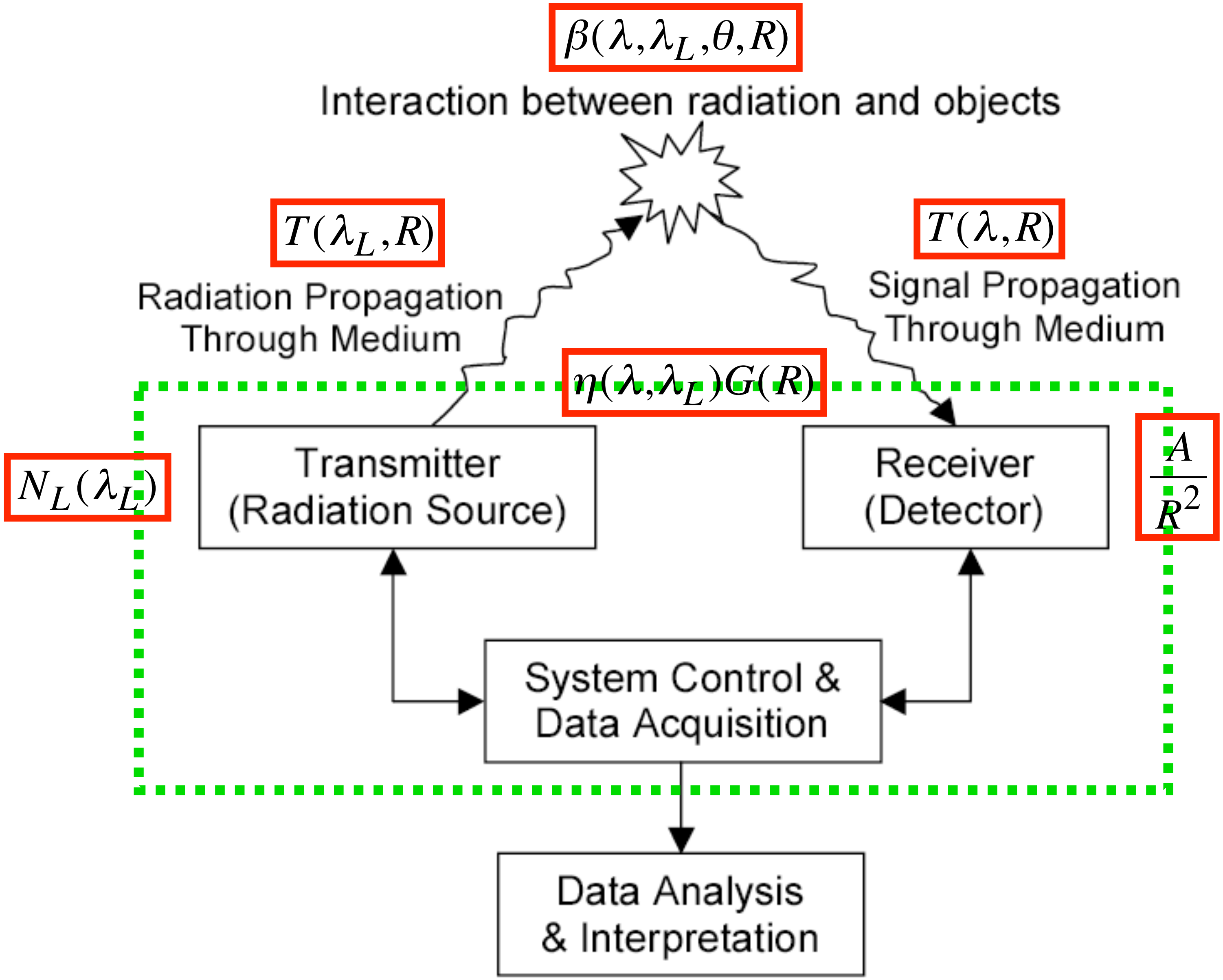
Lecture 08. Solutions of Lidar Equations

