Lecture 10. Temperature Lidar (1)

- How to measure temperature?
- Review of Techniques for Temperature Measurements
- Doppler Technique for Temperature and Wind Measurements
- Resonance Fluorescence Na Doppler Lidar
- Summary
How to Measure Temperature?

Use temperature-dependent effects or phenomena

- **Doppler Technique** - Doppler broadening (not only for Na, K, and Fe, but also for Rayleigh scattering, as long as Doppler broadening dominate and can be detected)

- **Boltzmann Technique** - population ratio (not only for Fe, but also for molecular spectroscopy in optical remote sensing and rotational Raman lidar)

- **Integration Technique (Rayleigh or Raman)** - integration lidar technique using ideal gas law and assuming hydrostatic equilibrium (not only for modern lidar, but also for cw searchlight and rocket falling sphere - some way to measure atmosphere number density)

- **Rotational Raman Technique** - temperature dependence of population ratio, similar to Boltzmann technique
Doppler Technique

Doppler Spectrum (Width and Shift) ⇒ Temperature and Radial Wind

\( \sigma_{D_{rms}} = \frac{\omega_0}{c} \sqrt{\frac{k_B T}{M}} = \frac{1}{\lambda_0} \sqrt{\frac{k_B T}{M}} = \frac{1}{M\lambda_0^2} \)

\( \Delta \omega = \omega - \omega_0 = -\bar{k} \cdot \bar{v} = -\omega_0 \frac{v \cos \theta}{c} \)

\( v' = v \left( 1 - \frac{v_R}{c} \right) = v - \frac{v_R}{\lambda} \)
Boltzmann Technique

\[ z^5F^0 \]

3d\(^6\)4s4p

372nm 100% 374nm 91% 368nm 9%

J'=1

J'=2

J'=3

J'=4

J'=5

J=0

J=1

J=2

J=3

J=4

Atomic Fe Energy Level

[Gelbwachs, 1994; Chu et al., 2002]

Maxwell-Boltzmann Distribution in Thermal-dynamic Equilibrium

\[
\frac{P_2(J = 3)}{P_1(J = 4)} = \frac{\rho_{Fe(374)}}{\rho_{Fe(372)}} = \frac{g_2}{g_1} \exp\left(-\frac{\Delta E}{k_B T}\right)
\]

\[ T = \frac{\Delta E / k_B}{\ln\left(\frac{g_2 \cdot P_1}{g_1 \cdot P_2}\right)} \]

\( P_1, P_2 \) -- Fe populations

\( g_1, g_2 \) -- Degeneracy

\( k_B \) -- Boltzmann constant

T -- Temperature

Population Ratio \( \Rightarrow \) Temperature
Rayleigh Integration Technique

**Hydrostatic Equation**
\[ dP = -\rho gdz \]

**Ideal Gas Law**
\[ P = \rho RT \]

**Temperature**
\[ T(z) = T(z_0) \frac{\rho(z_0)}{\rho(z)} + \frac{1}{R} \int_{z}^{z_0} g(r) dr \frac{\rho(r)}{\rho(z)} \]

- \( T(z_0) \) - Seeding Temperature;
- \( \rho \) - number density
- \( R \) - gas constant for dry air;
- \( g \) - gravitational acceleration

**Lidar Backscatter Ratio \( \Rightarrow \) Relative Density \( \Rightarrow \) Temperature (at different altitudes)**
Temperature can be derived from the ratio of two pure Rotational Raman line intensities. This is essentially the same principle as Boltzmann temperature technique!
Temperature Techniques

- **75-120km**: resonance fluorescence Doppler technique (Na, K, Fe) & Boltzmann technique (Fe, OH, O₂)
- **30-90km**: Rayleigh integration technique & Rayleigh Doppler technique
- **Below 30 km**: scattering Doppler technique and Raman (Boltzmann and integration) technique
- **Boundary layer**: DIAL, HSRL, Rotational Raman
Doppler Technique to Measure Temperature and Wind

- Doppler effect is commonly experienced by moving particles, such as atoms, molecules, and aerosols. It is the apparent frequency change of radiation or wave that is perceived by the particles moving relative to the source of the radiation or wave. This is called Doppler shift.

