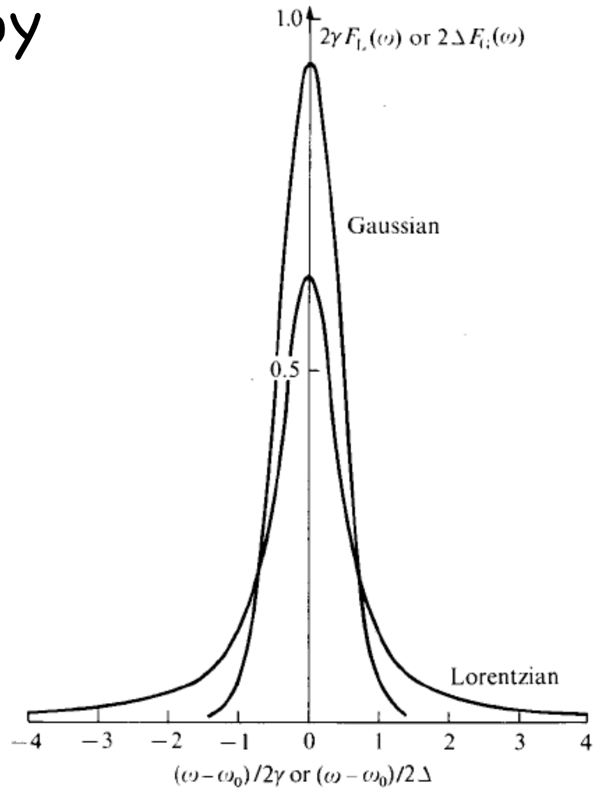
$$\omega = \omega_0 - \vec{k} \cdot \vec{v} \quad \Longrightarrow \quad \Delta\omega = \omega - \omega_0 = -\vec{k} \cdot \vec{v} = -\omega_0 \frac{v\cos\theta}{c}$$

where ω_0 is the radiation frequency at rest, ω is the shifted frequency, k is the wave vector of the radiation ($k=2\pi/\lambda$), and v is the particle velocity.

Doppler Technique to Measure Temperature and Wind

- Due to particles' thermal motions in the atmosphere, the distribution of perceived frequencies for all particles mirrors their velocity distribution. According to the Maxwellian velocity distribution, the perceived frequencies by moving particles has a Gaussian lineshape, given by

$$\exp\left(-\frac{Mv_z^2}{2k_BT}\right)dv_z = \exp\left\{-\frac{Mc^2(\omega-\omega_0)^2}{2\omega_0^2 k_B T}\right\}\frac{c}{\omega_0}d\omega$$

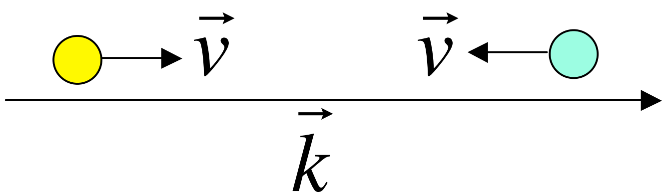


- The peak is at $\omega = \omega_0$ and the rms width is give by

$$\sigma_{rms} = \frac{\omega_0}{c}\sqrt{\frac{k_BT}{M}} = \frac{1}{\lambda_0}\sqrt{\frac{k_BT}{M}}$$

Doppler Shift For Wind Measurement

$$\Delta\omega = \omega - \omega_0 = -\vec{k}\cdot\vec{v} = -\omega_0 \frac{v\cos\theta}{c}$$