- HWK Report #1
- Solution for scattering form lidar equation
- Solution for fluorescence form lidar equation
- Solution for differential absorption lidar equation
- Solution for resonance fluorescence lidar
- Solution for Rayleigh and Mie lidar
- Summary

HWK Report #1



$\beta(\lambda,\lambda_L,\theta,R)$
Interaction between radiation and objects
$T(\lambda_L,R)$
Radiation Propagation Through Medium
$T(\lambda,R)$
Signal Propagation Through Medium
$\eta(\lambda,\lambda_L)G(R)$
$N_L(\lambda_L)$
Transmitter (Radiation Source)
Receiver (Detector)
$\frac{A}{R^2}$
System Control & Data Acquisition
Data Analysis & Interpretation

General Lidar Equation

Assumptions: independent and single scattering

$$N_S(\lambda, R) = N_L(\lambda_L) \cdot \left[\beta(\lambda, \lambda_L, \theta, R)\Delta R\right] \cdot \frac{A}{R^2} \cdot \left[T(\lambda_L, R)T(\lambda, R)\right] \cdot \left[\eta(\lambda, \lambda_L)G(R)\right] + N_B$$

- N_S - expected photon counts detected at λ and distance R;
- 1st term - number of transmitted laser photons;
- 2nd term - probability that a transmitted photon is scattered by the scatters into a unit solid angle at angle θ;
- 3rd term - probability that a scatter photon is collected by the receiving telescope;
- 4th term - light transmission during light propagation from laser source to distance R and from distance R to receiver;
- 5th term - overall system efficiency;
- 6th term - background and detector noise.

More in General Lidar Equation

$$N_S(\lambda,R)=\left[\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right]\cdot\left[\beta(\lambda,\lambda_L,\theta,R)\Delta R\right]\cdot\frac{A}{R^2}\cdot\left[T(\lambda_L,R)T(\lambda,R)\right]\cdot\left[\eta(\lambda,\lambda_L)G(R)\right]+N_B$$

N_S (R) - expected received photon number from a distance R

P_L - transmitted laser power, λ_L - laser wavelength

Δt - integration time,

h - Planck's constant, c - light speed

β(R) - volume scatter coefficient at distance R for angle θ,

ΔR - thickness of the range bin

A - area of receiver,

T(R) - one way transmission of the light from laser source to distance R or from distance R to the receiver,

η - system optical efficiency,

G(R) - geometrical factor of the system,

N_B - background and detector noise photon counts.

Solution for Scattering Form Lidar Equation

- Scattering form lidar equation

$$N_S(\lambda,z) = \left[\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right] \cdot \left[\beta(\lambda,\lambda_L,z)\Delta z\right] \cdot \left(\frac{A}{z^2}\right) \cdot \left[T(\lambda_L,z)T(\lambda,z)\right] \cdot \left[\eta(\lambda,\lambda_L)G(z)\right] + N_B$$

- Solution for scattering form lidar equation

$$\beta(\lambda,\lambda_L,z) = \frac{N_S(\lambda,z) - N_B}{\left[\frac{P_L(\lambda_L)\Delta t}{hc/\lambda_L}\right]\Delta z \left(\frac{A}{z^2}\right)\left[T(\lambda_L,z)T(\lambda,z)\right]\left[\eta(\lambda,\lambda_L)G(z)\right]}$$

Solution for Fluorescence Form Lidar Equation

- Fluorescence form lidar equation

$$N_S(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda,z)n_c(z)R_B(\lambda)\Delta z\right)\left(\frac{A}{4\pi z^2}\right)\left(T_a^2(\lambda,z)T_c^2(\lambda,z)\right)\left(\eta(\lambda)G(z)\right)+N_B$$

- Solution for fluorescence form lidar equation

$$n_c(z) = \frac{N_S(\lambda,z)-N_B}{\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda)R_B(\lambda)\Delta z\right)\left(\frac{A}{4\pi z^2}\right)\left(\eta(\lambda)T_a{}^2(\lambda,z)T_c^2(\lambda,z)G(z)\right)}$$

