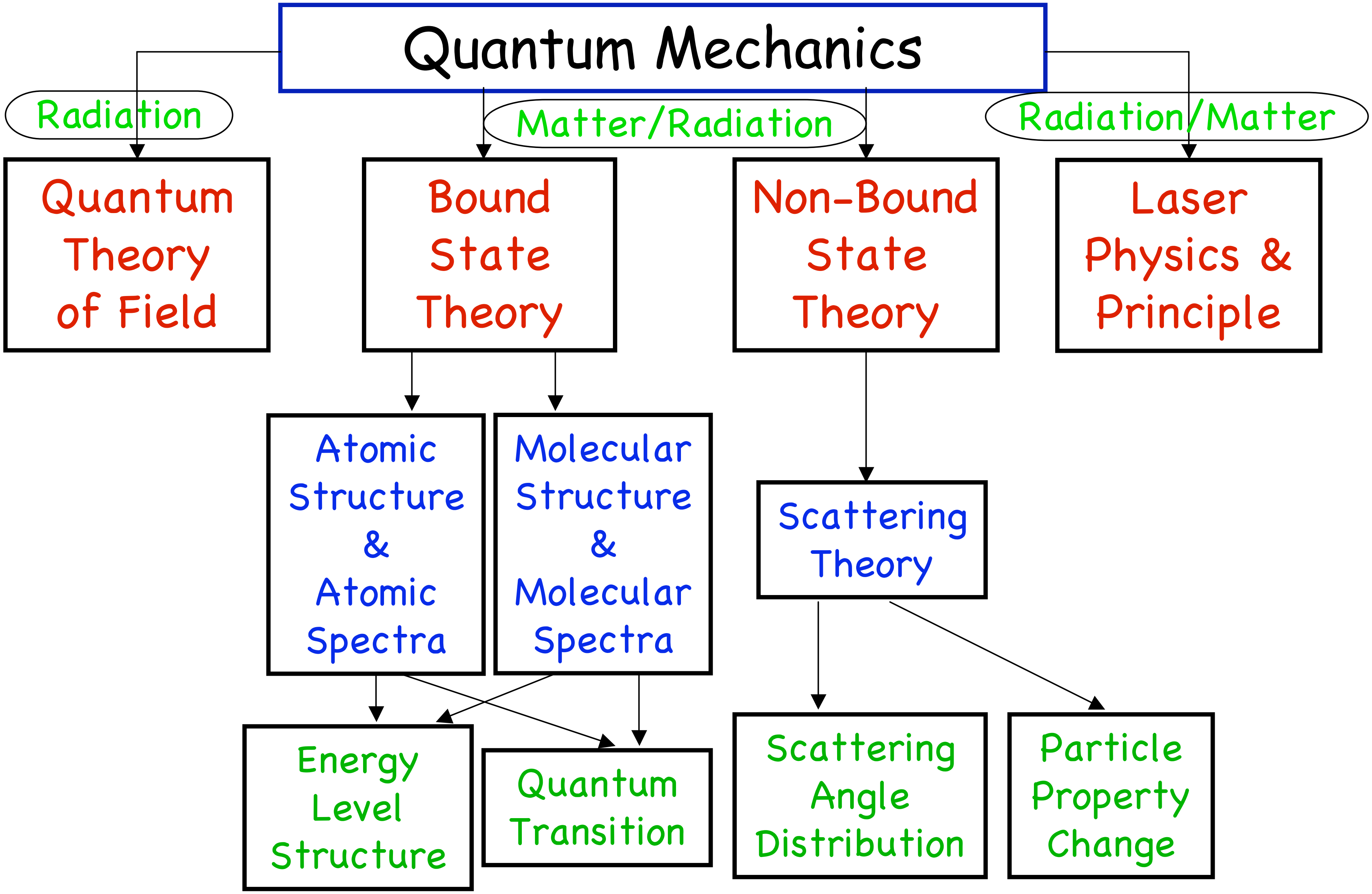
Quantum Physics



Lecture 4. Quantum Physics in Lidar Principle

- Quantum theory of light
- Basic concepts: eigenstate and superposition
- Equations of motion: Schrödinger equation
- Stationary states: Atomic structure