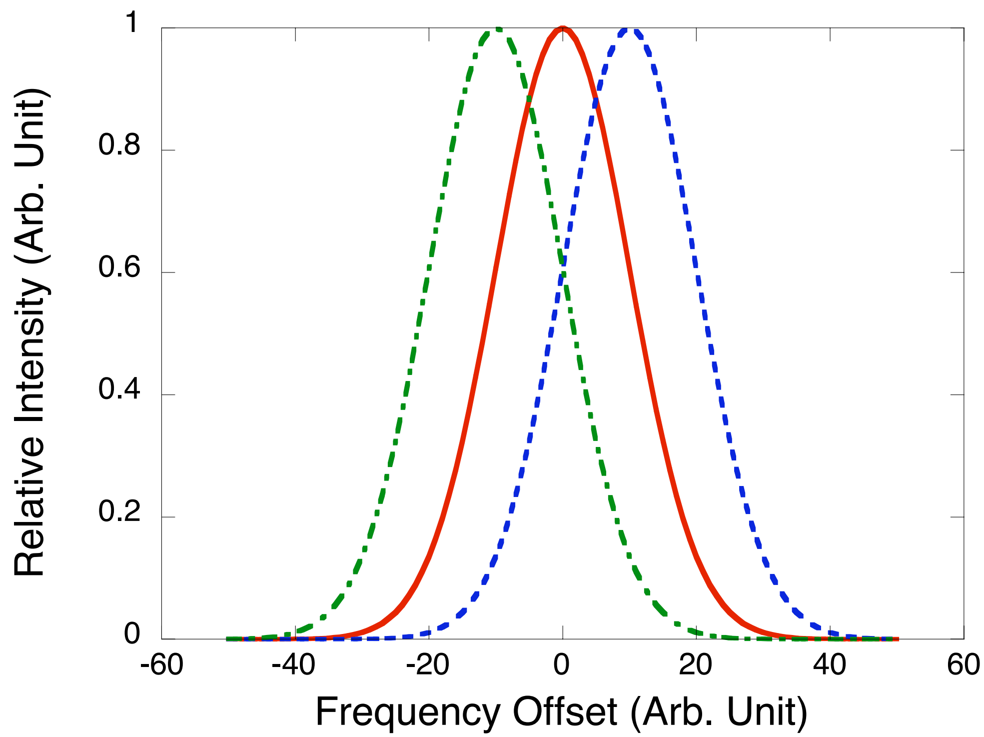


Same direction

- red shift

Opposite direction

- blue shift



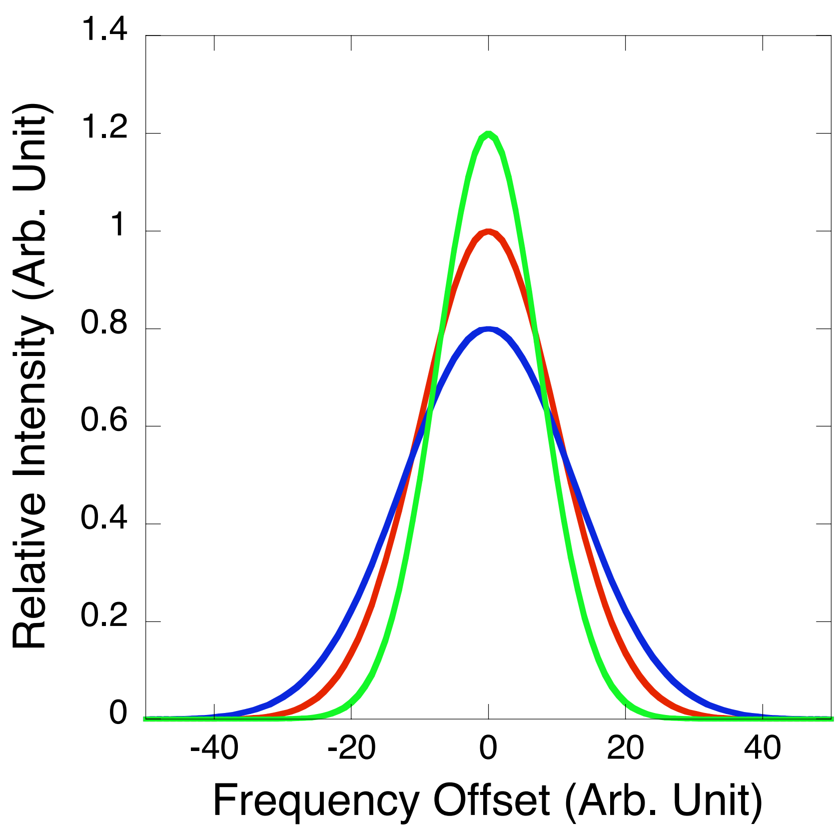
❑ The velocity measurements of lidar, radar, and sodar all base on the Doppler shift principle !

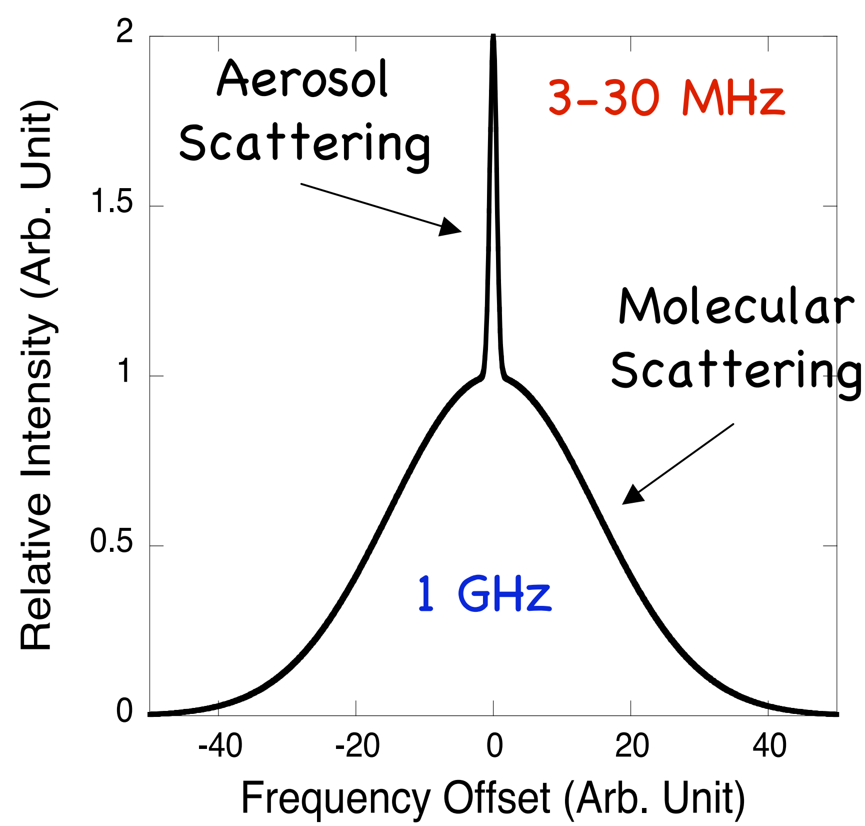
Doppler Broadening For Temperature

$$\sigma_{rms} = \frac{\omega_0}{c}\sqrt{\frac{k_B T}{M}} = \frac{1}{\lambda_0}\sqrt{\frac{k_B T}{M}}$$

