Lecture 19. Temperature Lidar (3)

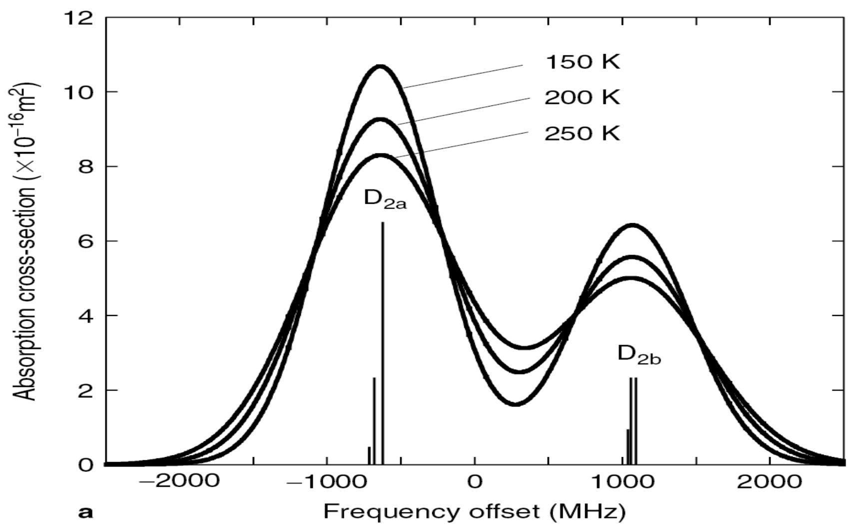
- Review of Doppler and Boltzmann Techniques
- Doppler Ratio Technique
- Accuracy versus Precision
- Error Analysis of Na Doppler Lidar

(Accuracy, Precision, Error Propagation)

- Rayleigh Integration Technique
- Error Analysis of Rayleigh Integration Lidar
- Rayleigh Lidar Instrumentation
- Summary

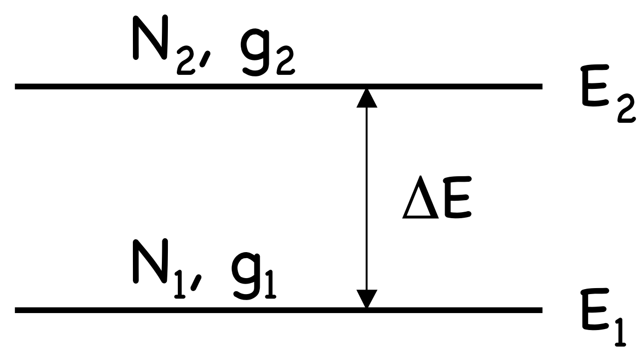
Review of Doppler & Boltzmann

- Doppler effect and Boltzmann distribution are two effects that are directly temperature-dependent. The Doppler technique and Boltzmann technique are "straight-forward" in the sense of deriving temperature or wind. However, the lidar architecture is usually complicated and sophisticated, due to the high demands on frequency accuracy, linewidth, and power combination.



$$\Delta\omega = \omega - \omega_0 = -\vec{k} \cdot \vec{v} = -\omega_0 \frac{v\cos\theta}{c}$$

$$\sigma_{rms} = \frac{\omega_0}{c}\sqrt{\frac{k_B T}{M}} = \frac{1}{\lambda_0}\sqrt{\frac{k_B T}{M}}$$



$$\frac{N_2}{N_1} = \frac{g_2}{g_1}\exp\left\{-\left(E_2 - E_1\right)/k_B T\right\}$$

$$T = \frac{\Delta E / k_B}{\ln\left(\frac{g_2}{g_1} \cdot \frac{N_1}{N_2}\right)}$$