Differential Absorption/Scattering Form

- For the laser with wavelength λ_{on} on the molecular absorption line

$$N_S(\lambda_{on},z) = N_L(\lambda_{on})\left[\beta_{sca}(\lambda_{on},z)\Delta z\right]\left(\frac{A}{z^2}\right)\exp\left[-2\int_0^z \overline{\alpha}(\lambda_{on},z')dz'\right]$$
$$\times\exp\left[-2\int_0^z \sigma_{abs}(\lambda_{on},z')n_c(z')dz'\right]\left[\eta(\lambda_{on})G(z)\right] + N_B$$

- For the laser with wavelength λ_{off} off the molecular absorption line

$$N_S(\lambda_{off},z) = N_L(\lambda_{off})\left[\beta_{sca}(\lambda_{off},z)\Delta z\right]\left(\frac{A}{z^2}\right)\exp\left[-2\int_0^z \overline{\alpha}(\lambda_{off},z')dz'\right]$$
$$\times\exp\left[-2\int_0^z \sigma_{abs}(\lambda_{off},z')n_c(z')dz'\right]\left[\eta(\lambda_{off})G(z)\right] + N_B$$

Differential Absorption/Scattering Form

- The ratio of photon counts from these two channels is a function of the differential absorption and scattering:

$$\boxed{\begin{aligned}\frac{N_S(\lambda_{on},z)-N_B}{N_S(\lambda_{off},z)-N_B} &= \frac{N_L(\lambda_{on})\beta_{sca}(\lambda_{on},z)}{N_L(\lambda_{off})\beta_{sca}(\lambda_{off},z)}\frac{\eta(\lambda_{on})}{\eta(\lambda_{off})} \\ &\times \exp\left\{-2\int_0^z\left[\overline{\alpha}(\lambda_{on},z')-\overline{\alpha}(\lambda_{off},z')\right]dz'\right\} \\ &\times \exp\left\{-2\int_0^z\left[\sigma_{abs}(\lambda_{on},z')-\sigma_{abs}(\lambda_{off},z')\right]n_c(z')dz'\right\}\end{aligned}}$$

$$\boxed{\Delta\sigma = \sigma_{abs}(\lambda_{on})-\sigma_{abs}(\lambda_{off})}$$

Solution for Differental Absorption Lidar Equation

- Solution for differential absorption lidar equation

$$n_c(z) = \frac{1}{2\Delta\sigma}\frac{d}{dz}\left\{ \begin{array}{l} \ln\left[\dfrac{N_L(\lambda_{on})\beta_{sca}(\lambda_{on},z)}{N_L(\lambda_{off})\beta_{sca}(\lambda_{off},z)}\dfrac{\eta(\lambda_{on})}{\eta(\lambda_{off})}\right] \\ \quad -\ln\left[\dfrac{N_S(\lambda_{on},z)-N_B}{N_S(\lambda_{off},z)-N_B}\right] \\ \quad -\left[\bar{\alpha}(\lambda_{on},z')-\bar{\alpha}(\lambda_{off},z')\right] \end{array} \right\}$$

$$\Delta\sigma = \sigma_{abs}(\lambda_{on}) - \sigma_{abs}(\lambda_{off})$$

Solution for
Resonance Fluorescence Lidar Equation

- Resonance fluorescence and Rayleigh lidar equations

$$N_S(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_{eff}(\lambda,z)n_c(z)R_B(\lambda)\Delta z\right)\left(\frac{A}{4\pi z^2}\right)\left(T_a^{\,2}(\lambda)T_c^2(\lambda,z)\right)\left(\eta(\lambda)G(z)\right) + N_B$$

$$N_R(\lambda,z_R) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\sigma_R(\pi,\lambda)n_R(z_R)\Delta z\right)\left(\frac{A}{z_R^{\,2}}\right)T_a^{\,2}(\lambda,z_R)\left(\eta(\lambda)G(z_R)\right) + N_B$$