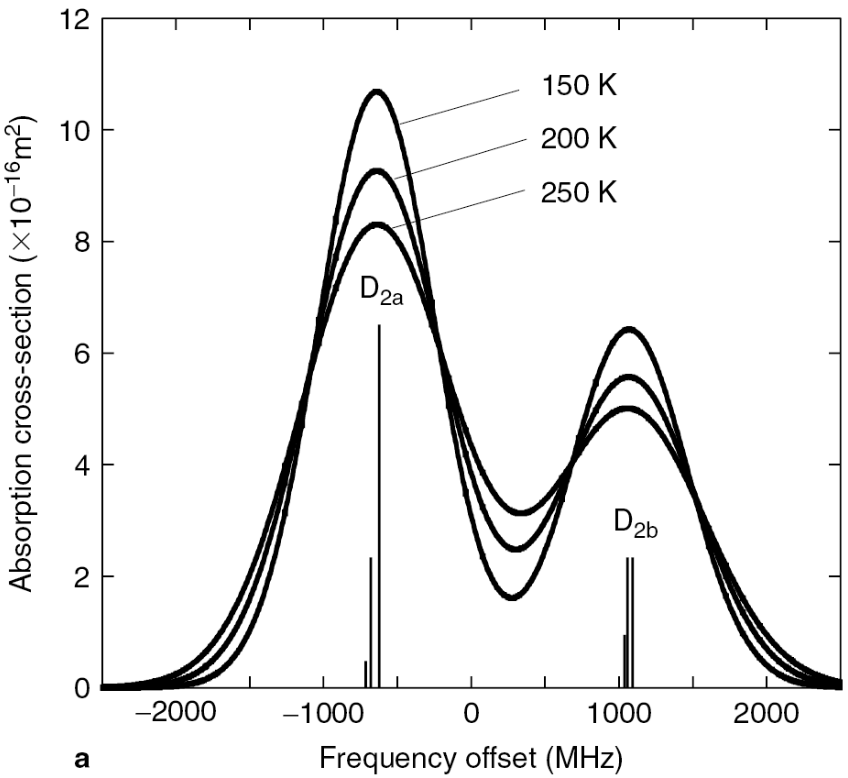
T ↗ ⇨ σ_{rms} ↗

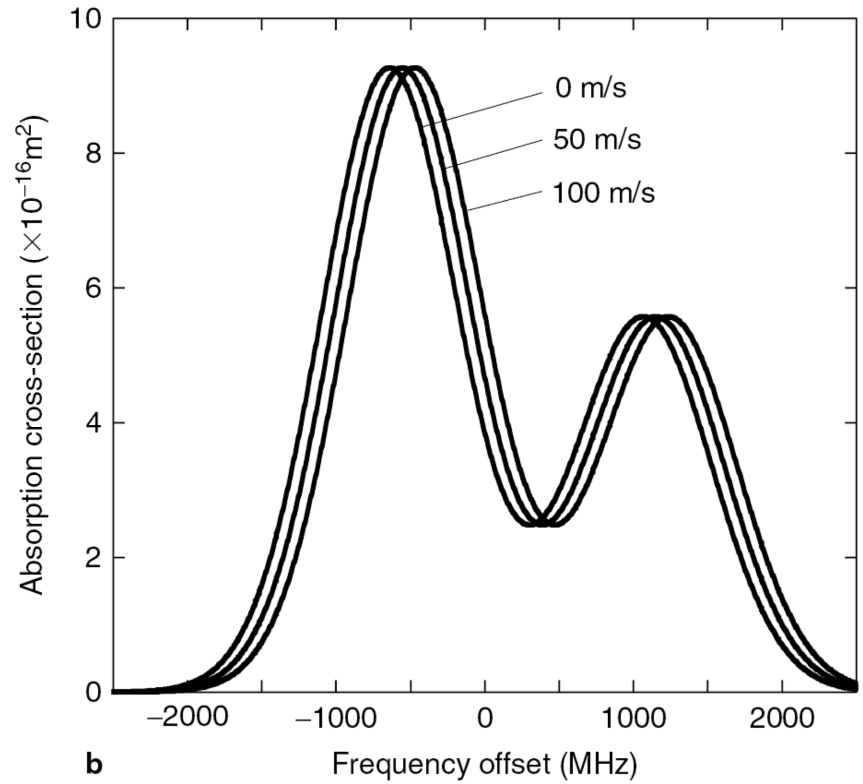
M ↗ ⇨ σ_{rms} ↘





Doppler Effect in Na D_2 Line Resonance Fluorescence

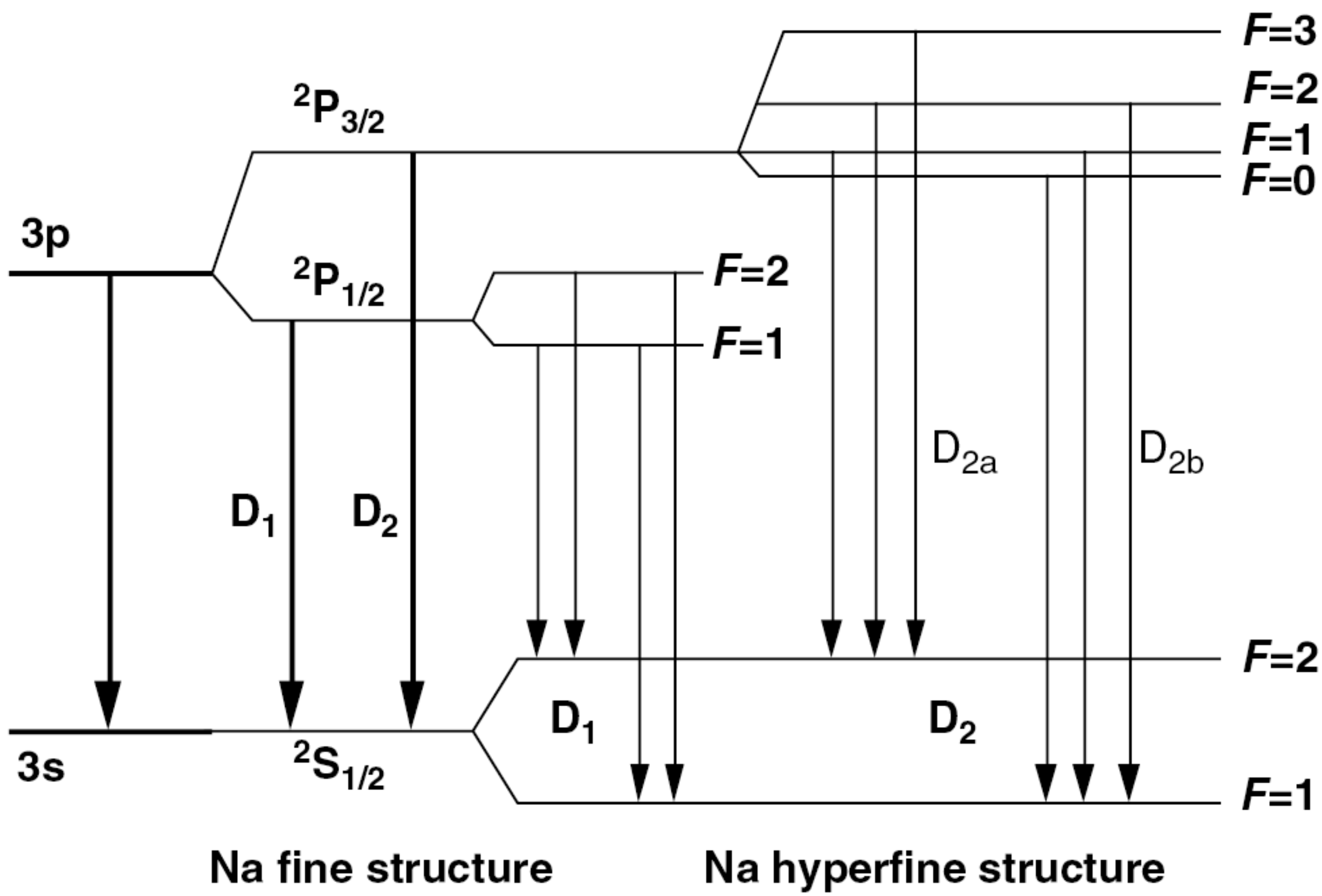




Na D_2 absorption linewidth is temperature dependent

Na D_2 absorption peak freq is wind dependent

Na Atomic Energy Levels



Na Atomic Parameters

Table 5.1 Parameters of the Na D_1 and D_2 Transition Lines