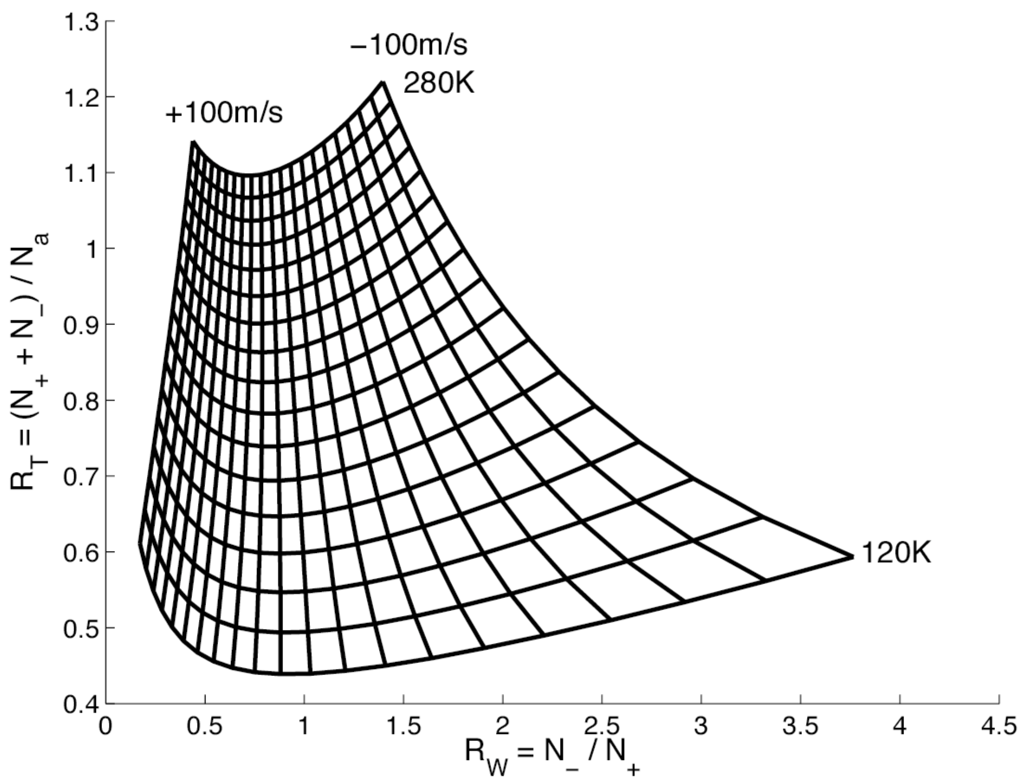
Doppler Ratio Technique

- From physics, the ratios of R_T and R_W are given by

$$R_T = \frac{N_{Norm}(f_+,z) + N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)} = \frac{\sigma_{eff}(f_+,z) + \sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}$$

$$R_W = \frac{N_{Norm}(f_-,z)}{N_{Norm}(f_+,z)} = \frac{\sigma_{eff}(f_-,z)}{\sigma_{eff}(f_+,z)}$$

- Compute Doppler calibration curves from physics



Doppler Ratio Technique

- From actual photon counts, we have the ratios calculated:

$$R_T = \frac{N_{Norm}(f_+,z) + N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}$$

$$= \frac{\left(\frac{N_S(f_+,z)-N_B}{N_S(f_+,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{E^2(f_+,z)} - \frac{n_R(z)}{n_R(z_R)}\right) + \left(\frac{N_S(f_-,z)-N_B}{N_S(f_-,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{E^2(f_-,z)} - \frac{n_R(z)}{n_R(z_R)}\right)}{\frac{N_S(f_a,z)-N_B}{N_S(f_a,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{E^2(f_a,z)} - \frac{n_R(z)}{n_R(z_R)}}$$

$$R_W = \frac{N_{Norm}(f_-,z)}{N_{Norm}(f_+,z)} = \frac{\frac{N_S(f_-,z)-N_B}{N_S(f_-,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{E^2(f_-,z)} - \frac{n_R(z)}{n_R(z_R)}}{\frac{N_S(f_+,z)-N_B}{N_S(f_+,z_R)-N_B}\frac{z^2}{z_R{}^2}\frac{1}{E^2(f_+,z)} - \frac{n_R(z)}{n_R(z_R)}}$$

- Look up these two ratios on the calibration curves to infer the corresponding Temperature and Wind.

Na Density Derivation

- The Na density can then be inferred either from the peak frequency signal or from a weighted average of all three frequency signals.
- The weighted effective cross-section is

$$\sigma_{eff\_wgt} = \sigma_a + \alpha\sigma_+ + \beta\sigma_-$$

where α and β are chosen so that

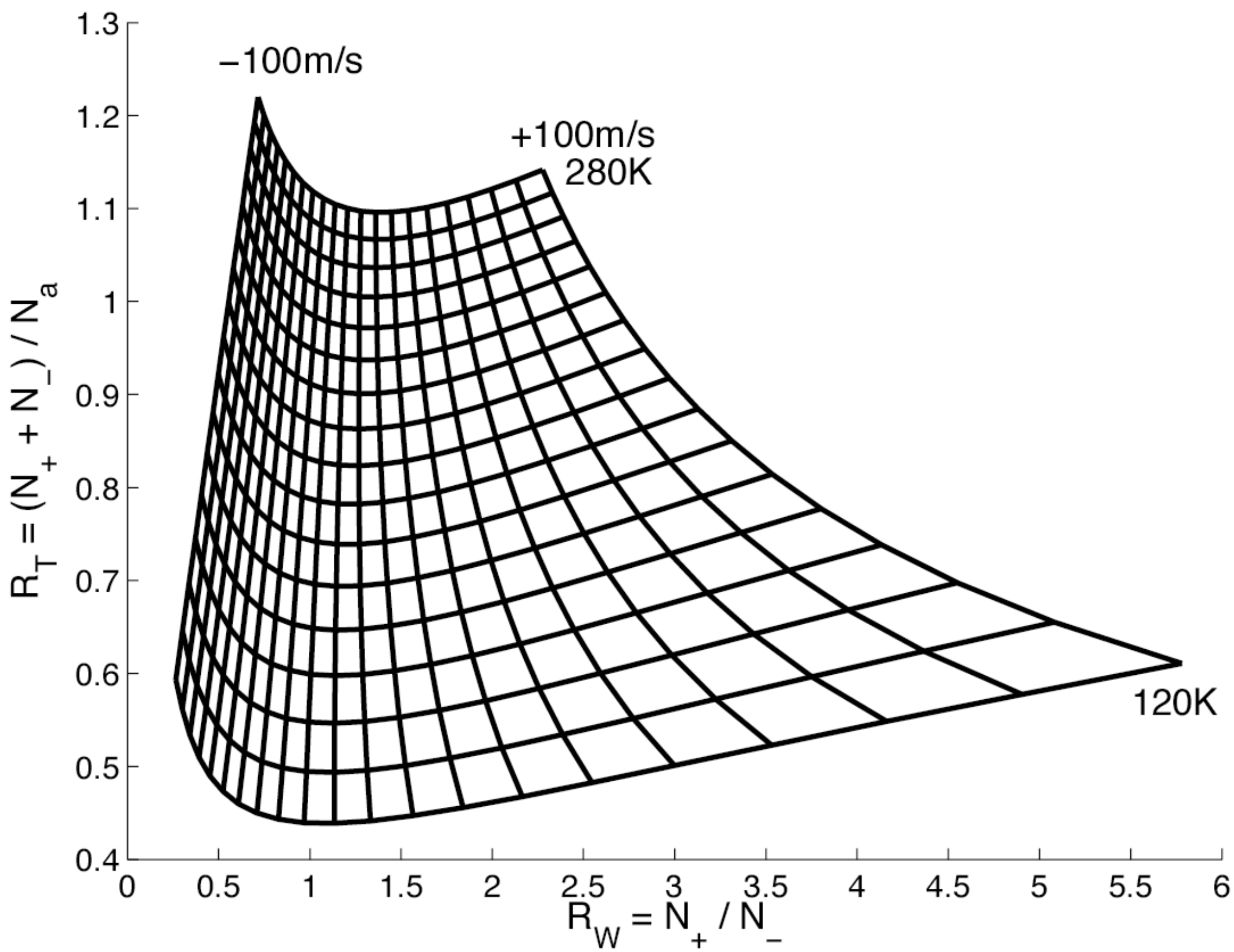
$$\frac{\partial \sigma_{eff\_wgt}}{\partial T} = 0; \qquad \frac{\partial \sigma_{eff\_wgt}}{\partial v_R} = 0$$