- Rayleigh normalization

$$\frac{n_c(z)}{n_R(z_R)} = \frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^{\,2}}\cdot\frac{4\pi\sigma_R(\pi,\lambda)}{\sigma_{eff}(\lambda,z)R_B(\lambda)}\cdot\frac{\cancel{T_a^{\,2}}(\lambda,z_R)\cancel{G(z_R)}}{\cancel{T_a^{\,2}}(\lambda,z)T_c^2(\lambda,z)\cancel{G(z)}}$$

- Solution for resonance fluorescence

$$\boxed{n_c(z) = n_R(z_R)\frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^{\,2}}\cdot\frac{4\pi\sigma_R(\pi,\lambda)}{\sigma_{eff}(\lambda,z)R_B(\lambda)}\cdot\frac{1}{T_c^2(\lambda,z)}}$$

Solution for Rayleigh and Mie Lidars

- Rayleigh and Mie (middle atmos) lidar equations

$$N_S(\lambda,z)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\beta_R(z)+\beta_{aerosol}(z)\right)\Delta z\left(\frac{A}{z^2}\right)T_a^{\,2}(\lambda,z)\left(\eta(\lambda)G(z)\right)+N_B$$

$$N_R(\lambda,z_R)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left(\beta_R(z_R)\Delta z\right)\left(\frac{A}{z_R^{\,2}}\right)T_a^{\,2}(\lambda,z_R)\left(\eta(\lambda)G(z_R)\right)+N_B$$

- Rayleigh normalization

$$\frac{\beta_R(z)+\beta_{aerosol}(z)}{\beta_R(z_R)}=\frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^{\,2}}\cdot\frac{\cancel{T_a^{\,2}(\lambda,z_R)}\cancel{G(z_R)}}{\cancel{T_a^{\,2}(\lambda,z)}\cancel{G(z)}}$$

- For Rayleigh scattering at z and z_R

$$\boxed{\frac{\beta_R(z)}{\beta_R(z_R)}=\frac{\sigma_R(z)n_{atm}(z)}{\sigma_R(z_R)n_{atm}(z_R)}=\frac{n_{atm}(z)}{n_{atm}(z_R)}}$$

Solution (Continued)

- Solution for Mie scattering in middle atmosphere

$$\beta_{aerosol}(z) = \beta_R(z_R)\left[\frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^{\,2}}-\frac{n_{atm}(z)}{n_{atm}(z_R)}\right]$$

$$\beta_R(\lambda,z_R,\pi) = 2.938\times10^{-32}\,\frac{P(z_R)}{T(z_R)}\cdot\frac{1}{\lambda^{4.0117}}\left(m^{-1}sr^{-1}\right)$$

- Rayleigh normalization when aerosols not present

$$\frac{\beta_R(z)}{\beta_R(z_R)} = \frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^{\,2}}\cdot\frac{\cancel{T_a^{\,2}}(\lambda,z_R)\cancel{G(z_R)}}{\cancel{T_a^{\,2}}(\lambda,z)\cancel{G(z)}}$$

- Solution for relative number density in Rayleigh lidar

$$RND(z) = \frac{n_{atm}(z)}{n_{atm}(z_R)} = \frac{\beta_R(z)}{\beta_R(z_R)} = \frac{N_S(\lambda,z)-N_B}{N_R(\lambda,z_R)-N_B}\cdot\frac{z^2}{z_R^{\,2}}$$

Summary

- Solutions of lidar equation can be obtained by solving the lidar equation directly if all the lidar parameters and atmosphere conditions are well known.
- Solutions for three forms of lidar equations are shown: scattering form, fluorescence form, and differential absorption form.
- However, system parameters and atmosphere conditions may vary frequently and are NOT well known to experimenters.
- A good solution is to perform Rayleigh normalization to cancel out most of the system and atmosphere parameters so that the essential and known parts can be solved.