Transition Line	Central Wavelength (nm)	Transition Probability ($10^8\ s^{-1}$)	Radiative Lifetime (nsec)	Oscillator Strength f_{ik}
D_1 ($^2P_{1/2} \rightarrow {}^2S_{1/2}$)	589.7558	0.614	16.29	0.320
D_2 ($^2P_{3/2} \rightarrow {}^2S_{1/2}$)	589.1583	0.616	16.23	0.641

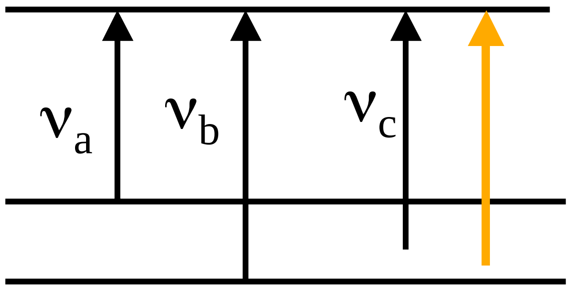
Group	$^2S_{1/2}$	$^2P_{3/2}$	Offset (GHz)	Relative Line Strength[a]
D_{2b}	$F=1$	$F=2$	1.0911	5/32
		$F=1$	1.0566	5/32
		$F=0$	1.0408	2/32
D_{2a}	$F=2$	$F=3$	−0.6216	14/32
		$F=2$	−0.6806	5/32
		$F=1$	−0.7150	1/32