  – interaction versus energy levels
- Transition: Atomic spectra
- Alkali spectra

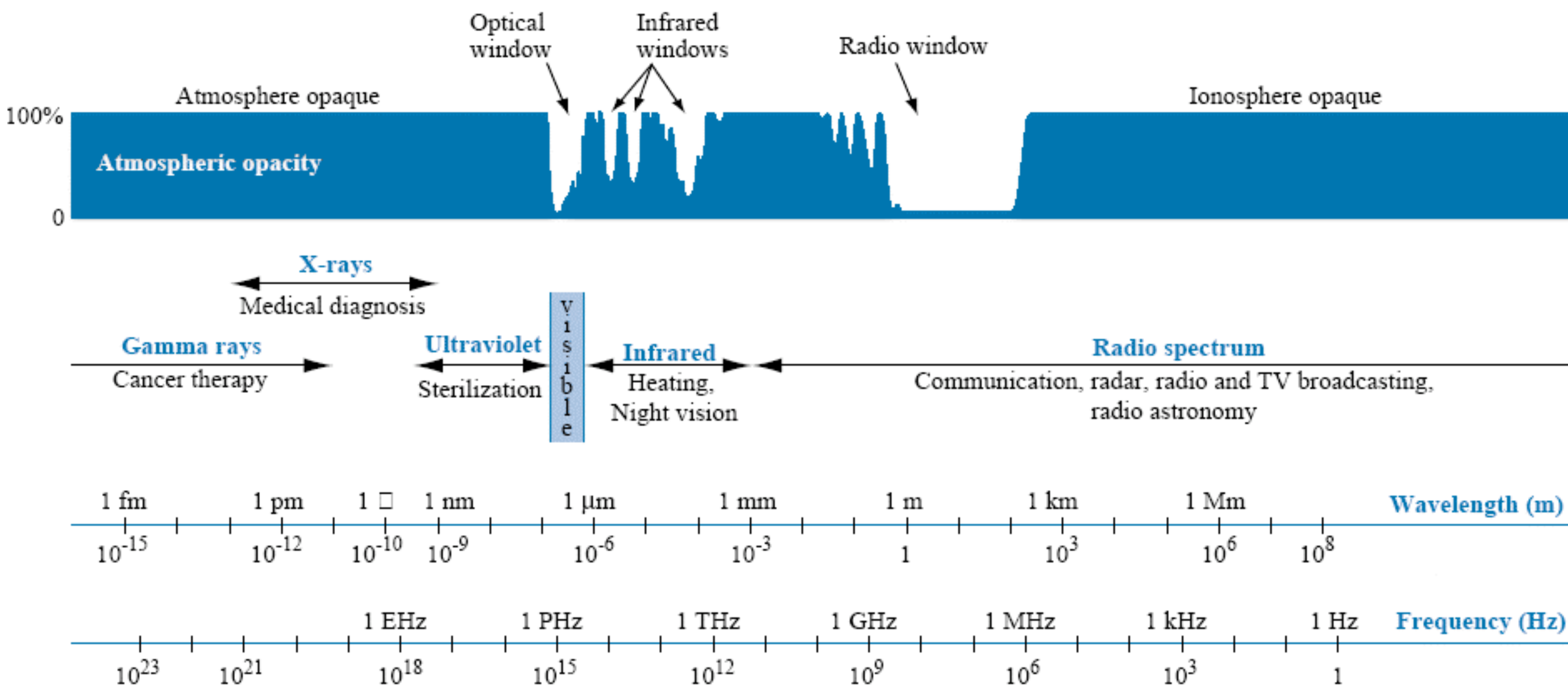
What is light: wave or particle?

Light is electromagnetic waves at optical frequency – light wave!

Light is photon streams with quantized energy and momentum.

How do we choose what to use?