- Doppler frequency shift is proportional to the radial velocity along the line of sight (LOS) of the radiation -

\[ \Delta \omega = \omega - \omega_0 = -k \cdot \vec{v} = -\omega_0 \frac{v \cos \theta}{c} \]

where \( \omega_0 \) is the radiation frequency at rest, \( \omega \) is the shifted frequency, \( k \) is the wave vector of the radiation (\( k=2\pi/\lambda \), and \( v \) is the particle velocity.
Doppler Technique to Measure Temperature and Wind

- Due to particles' thermal motions in the atmosphere, the distribution of perceived frequencies for all particles mirrors their velocity distribution. According to the Maxwellian velocity distribution, the perceived frequencies by moving particles has a Gaussian lineshape, given by

\[
\exp\left(-\frac{M v_z^2}{2k_B T}\right) dv_z = \exp\left\{-\frac{M c^2 (\omega - \omega_0)^2}{2\omega_0^2 k_B T}\right\} \frac{c}{\omega_0} d\omega
\]

- The peak is at \( \omega = \omega_0 \) and the \( \text{rms} \) width is given by

\[
\sigma_{\text{rms}} = \frac{\omega_0}{c} \sqrt{\frac{k_B T}{M}} = \frac{1}{\lambda_0} \sqrt{\frac{k_B T}{M}}
\]
Doppler Shift For Wind Measurement

\[ \Delta \omega = \omega - \omega_0 = -\vec{k} \cdot \vec{v} = -\omega_0 \frac{v \cos \theta}{c} \]

- Same direction: red shift
- Opposite direction: blue shift

The velocity measurements of lidar, radar, and sodar all base on the Doppler shift principle!
Doppler Broadening For Temperature

\[ \sigma_{rms} = \frac{\omega_0}{c} \sqrt{\frac{k_B T}{M}} = \frac{1}{\lambda_0} \sqrt{\frac{k_B T}{M}} \]

\[ T \uparrow \Rightarrow \sigma_{rms} \uparrow \]

\[ M \downarrow \Rightarrow \sigma_{rms} \downarrow \]

![Graphs showing relative intensity vs. frequency offset for aerosol and molecular scattering, with indicated frequency ranges 3-30 MHz and 1 GHz.](image-url)
Doppler Effect in Na $D_2$ Line
Resonance Fluorescence

Na $D_2$ absorption linewidth is temperature dependent

Na $D_2$ absorption peak freq is wind dependent
Na Atomic Parameters

Table 5.1 Parameters of the Na D₁ and D₂ Transition Lines

<table>
<thead>
<tr>
<th>Transition Line</th>
<th>Central Wavelength (nm)</th>
<th>Transition Probability (10⁸ s⁻¹)</th>
<th>Radiative Lifetime (nsec)</th>
<th>Oscillator Strength f_{ik}</th>
</tr>
</thead>
<tbody>
<tr>
<td>D₁ (²P_{1/2}→²S_{1/2})</td>
<td>589.7558</td>
<td>0.614</td>
<td>16.29</td>
<td>0.320</td>
</tr>
<tr>
<td>D₂ (²P_{3/2}→²S_{1/2})</td>
<td>589.1583</td>
<td>0.616</td>
<td>16.23</td>
<td>0.641</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group</th>
<th>²S_{1/2}</th>
<th>²P_{3/2}</th>
<th>Offset (GHz)</th>
<th>Relative Line Strength a</th>
</tr>
</thead>
<tbody>
<tr>
<td>D₂b</td>
<td>F = 1</td>
<td>F = 2</td>
<td>1.0911</td>
<td>5/32</td>
</tr>
<tr>
<td></td>
<td>F = 1</td>
<td></td>
<td>1.0566</td>
<td>5/32</td>
</tr>
<tr>
<td></td>
<td>F = 0</td>
<td></td>
<td>1.0408</td>
<td>2/32</td>
</tr>
<tr>
<td>D₂a</td>
<td>F = 2</td>
<td></td>
<td>-0.6216</td>
<td>14/32</td>
</tr>
<tr>
<td></td>
<td>F = 3</td>
<td></td>
<td>-0.6806</td>
<td>5/32</td>
</tr>
<tr>
<td></td>
<td>F = 1</td>
<td></td>
<td>-0.7150</td>
<td>1/32</td>
</tr>
</tbody>
</table>

Doppler-Free Saturation–Absorption Features of the Na D₂ Line

<table>
<thead>
<tr>
<th>f_α (MHz)</th>
<th>f_c (MHz)</th>
<th>f_b (MHz)</th>
<th>f_+ (MHz)</th>
<th>f_- (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-651.4</td>
<td>187.8</td>
<td>1067.8</td>
<td>-21.4</td>
<td>-1281.4</td>
</tr>
</tbody>
</table>

aRelative line strengths are in the absence of a magnetic field or the spatial average. When Hanle effect is considered in the atmosphere, the relative line strengths will be modified depending on the geomagnetic field and the laser polarization.
Na Spectroscopy

\[ \nu_c = \frac{\nu_a + \nu_b}{2} \]

\[ \sigma_{\text{eff}}(\nu) = \frac{1}{\sqrt{2\pi}\sigma_e} \frac{e^2f}{4\epsilon_0 m_e c} \sum_{n=1}^{6} A_n \exp \left( - \frac{\left[ \nu_n - \nu \left( 1 - \frac{v_R}{c} \right) \right]^2}{2\sigma_e^2} \right) \]
Metrics: Scanning Technique