Doppler-Free Saturation–Absorption Features of the Na D_2 Line

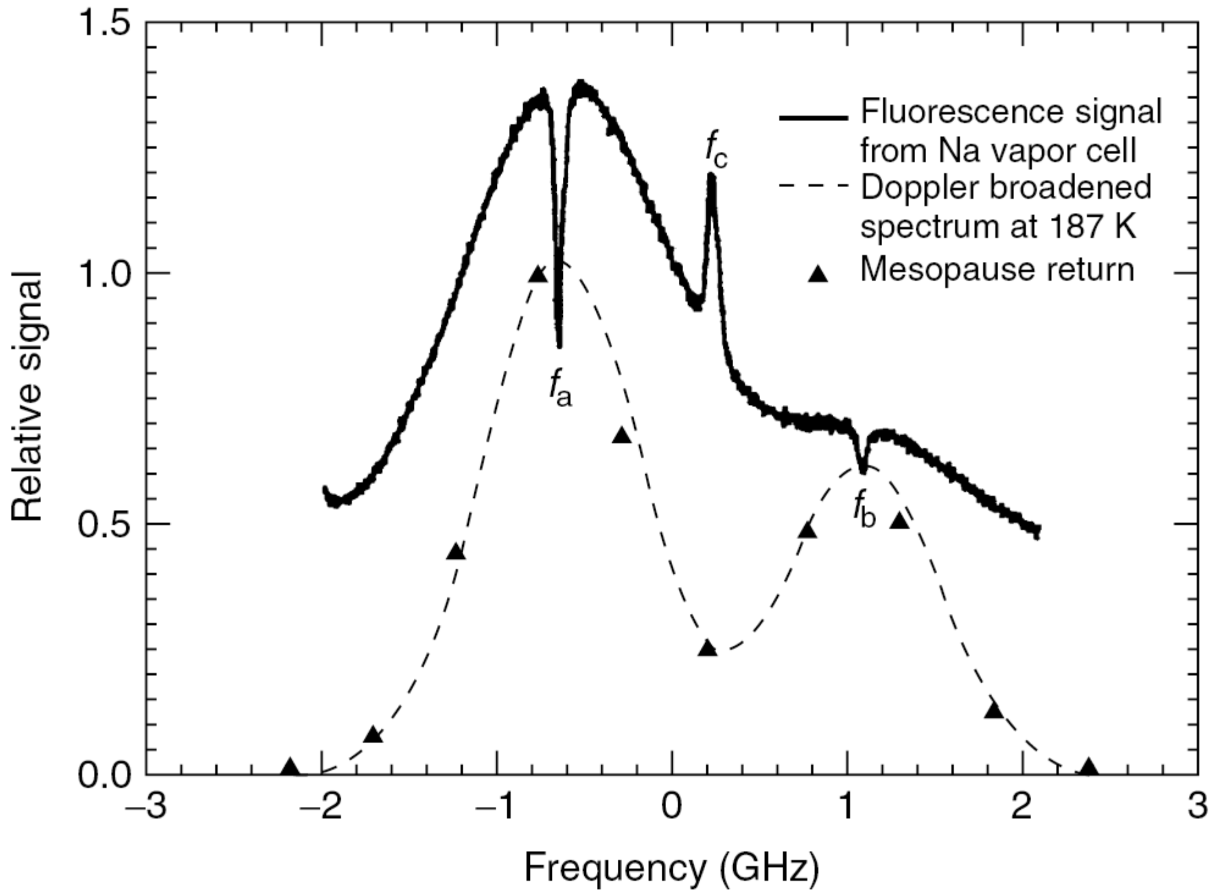
f_a (MHz)	f_c (MHz)	f_b (MHz)	f_+ (MHz)	f_- (MHz)
−651.4	187.8	1067.8	−21.4	−1281.4

[a] Relative line strengths are in the absence of a magnetic field or the spatial average. When Hanle effect is considered in the atmosphere, the relative line strengths will be modified depending on the geomagnetic field and the laser polarization.

Na Spectroscopy



$$\nu_c = (\nu_a + \nu_b)/2$$



$$\sigma_{\mathrm{eff}}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{e}}} \frac{e^2 f}{4\epsilon_0 m_{\mathrm{e}} c} \sum_{n=1}^{6} A_n \exp\left(- \frac{\left[\nu_n - \nu\left(1 - \frac{v_{\mathrm{R}}}{c}\right)\right]^2}{2\sigma_{\mathrm{e}}^2} \right)$$

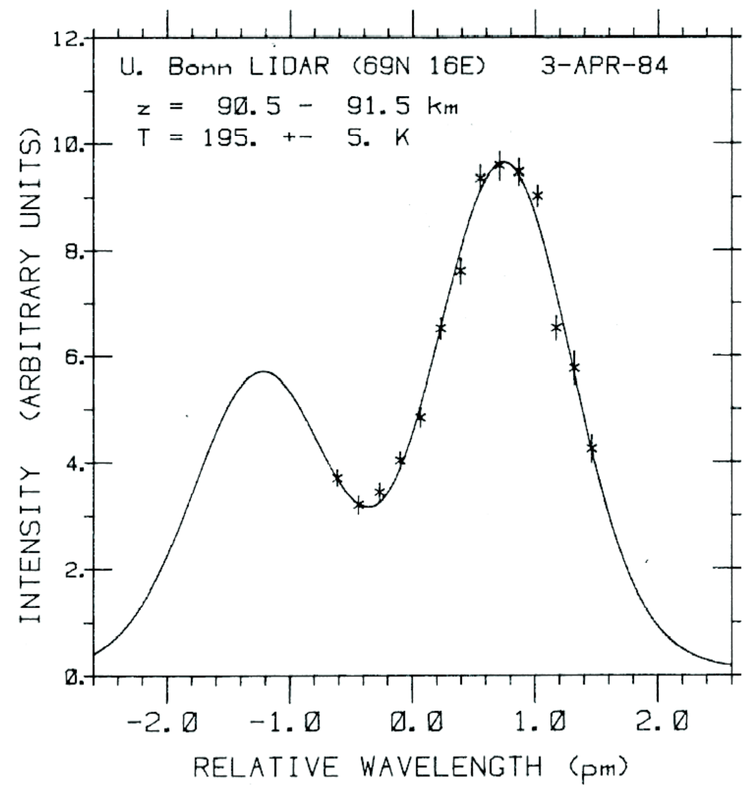
Metrics: Scanning Technique

$$N_{Na}(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda)n_{Na}(z)\Delta z\right)\left(\frac{A}{4\pi z^2}\right)\left(\eta(\lambda)T_a^{\ 2}(\lambda)E^2(\lambda,z)G(z)\right)$$