Light as Electromagnetic Waves



Spectrum of electromagnetic waves

Visible wavelength is 400-700 nm, corresponding to 10^{14} Hz

Light compose of Photons

- Plank's quantization to explain blackbody radiation

$$\rho(\nu) = \frac{8\pi\nu^2}{c^3} \cdot \frac{h\nu}{e^{h\nu/kT} - 1}$$

$\rho(\nu)$ is the radiant energy density within a cavity in thermal equilibrium at temperature T

K is the Boltzmann constant = 1.381 x 10^{-23} J/K

ν is frequency, and c is the speed of light

h is the famous Plank constant = 6.626 x 10^{-34} Js

Plank found that the energy of an atomic oscillator in the walls of a blackbody needed to be quantized to integer numbers of $h\nu$ in order to describe the observed curve.

Photon: Quanta of Light

The quantized nature of electromagnetic field: the energy and momentum are not continuous, but are integer values of minimum quanta - the photons.

The particle-properties and wave-properties of light are connected by the following relationship

Photon energy $$E = h\nu$$

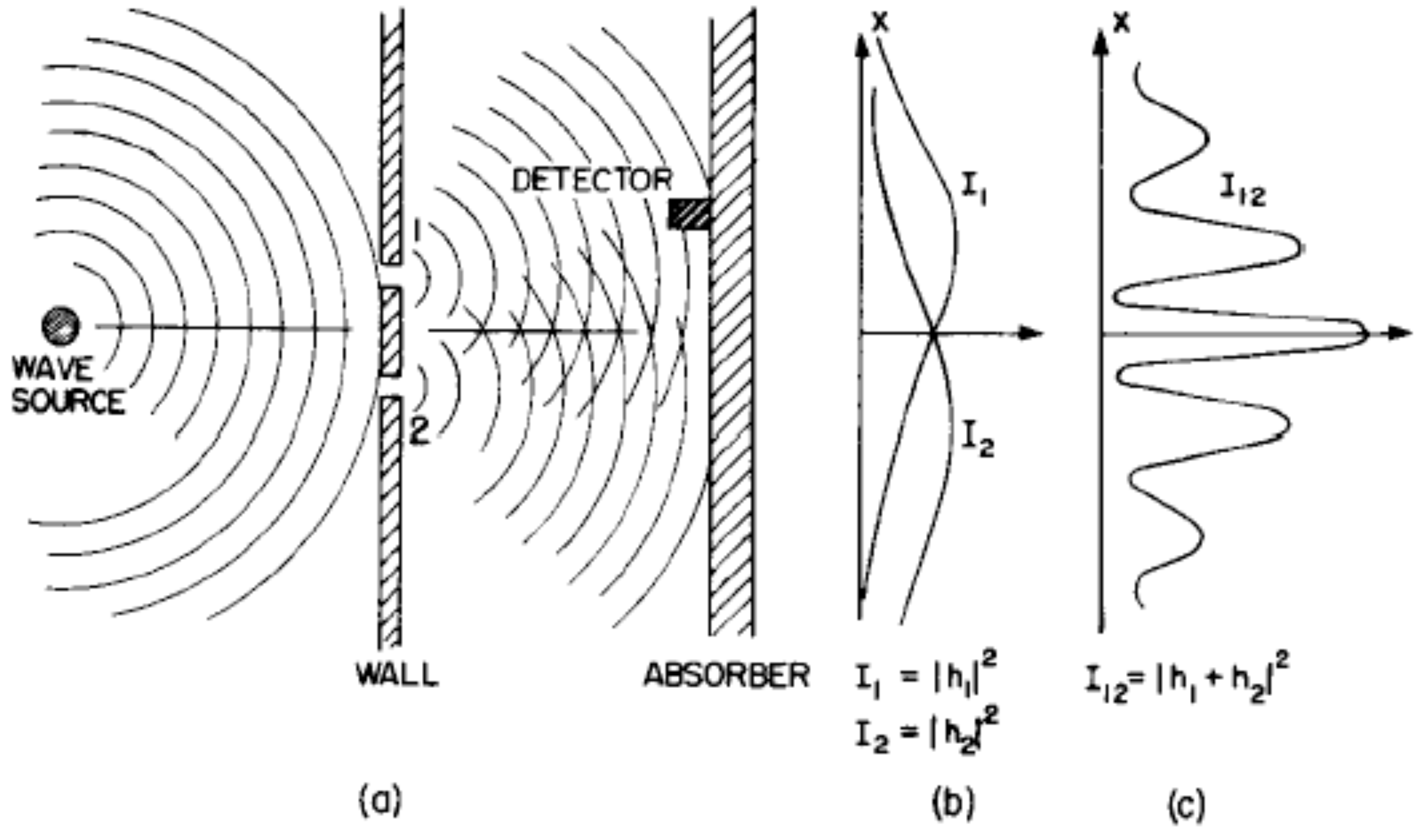
Photon momentum $$P = h / \lambda$$

ν is the light frequency and λ is the light wavelength

Light: Wave-Particle Duality

- Experimental proof of wave nature of light:
  - Young's double-split interference
  - Light diffraction, polarization
- Experimental proof of particle nature of light:
  - Blackbody radiation
  - Photo-electric effect
  - Compton scattering
  - Lamb-shift in H atom