\[ N_{Na}(\lambda,z) = \left( \frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) \left( \sigma_{eff}(\lambda)n_{Na}(z)\Delta z \right) \left( \frac{A}{4\pi z^2} \right) \left( \eta(\lambda)T_a^2(\lambda)E^2(\lambda,z)G(z) \right) \]

\[ N_R(\lambda,z_R) = \left( \frac{P_L(\lambda)\Delta t}{hc/\lambda} \right) \left( \sigma_R(\pi,\lambda)n_R(z_R)\Delta z \right) \left( \frac{A}{z_R^2} \right) \left( \eta(\lambda)T_a^2(\lambda,z_R)G(z_R) \right) \]

\[ \sigma_{eff}(\lambda,z) = \frac{C(z)}{E^2(\lambda,z)N_{Na}(\lambda,z)} \]

where \[ C(z) = \frac{\sigma_R(\pi,\lambda)n_R(z_R)4\pi z^2}{n_{Na}(z)z_R^2} \]

\[ \eta(\lambda) = \begin{cases} \frac{\lambda^2}{8\pi^2} & \lambda > 0 \\ \frac{\lambda^2}{8\pi^2} & \lambda < 0 \end{cases} \]

\[ T_a = \begin{cases} 1 & \lambda > 0 \\ 0 & \lambda < 0 \end{cases} \]
Scanning Na Lidar Results

U. Bonn LIDAR (69°N 16°E)  3. April 1984
\[ R_T(z) = \frac{N_{\text{norm}}(f_c, z, t_1)}{N_{\text{norm}}(f_a, z, t_2)} = \frac{\sigma_{\text{eff}}(f_c, z)n_{\text{Na}}(z, t_1)}{\sigma_{\text{eff}}(f_a, z)n_{\text{Na}}(z, t_2)} \approx \frac{\sigma_{\text{eff}}(f_c, z)}{\sigma_{\text{eff}}(f_a, z)} \]

Gaussian lineshape

\[ f_c = 187.8 \text{ MHz} \]
\[ f_a = -651.4 \text{ MHz} \]

\[ N_{\text{norm}}(f, z, t) = \frac{N_{\text{Na}}(f, z, t)}{N_{R}(f, z, t)E^2(f, z)} \]

\[ N_{\text{norm}}(f, z, t) = \frac{\sigma_{\text{eff}}(f)n_{\text{Na}}(z)}{\sigma_{R}(\pi, f)n_{R}(z R)} \frac{z_R^2}{4\pi z^2} \]
**Metrics: 3-Frequency Technique**

\[
R_T(z) = \frac{N_{\text{norm}}(f_+, z, t_1) + N_{\text{norm}}(f_-, z, t_2)}{N_{\text{norm}}(f_a, z, t_3)} 
\approx \frac{\sigma_{\text{eff}}(f_+, z) + \sigma_{\text{eff}}(f_-, z)}{\sigma_{\text{eff}}(f_a, z)}
\]

\[
R_W(z) = \frac{N_{\text{norm}}(f_-, z, t_2)}{N_{\text{norm}}(f_+, z, t_1)} \approx \frac{\sigma_{\text{eff}}(f_-, z)}{\sigma_{\text{eff}}(f_+, z)}
\]

- Gaussian lineshape
  - \(f_a = -651.4 \text{ MHz}\)
  - \(f_+ = -21.4 \text{ MHz}\)
  - \(f_- = -1281.4 \text{ MHz}\)

- \(v_R = 0 \text{ m/s}\)
- Temperature (K)

- Gaussian lineshape
  - \(f_a = -651.4 \text{ MHz}\)
  - \(f_+ = -21.4 \text{ MHz}\)
  - \(f_- = -1281.4 \text{ MHz}\)

- \(T = 200 \text{ K}\)
- Radial wind velocity (m/s)
Na Doppler Lidar Calibration

\[ R_T = \frac{N_+ + N_-}{N_a} \]

\[ R_W = \frac{N_+ - N_-}{N_a} \]

-100 m/s
+100 m/s
280K
120K
**Summary**

- The key point to measure temperature and wind is to find and use temperature-dependent and wind-dependent effects and phenomena to make measurements.
- Doppler technique utilizes the Doppler effect (frequency shift and linewidth broadening) by moving particles to infer wind and temperature information.
- It is widely applied in lidar, radar and sodar technique as well as passive optical remote sensing.
- Resonance fluorescence Doppler lidar technique applies scanning or ratio technique to infer the temperature and wind from the Doppler spectroscopy, while the Doppler spectroscopy is inferred from intensity ratio at different frequencies.