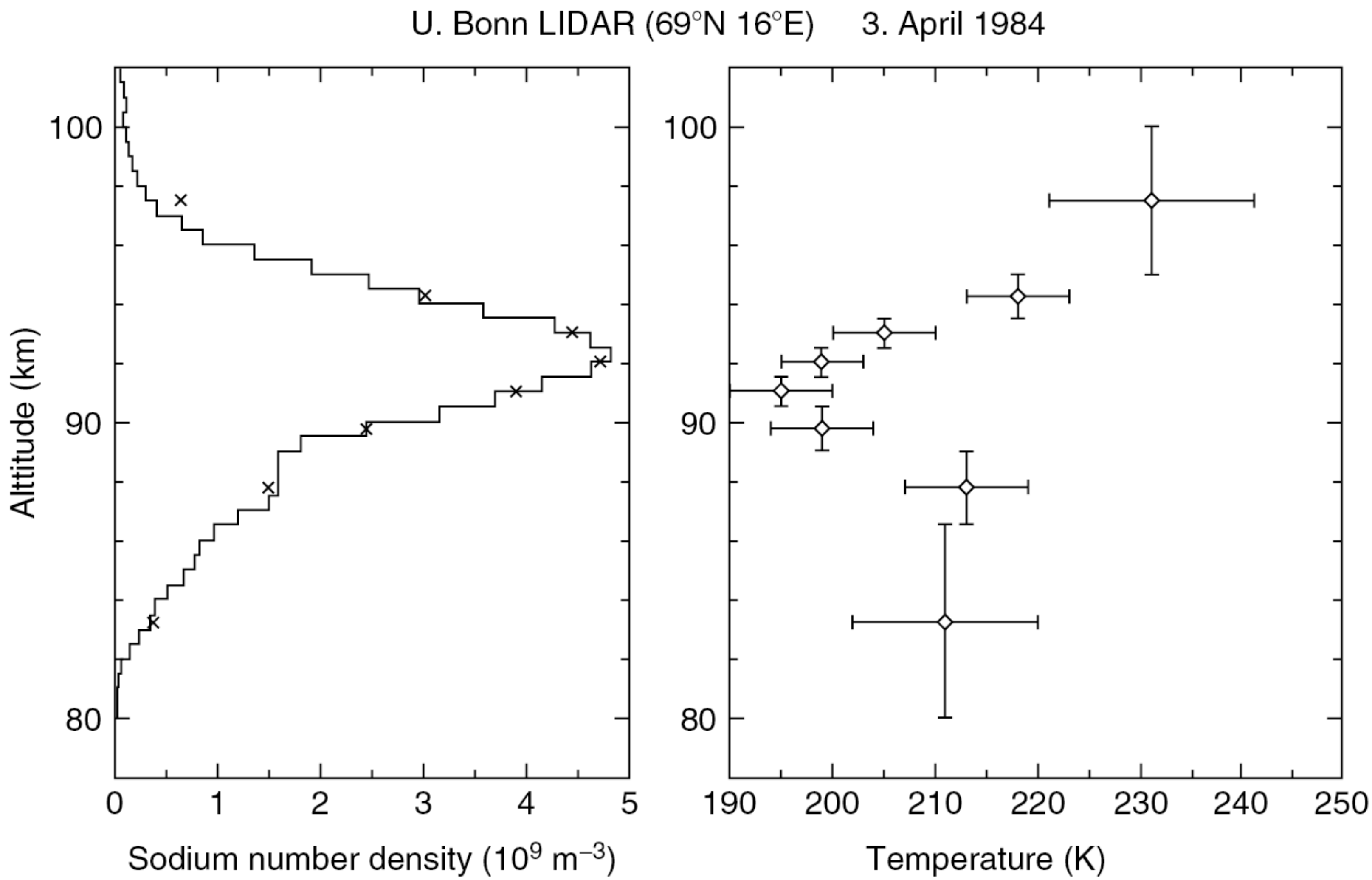
$$N_R(\lambda,z_R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_R(\pi,\lambda)n_R(z_R)\Delta z\right)\left(\frac{A}{z_R^{\ 2}}\right)\left(\eta(\lambda)T_a^{\ 2}(\lambda,z_R)G(z_R)\right)$$

$$\sigma_{eff}(\lambda,z) = \frac{C(z)}{E^2(\lambda,z)}\frac{N_{Na}(\lambda,z)}{N_R(\lambda,z_R)}$$

where $C(z) = \dfrac{\sigma_R(\pi,\lambda)n_R(z_R)}{n_{Na}(z)}\dfrac{4\pi z^2}{z_R^{\ 2}}$

