ASEN 6519. Lidar Remote Sensing
HWK Project #6 – Lidar Simulation for Range-Resolved Signals with Error Analysis

This project is to simulate the range-resolved K Doppler lidar signals with temperature error analysis by integrating the knowledge we gained through the lidar class. The knowledge includes our understanding of (1) different scattering processes; (2) the lidar equation and the lidar remote sensing procedure; (3) the roles of atmospheric number density and atomic species density in the lidar equation; (4) the signal-to-noise ratio (SNR) related to the uncertainty caused by photon noise; (5) the ratio technique for temperature derivation; (6) temperature error analysis for photon noise errors, etc.

This project contains the following tasks (related parameters are listed at the end of the assignment) –

1. Understand the K spectroscopy and calculate the absorption and effective scattering cross-sections for the K Doppler lidar, considering the 4 hyperfine lines of K D1 line and 2 isotopes of K (i.e., $^{39}$K and $^{41}$K). Do this for T = 200 K and $V_R = 0$ m/s. Plot the absorption and effective cross-sections versus frequency offset (-1500 to +1500 MHz).

2. Simulate the range-resolved photon count return from 30-150 km by a K Doppler lidar, considering Rayleigh scattering signal from all altitudes, K resonance fluorescence signal from 70-115 km, and background photon counts for all altitudes. Plot the range-resolved photon count profiles. (Put altitude to x-axis)

   The K Doppler lidar uses 3-frequency technique, similar to the Na Doppler lidar (see our textbook and lecture notes for the details of the Arecibo K lidar). Please simulate the entire 30-150 km profiles for the peak frequency and simulate the K signals from 75-115 km for all three frequencies.

   The atmospheric number density and temperature profiles are taken from a MSIS00 model profile (posted at the class website). The K layer can be simulated using a Gaussian distribution with the peak at 91 km, rms width of 4.7 km, and column abundance of $6 \times 10^7$ cm$^{-2}$. The background count is 10/1000 shots/km. The temperature profile in the K layer is taken from the same MSIS00 model profile. The radial wind in K layer is $V_R = 0$ m/s.

3. Add the photon noise with Possion distribution to the range-resolved lidar profile (30-150 km) for the peak frequency to simulate the actual lidar returns with photon noise. Try this for 10, 1000, 10,000 shots of integration. Plot the photon count profiles. (Put altitude to x-axis)

4. Derive the signal-to-noise ratio (SNR) equation when considering the Rayleigh, K, and background photon counts, and then calculate SNR for the range-resolved photon count profile from 30-150 km. Plot the SNR profile in absolute number and dB, respectively. (Put altitude to x-axis)

5. Calculate the temperature metrics from the simulated 3-frequency K signals and from the calculated effective cross-sections using equation (1) and (2). Plot the two $R_T$ versus altitude side by side (put altitude to y-axis). Are they the same?
\[ R_T = \frac{N_+ + N_-}{N_a} \quad (1), \quad R_T = \frac{\sigma_{\text{eff}} (f_+, T, V_R) + \sigma_{\text{eff}} (f_-, T, V_R)}{\sigma_{\text{eff}} (f_a, T, V_R)} \quad (2) \]

(6) Using Eq. (2) of \( R_T \), derive the temperature error coefficient \( \frac{R_T}{\partial R_T / \partial T} \). It is easy to derive this numerically. Again, the temperature profile in K layer is from the MSIS90 profile. Plot the temperature error coefficient versus altitude and side-by-side plot the temperature versus altitude (put altitude to y-axis).

You may also realize that this temperature error coefficient is the reciprocal of temperature sensitivity \( S_T \), which you have derived for the Na Doppler lidar in Project #3.

(7) Derive the following error equation for \( \Delta R_T / R_T \) (i.e., Eq. (5.83) in our textbook), and then calculate and plot \( \Delta R_T / R_T \) versus altitude from photon counts.

\[ \frac{\Delta R_T}{R_T} = \left( \frac{1 + \frac{1}{R_T}}{\left( N_{f_a} \right)^{1/2}} \right)^{1/2} \left[ 1 + \frac{B}{N_{f_a}} \frac{1 + \frac{2}{R_T}}{\left( 1 + \frac{1}{R_T} \right)^{1/2}} \right] \quad (3) \]

(8) From Steps (7) and (8), derive the temperature error caused by the photon noise. Plot the temperature error \( \Delta T \) vs altitude, and side-by-side plot the K photon count profile.