- The Na density is then calculated by

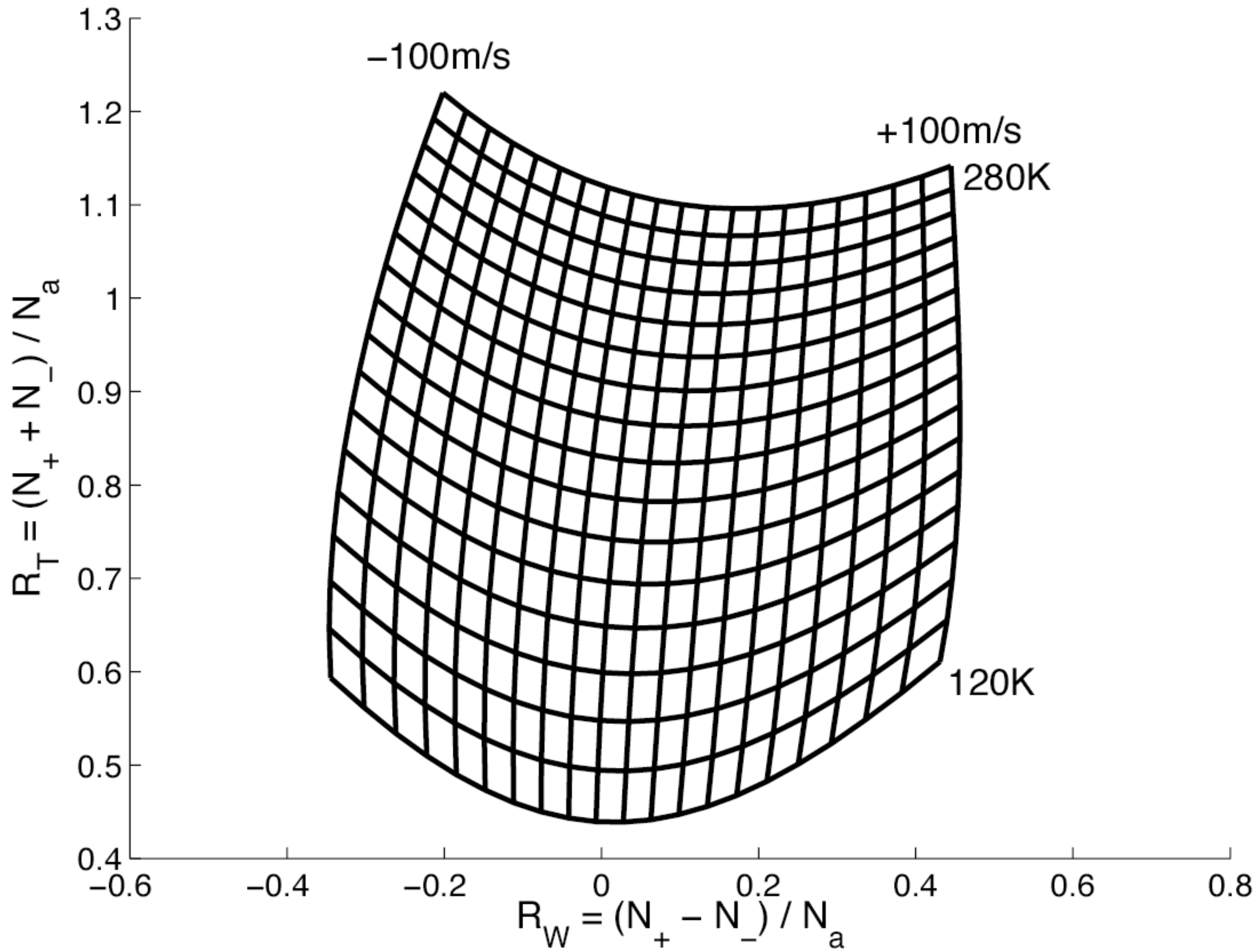
$$n_{Na}(z) = 4\pi n_R(z_R)\sigma_R \frac{z^2}{z_R^2} \frac{N_{norm}(f_a, z) + \alpha N_{norm}(f_+, z) + \beta N_{norm}(f_-, z)}{\sigma_a + \alpha\sigma_+ + \beta\sigma_-}$$