Young's Interference Experiment

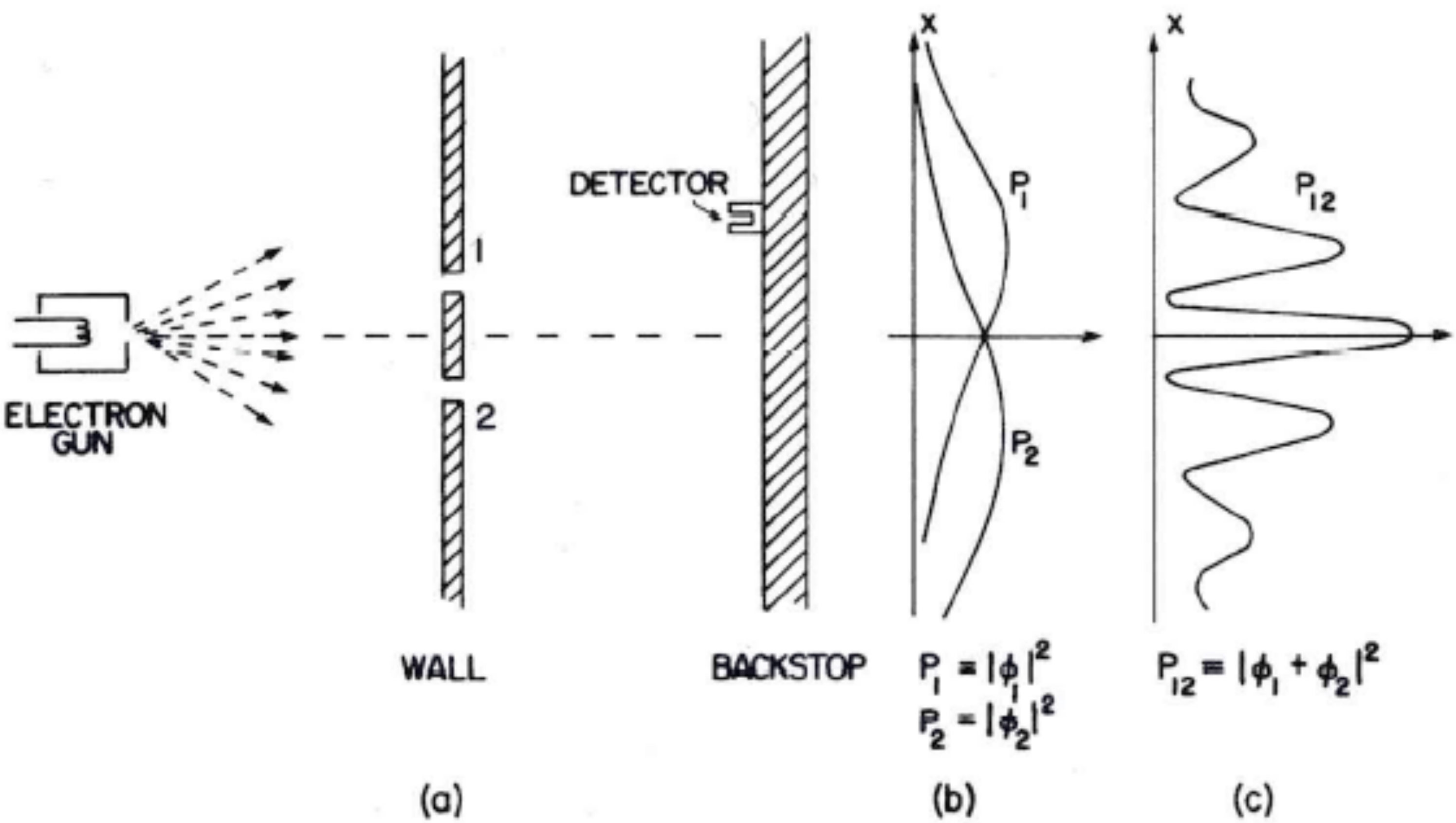


Interference experiment with light

Wave-Particle Duality of Light

- Essential particle nature is that energy and momentum are quantized.
- Essential wave nature is that the wave function represents the probability amplitude of single particle in certain state. The wave function can have wave interference.
- Statistical manner of the wave function can connect the particle nature with the wave nature.

Particle's Diffraction & Interference



Interference experiment with electrons

de Broglie Matter Wave

- Matter wave – de Broglie wave: all matter particles possess a dual (wave-particle) nature.