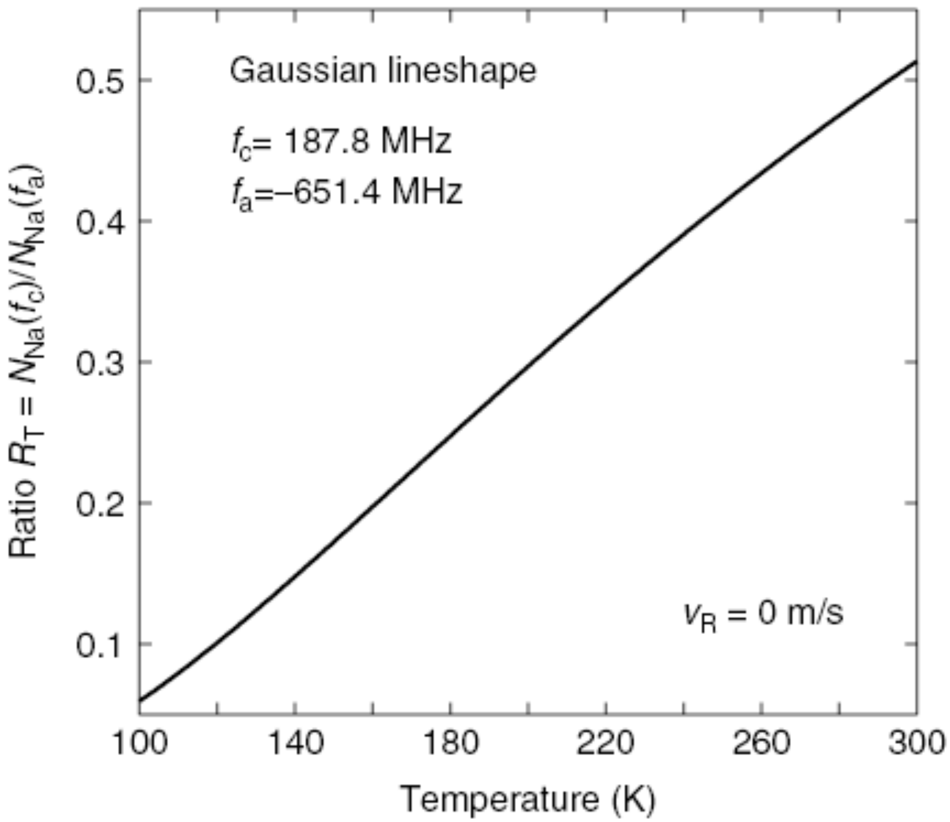


Scanning Na Lidar Results



Metrics: 2-Frequency Technique

$$R_T(z) = \frac{N_{norm}(f_c, z, t_1)}{N_{norm}(f_a, z, t_2)} = \frac{\sigma_{eff}(f_c, z) n_{Na}(z, t_1)}{\sigma_{eff}(f_a, z) n_{Na}(z, t_2)} \approx \frac{\sigma_{eff}(f_c, z)}{\sigma_{eff}(f_a, z)}$$



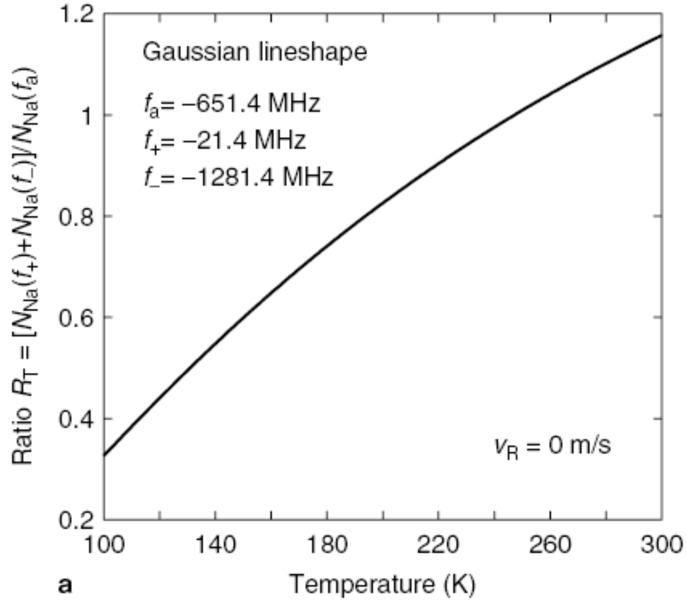
$$N_{norm}(f, z, t) = \frac{N_{Na}(f, z, t)}{N_R(f, z, t) E^2(f, z)}$$

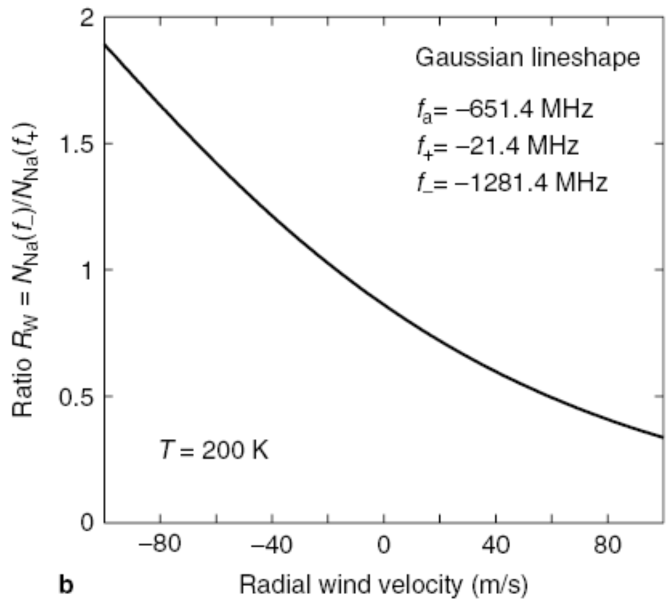
$$N_{norm}(f, z, t) = \frac{\sigma_{eff}(f) n_{Na}(z)}{\sigma_R(\pi, f) n_R(z_R)} \frac{z_R^2}{4\pi z^2}$$