Comparison of Different R_W Metrics

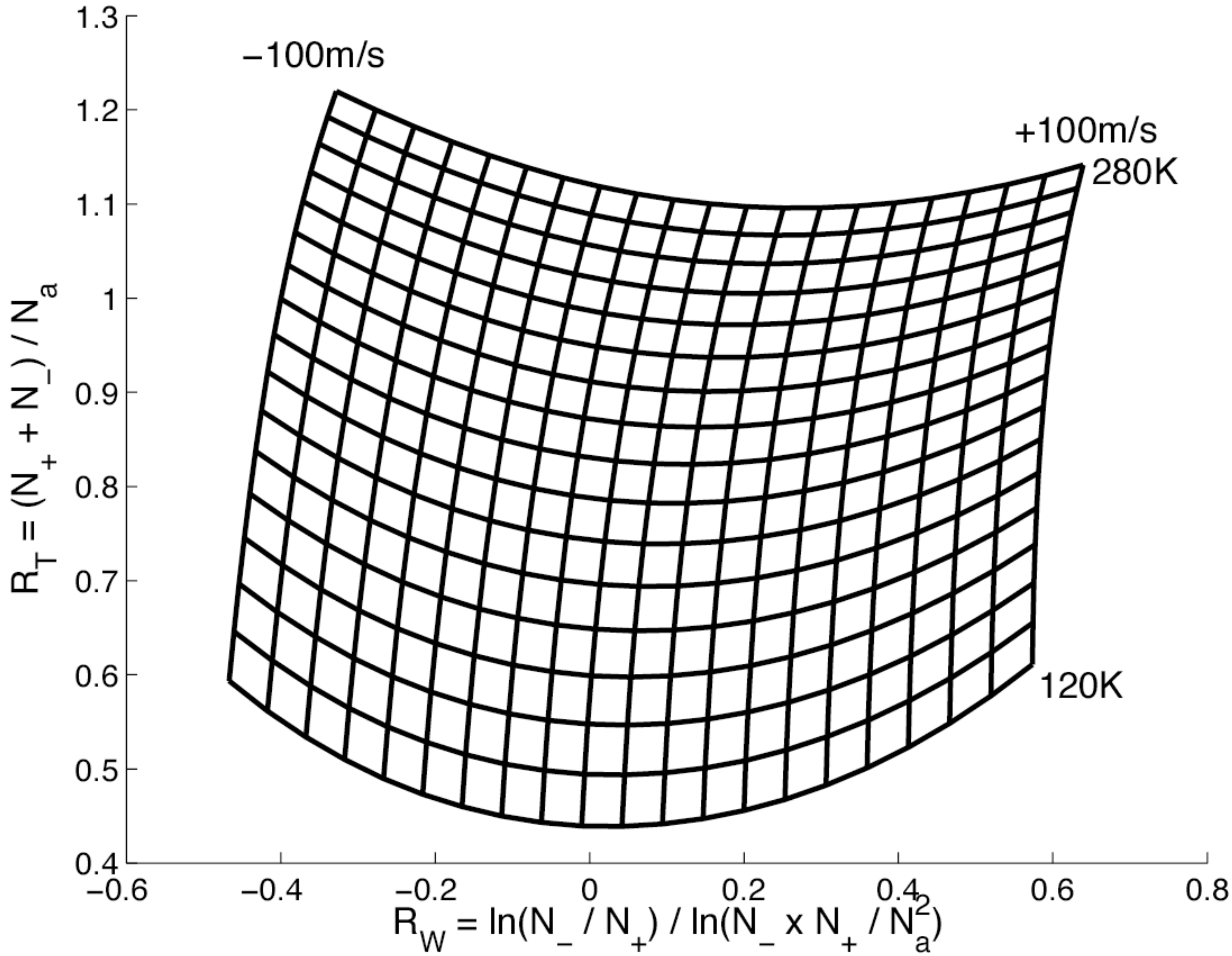
- Different metrics of R_W result in different wind sensitivities
- The ratio $R_W = N_+/N_-$ has inhomogeneous sensitivity



❑ CSU ratio R_W= $(N_+ - N_-)/N_a$ has much better uniformity than the simplest ratio



❑ UIUC ratio $R_W = \ln(N_- / N_+)/\ln(N_- \times N_+ / N_a^2)$ has good uniformity