- The wavelength and momentum of a particle are related by the following equation

$$\lambda = h / P, \qquad \nu = E / h$$

λ is the wavelength of the matter wave
h is the Plank constant = 6.626×10^{-34} Js
P is the momentum of the particle
ν is the frequency of the matter wave
E is the energy of the particle

de Broglie Matter Wave

$$\lambda = \frac{h}{P} = \frac{h}{mv}\sqrt{1 - \frac{v^2}{c^2}}$$

λ is the wavelength of the matter wave

h is the Plank constant = 6.626×10^{-34} Js

P is the momentum of the particle

m is the mass of the particle

v is the speed of the particle

c is the speed of light

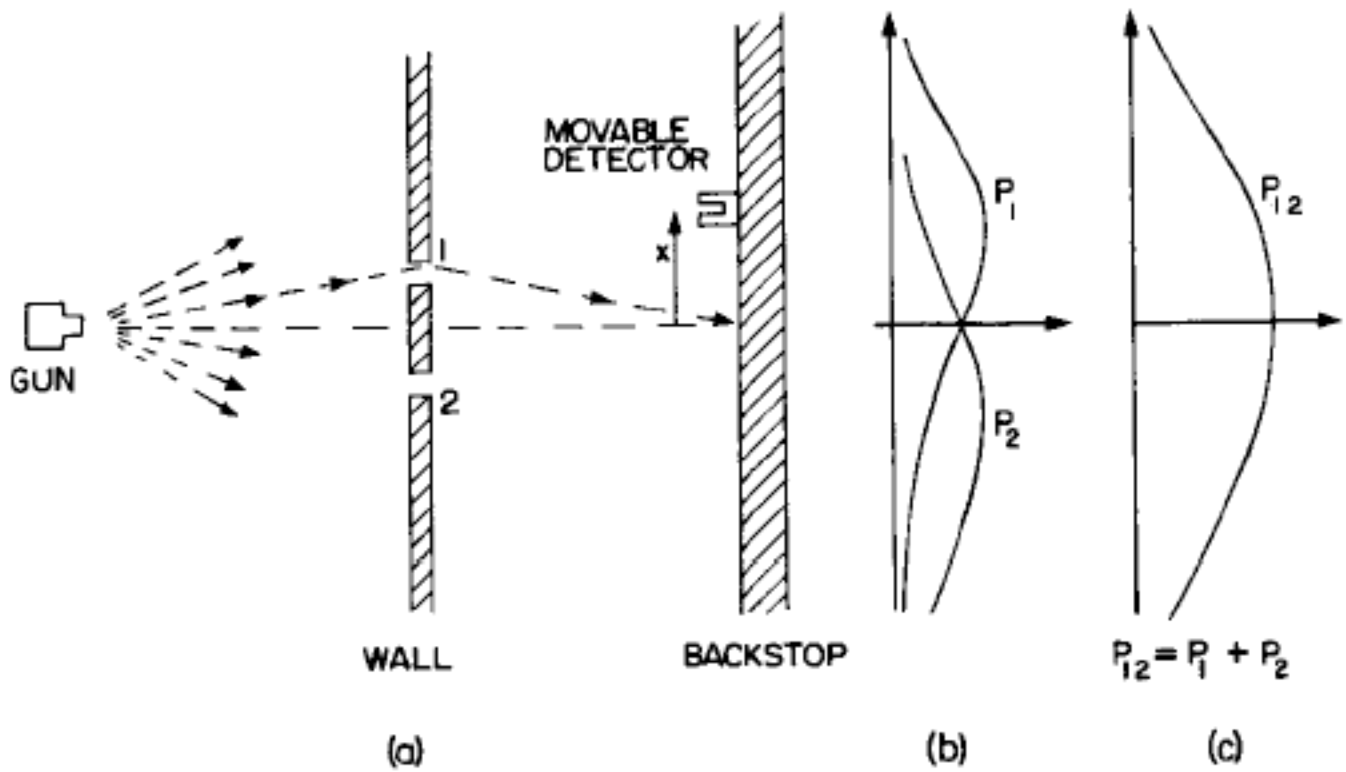
Dual Nature of Matter and Radiation
Wave-Particle Duality