The temperature error is given by

\[ \Delta T = \frac{\partial T}{\partial R_T} \Delta R_T = \frac{R_T}{\partial R_T / \partial T} \Delta R_T \quad (4) \]

Plot the temperature \( T \) with errors (i.e., \( T \pm \Delta T \)) profile vs altitude (put altitude to y-axis). Try this for 1000, 20,000, and 100,000 shots of integration to see how the temperature errors change.

With this code, you may also vary lidar parameters, e.g., the telescope diameter, the detector quantum efficiency, or laser power, etc. to see how they affect the temperature errors under the same integration shots.

(9) Na Doppler lidar error analysis is very similar to the above procedure. In real data processing, we usually simplify the temperature error coefficient, i.e., do not count in the coefficient variation with operating points (i.e., \( T \) and \( V \) values) but use a nominal coefficient at \( T = 200 \) K and \( V_R = 0 \) m/s to estimate the errors. This nominal temperature coefficient is 202.8 for the Na Doppler lidar. Thus, the Na lidar temperature error can be estimated as Eq. (5), where \( \Delta R_T / R_T \) is also given by Eq. (3)

\[ \Delta T = 202.8 \times \frac{\Delta R_T}{R_T} \quad (5) \]

Please implement Eq. (5) into your Project #5 data processing code to calculate the Na temperature error and then plot the temperature with error bars for .001 profile. (Put altitude to y-axis) The photon counts used in the error analysis should be raw photon counts, i.e., without PMT, chopper, and range corrections.
Related atomic parameters are

- Molecular weight of $^{39}$K: 38.9637069
- Molecular weight of $^{41}$K: 40.96182597
- Molecular weight of standard K: 39.0983
- Abundance of $^{39}$K = 0.932581;
- Abundance of $^{41}$K = 0.067302;
- K D1 line central wavelength: 770.1088 nm (in vacuum)
- K D1 line $A_{\text{li}} = 0.382 \times 10^8$ (s$^{-1}$)
- K D1 line oscillator strength: 0.340

\[
\text{freq}_i^{39}(1) = 310.00983\text{e}^6; \quad \% \text{ relative to the D1 line center}
\]
\[
\text{freq}_i^{39}(2) = 252.84983\text{e}^6;
\]
\[
\text{freq}_i^{39}(3) = -151.7099\text{e}^6;
\]
\[
\text{freq}_i^{39}(4) = -208.8699\text{e}^6;
\]
\[
\text{freq}_i^{41}(1) = 405\text{e}^6;
\]
\[
\text{freq}_i^{41}(2) = 375\text{e}^6;
\]
\[
\text{freq}_i^{41}(3) = 151\text{e}^6;
\]
\[
\text{freq}_i^{41}(4) = 121\text{e}^6;
\]
\[
\text{strength}(1) = 5/16;
\]
\[
\text{strength}(2) = 1/16;
\]
\[
\text{strength}(3) = 5/16;
\]
\[
\text{strength}(4) = 5/16;
\]

Related Arecibo K Doppler lidar parameters are

- Laser RMS linewidth: 70 MHz
- Laser central frequency at K D1a: -180 MHz (relative to the line center)
- Acousto-Optic frequency shift: 477.6 MHz
- Laser frequency chirp (offset): 0 MHz (for this project, in reality it is not zero)
- Laser pulse energy: 100 mJ
- Laser repetition rate: 30.55 Hz
- Laser wavelength: 770.1088 nm (in vacuum)
- Transmitter mirror reflectivity: 99.8% for each mirror and total of 3 mirrors
- Telescope primary mirror diameter: 80 cm
- Primary mirror reflectivity: 91%
- Fiber throughput: 75%
- Transmission of receiver optics: 74%
- Interference filter peak transmission: 80%
- PMT quantum efficiency: 15%
- Geometric factor for above 20 km: 1

Related atmosphere parameters are

- Lower atmosphere transmission (from ground to 30 km) at 770 nm: 80%
- Atmospheric transmission from 30 to 75 km: 100%
- Atmospheric number density: taken from MSIS00 number density
- Atmospheric temperature: taken from MSIS00 number density
- K layer: Gaussian, peak at 91 km, rms width of 4.7 km
- K layer column abundance: $6 \times 10^7$ cm$^{-2}$
- K layer transmission/extinction: should be considered from the K absorption aspect

You are required to show your MatLab or equivalent code with your simulation results.