Sensitivity for T and W

- The temperature and wind sensitivities are defined as

$$S_T = \frac{\partial R_T / \partial T}{R_T} = \frac{\Delta R_T / R_T}{\Delta T}$$

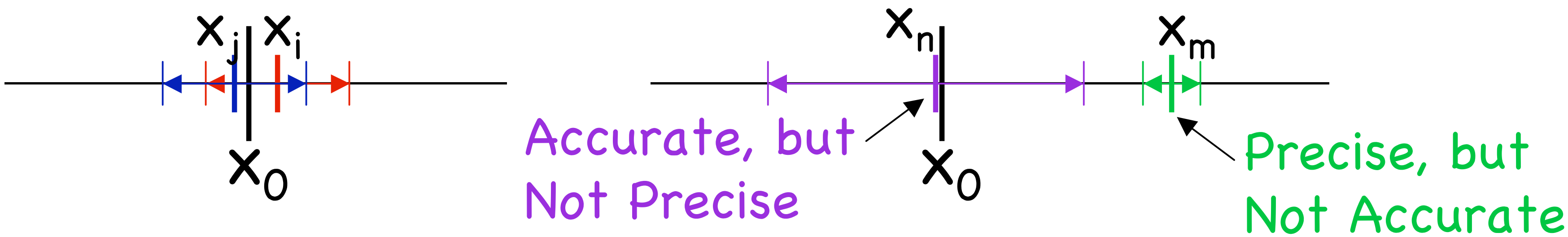
$$S_W = \frac{\partial R_W / \partial W}{R_W} = \frac{\Delta R_W / R_W}{\Delta W}$$

- PDA frequency offset: usually nonzero, so must be taken into account. For AR1102 data, the freq offset is 10.27MHz.

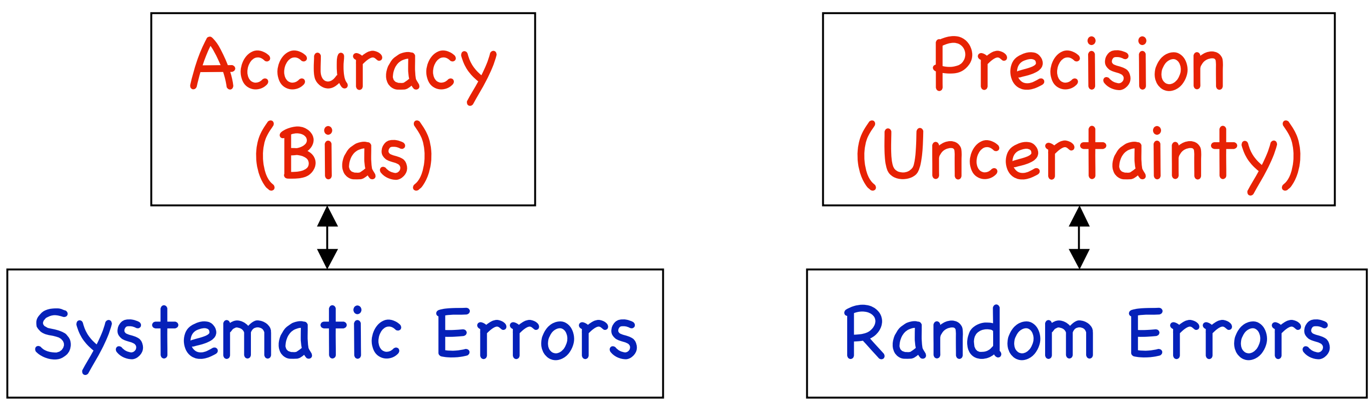
Actual laser freq = CW laser freq + PDA freq offset

Lidar Error Analysis

- For any measurement, the results are commonly supposed to be a mean value with a confidence range: $x_i \pm \Delta x_i$



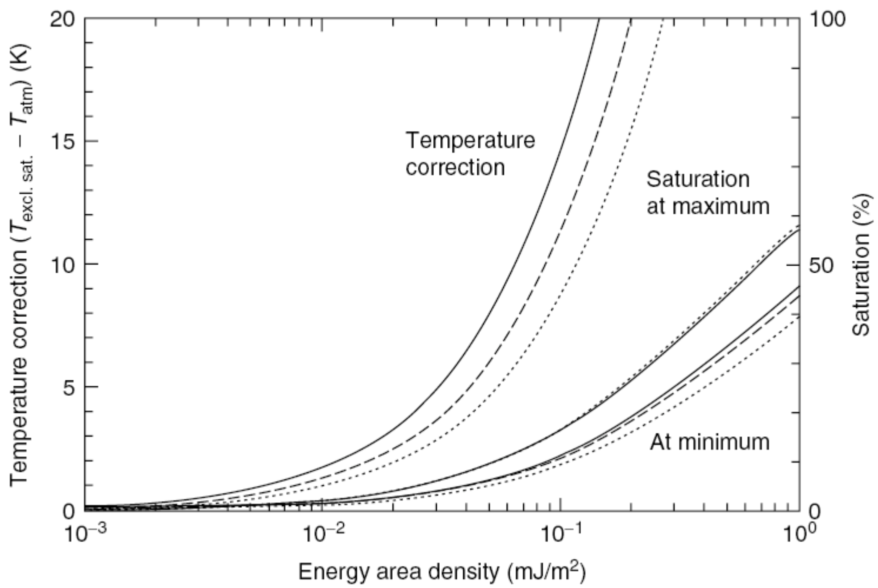
- In error analysis, accuracy and precision are two different concepts. Accuracy is concerned about bias, i.e., how far away is the measurement result from the true valu? Precision is concerned about uncertainty, i.e., how certain or how sure are we about the measurement result?