- The wave-particle duality is universal to all radiations (like light) and matter (like all particles).

- The association of particles with waves is applicable to all matter. All kind of particles are associated with waves in this way and conversely all wave motion is associated with particles.

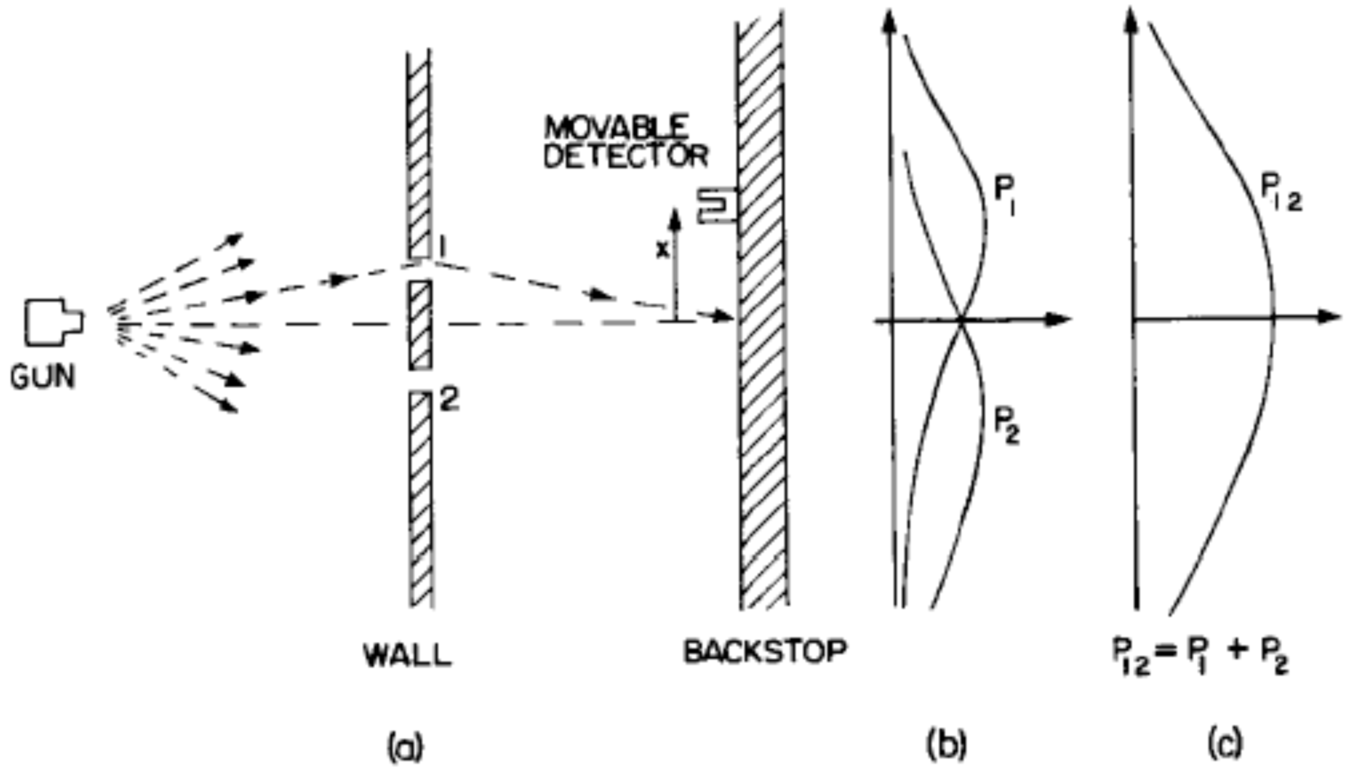
- All particles can be made to exhibit interference effects and all wave motion has its energy in the form of quanta.