Metrics: 3-Frequency Technique

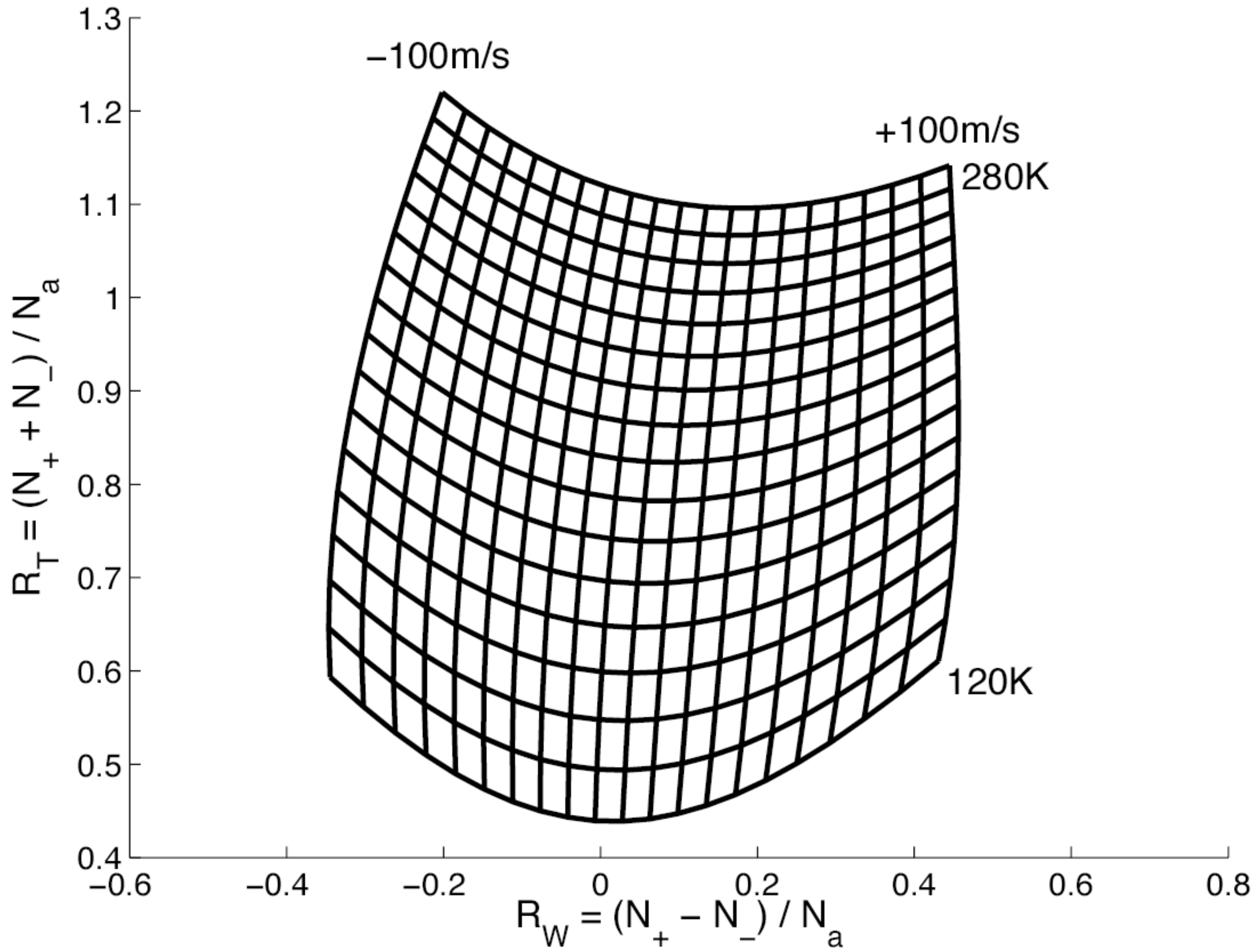
$$R_{\mathrm{T}}(z) = \frac{N_{\mathrm{norm}}(f_+, z, t_1) + N_{\mathrm{norm}}(f_-, z, t_2)}{N_{\mathrm{norm}}(f_{\mathrm{a}}, z, t_3)} \approx \frac{\sigma_{\mathrm{eff}}(f_+, z) + \sigma_{\mathrm{eff}}(f_-, z)}{\sigma_{\mathrm{eff}}(f_{\mathrm{a}}, z)}$$

$$R_{\mathrm{W}}(z) = \frac{N_{\mathrm{norm}}(f_-, z, t_2)}{N_{\mathrm{norm}}(f_+, z, t_1)} \approx \frac{\sigma_{\mathrm{eff}}(f_-, z)}{\sigma_{\mathrm{eff}}(f_+, z)}$$





Na Doppler Lidar Calibration



Summary

- The key point to measure temperature and wind is to find and use temperature-dependent and wind-dependent effects and phenomena to make measurements.
- Doppler technique utilizes the Doppler effect (frequency shift and linewidth broadening) by moving particles to infer wind and temperature information.
- It is widely applied in lidar, radar and sodar technique as well as passive optical remote sensing.
- Resonance fluorescence Doppler lidar technique applies scanning or ratio technique to infer the temperature and wind from the Doppler spectroscopy, while the Doppler spectroscopy is inferred from intensity ratio at different frequencies.