Error Analysis: Accuracy

- Systematic errors determine the measurement accuracy.
- Possible sources: imprecise information of (1) atomic absorption cross-section, (2) laser absolute frequency calibration, (3) laser lineshape, (4) receiver filter function, (5) geometric factor.
- Determination of $\sigma_{abs}(\nu)$: Hanle effect, Na layer saturation, and optical pumping effect.

Hanle effect modified A_n:
5, 5, 2, 14, 5, 1 →
5, 5.48, 2, 15.64, 5, 0.98



Na Layer Saturation

- Absolute laser frequency calibration and laser lineshape.
- Receiver filter function and geometric factor.

Error Analysis: Precision

- Random errors determine the measurement precision.
- Possible sources: (1) random uncertainty associated with laser jitter and electronic jitter, (2) shot noise associated with photon-counting system. The latter ultimately limits the precision because of the statistic nature of photon-detection processes.
- In normal lidar photon counting, photon counts obey Poisson distribution. Therefore, for a given photon count N, the corresponding uncertainty is $\Delta N = \sqrt{N}$
- For three frequency technique, the relative errors are

$$\frac{\Delta R_T}{R_T} = \frac{\left(1+\frac{1}{R_T}\right)^{1/2}}{\left(N_{f_a}\right)^{1/2}}\left[1+\frac{B}{N_{f_a}}\frac{\left(1+\frac{2}{R_T{}^2}\right)}{\left(1+\frac{1}{R_T}\right)}\right]^{1/2}$$

$$\frac{\Delta R_W}{R_W} = \frac{\left(1+\frac{1}{R_W}\right)^{1/2}}{\left(N_{f_+}\right)^{1/2}}\left[1+\frac{B}{N_{f_+}}\frac{\left(1+\frac{1}{R_W{}^2}\right)}{\left(1+\frac{1}{R_W}\right)}\right]^{1/2}$$

Calculation of Errors: Error Propagation

- Systematic and random errors will propagate to the measurement errors of temperature and wind. T and W errors can be derived by the use of differentials of the corresponding ratios R_T and R_W.

- For 2-frequency technique,

$$R_T(f_a, f_c, T, v_R, \sigma_L) = \frac{\sigma_{eff}(f_c, T, v_R, \sigma_L)}{\sigma_{eff}(f_a, T, v_R, \sigma_L)}$$

- Temperature errors are given by the derivatives

$$\Delta T = \frac{\partial T}{\partial R_T}\Delta R_T + \frac{\partial T}{\partial f_a}\Delta f_a + \frac{\partial T}{\partial f_c}\Delta f_c + \frac{\partial T}{\partial \sigma_L}\Delta \sigma_L + \frac{\partial T}{\partial v_R}\Delta v_R$$

- Using implicit differentiation, we have

$$\Delta T = \Delta R_T\left(\frac{\partial R_T}{\partial R_T}\bigg/\frac{\partial R_T}{\partial T}\right) + \Delta f_a\left(\frac{\partial R_T}{\partial f_a}\bigg/\frac{\partial R_T}{\partial T}\right) + \Delta f_c\left(\frac{\partial R_T}{\partial f_c}\bigg/\frac{\partial R_T}{\partial T}\right) + \Delta \sigma_L\left(\frac{\partial R_T}{\partial \sigma_L}\bigg/\frac{\partial R_T}{\partial T}\right) + \Delta v_R\left(\frac{\partial R_T}{\partial v_R}\bigg/\frac{\partial R_T}{\partial T}\right)$$

Calculation of Errors: Error Propagation

- The derivatives of R_T to each system parameters are

$$\frac{\partial R_T}{\partial x} = R_T \left[\frac{\partial \sigma_{eff}(f_c)/\partial x}{\sigma_{eff}(f_c)} - \frac{\partial \sigma_{eff}(f_a)/\partial x}{\sigma_{eff}(f_a)} \right]$$

- For example, the uncertainty in R_T caused by photon noise results in the temperature error:

$$\Delta T = \frac{\partial T}{\partial R_T} \Delta R_T = \frac{\Delta R_T}{R_T} \left[\frac{\partial \sigma_{eff}(f_c)/\partial T}{\sigma_{eff}(f_c)} - \frac{\partial \sigma_{eff}(f_a)/\partial T}{\sigma_{eff}(f_a)} \right]^{-1}$$

Where $\Delta R_T/R_T$ is determined photon counts of both signals and background, and the bracket gives the coefficient of ΔT to $\Delta R_T/R_T$.

- This differentiation of metric ratio method applies to both systematic and random errors, depending on the error sources: are they systematic bias or random jitter?
- For example, the error in f_a can be systematic bias and random jitter, which will lead to systematic error and uncertainty, respectively.