Interference Experiment



Interference experiment with bullets

Why can't we see the wave behavior of normal particles in daily life?



Interference experiment with bullets

Quantum Mechanics Concepts

- States of a dynamical system at an instant of time: state vector
- Physical observables: operator

(energy, momentum, angular momentum, etc.)

- Measurement process corresponds to operation on the state vector by the operator
- Eigenstates and eigenvalues
- Principle of superposition of states
- Probability amplitude

Essential Concept of Quantum Mechanics

- Superposition of states is the key of quantum mechanics
- Polarization experiment as an example

Equations of Motion

Schrödinger Equation:

$$i\hbar \frac{d}{dt}\left|\psi\right\rangle = H(t)\left|\psi\right\rangle$$

H is the Hamiltonian function (operator)

$$i\hbar \frac{d}{dt}\left|\psi\right\rangle = \left[-\frac{h^2}{2\mu}\nabla^2 + V\right]\left|\psi\right\rangle$$

Stationary States

$$H|\psi\rangle = E|\psi\rangle$$

$$\left[-\frac{h^2}{2\mu}\nabla^2 + V\right]|\psi\rangle = E|\psi\rangle$$

Solve the time-independent Schrodinger equation to find the eigenstates and eigenvalues

Interaction between Electron and Nuclear

- Coulomb force between the electron and the nucleus (centripetal)
- Centripetal static-electric field produced by other electrons
- Magnetic interaction between electron spin and electron orbital angular momentum
- Influence of nucleus
- Influence of external fields