Rayleigh Integration Technique

- Molecular scattering is proportional to the atmospheric density, so the temperature profile can be derived from the relative atmospheric density profile using the Rayleigh lidar technique.
- The use of high power lasers with the Rayleigh technique was pioneered by *Hauchecorne and Chanin* [1980]. The data analysis approach is very similar to that employed by *Elterman* in the early 1950s to measure stratospheric temperatures with search light technique [*Elterman*, 1951, 1953, 1954].
- This involves integrating the relative density profile downward using a starting temperature at the highest altitude in combination with the hydrostatic equation and the ideal gas law. The starting temperature may be chosen from a model because when the equation has been integrated downward by about one and half scale heights (atmospheric scale height is the altitude range in which density decreases by the factor 1/*e*), the calculated temperature is relatively insensitive to the starting estimation.

Rayleigh Integration Technique

- The hydrostatic equation $$\mathrm{d}P(z) = -\rho(z)g(z)\,\mathrm{d}z$$
- Ideal gas law $$P(z) = \frac{\rho(z)RT(z)}{M(z)}$$
- Integration from the upper altitude yields

$$T(z) = T(z_0)\frac{\rho(z_0)}{\rho(z)}\frac{M(z)}{M(z_0)} + \frac{M(z)}{R}\int_z^{z_0}\frac{\rho(z')g(z')}{\rho(z)}\,\mathrm{d}z'$$

T(z) = atmospheric temperature profile (K)

P(z) = atmospheric pressure profile (mbar)

ρ(z) = atmospheric mass density profile (kg/m^3)

g(z) = gravitational acceleration (m/s^2)

M(z) = mean molecular weight of the atmosphere

R = universal gas constant (8.31432 J/mol/K))

z_0 = altitude of the upper level starting temperature (m)

Rayleigh Integration Technique

- Atmos mass density vs number density $\rho(z) = n(z)M(z)/N_A$

where N_A is the Avogadro constant

- Thus, we have

$$T(z) = T(z_0)\frac{n(z_0)}{n(z)} + \frac{M(z)}{R}\int_z^{z_0} \frac{n(z')M(z')g(z')}{n(z)M(z)}\,\mathrm{d}z'$$

- Below 100 km for the well-mixed atmosphere, we have M(z) = M(z'), so they cancel out in the integration

$$T(z) = T(z_0)\frac{n(z_0)}{n(z)} + \frac{M(z)}{R}\int_z^{z_0} \frac{n(z')g(z')}{n(z)}\,\mathrm{d}z'$$

- Number density ratio (relative number density)

⇨ Temperature profile

Error Analysis for Rayleigh Tech

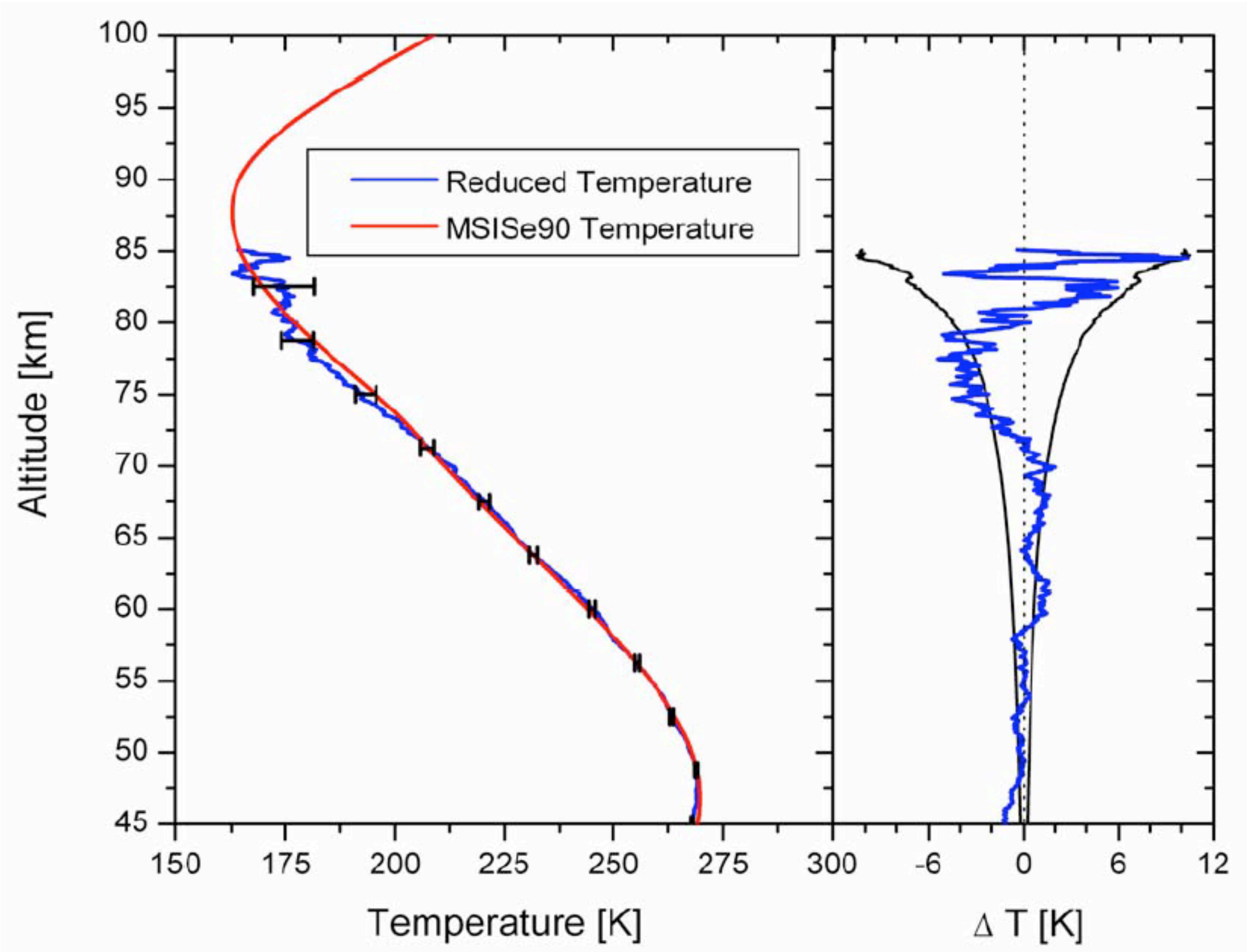
- The uncertainty is determined by the photon noise and upper altitude temperature $T(z_0)$. The variance of derived temperature is given by