Principal, Angular Momentum, Spin

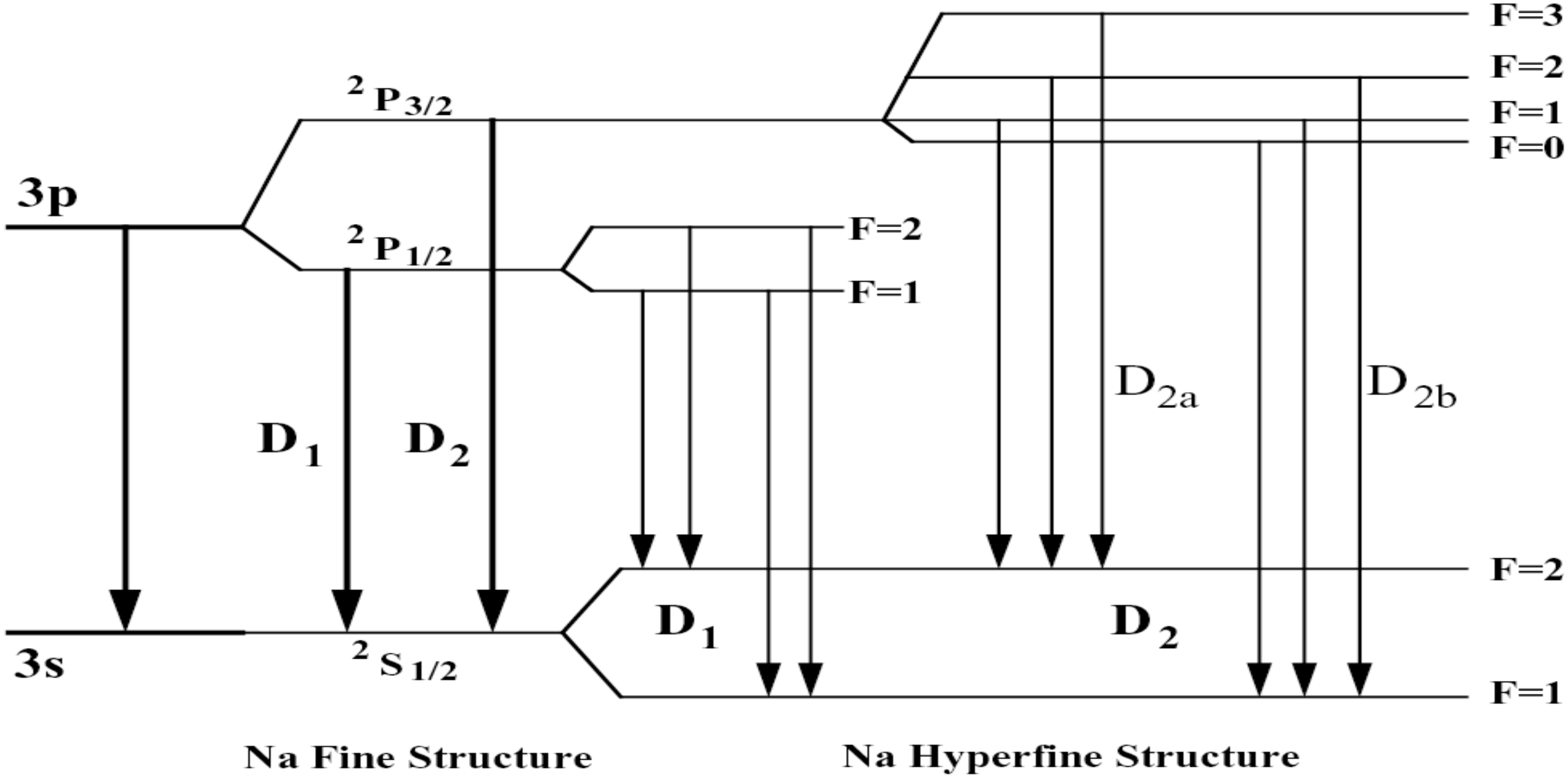
- Electronic states (energy levels)
- Fine structure
- Hyper-fine structure
- Magnetic energy levels
- Shift of energy levels by external fields

Quantum Transition

- Electric-dipole transition between electronic energy levels
- Magnetic-dipole transition between fine structure
- Electric-multiple-moment transition

Selection Rules

Alkali Atom Energy Level



Energy Level Diagram of Atomic Na

Summary

Reference Books

For optics, please refer to

Optics, by Eugene Hecht

For quantum mechanics, please refer to

The principles of Quantum Mechanics,

by P. A. M. Dirac, 4th edition, 1967.

Quantum Electronics, by Amnon Yariv

For atomic structure and spectra,

Laser and atomic spectra, by A. Corney

For laser, Lasers, by Peter W. Milonni

Homework for quantum theory of light

1. The light intensity for sunlight arriving at the top of Earth's atmosphere ($1.5x10^{11}$m from the Sun) is about $1.4kW/m^2$.
(a) Compute the average radiation pressure exerted on a metal reflector facing the Sun.
(b) Approximate the average radiation pressure at the surface of the Sun whose diameter is $1.4x10^9$m.

2. A pulsed laser (532 nm) runs at a repetition rate of 50 Hz. Each single pulse energy is 300 mJ.
(a) During 10 minutes operation time, how many photons are shot out?
(b) How much is the total momentum of photons sent out?
(c) The beam divergence is 1-mrad, and hits on a reflector mirror 100m away from the laser (assuming 100% reflectivity). How much radiation pressure does the laser beam exert on the mirror?

3. Compute the de Broglie wavelength for an electron traveling at 1000 m/s. Then assess to show the wave nature of an electron, what kind of dimension slit do you need?