$$\mathrm{var}[T(z)] \approx \frac{T^2(z)}{N_R(z)} + \left\{ \mathrm{var}[T(z_0)] + \frac{T^2(z_0)}{N_R(z_0)} \right\} \exp[-2(z_0 - z)/H]$$

- After 1-2 scale height, the error introduced by $T(z_0)$ is not important anymore. So the temperature error is mainly determined by the photon counts and their noise.

- The accuracy of Rayleigh temperature is largely determined by the hardware - whether PMT is saturated and whether the saturation causes non-flat background, etc.

Sample of Rayleigh T and Errors



Courtesy of Josh Herron and Prof. Vincent Wickwar @ USU

Rayleigh Lidar Instrumentation

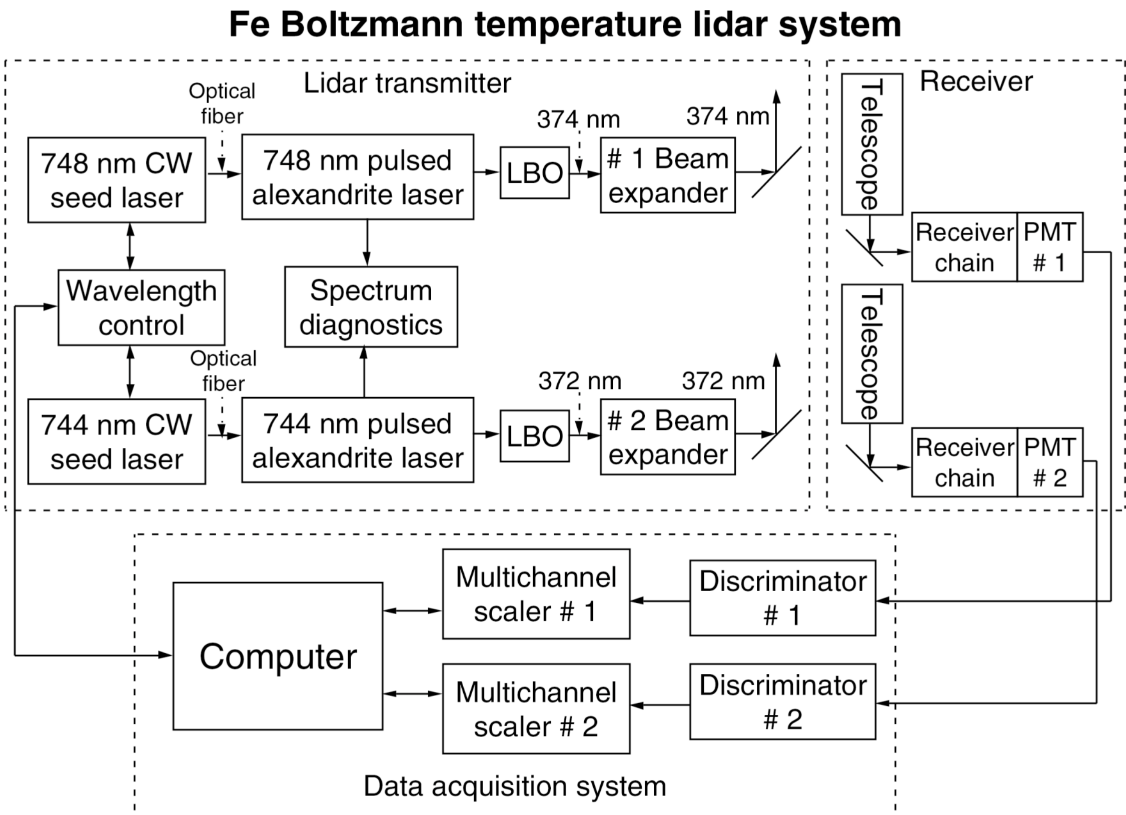
- Typical Rayleigh temperature lidar utilizes the commercial Nd:YAG laser system as it provides robust laser power and operation. But (short wavelength) resonance fluorescence lidar, like Fe Boltzmann lidar, also functions as a Rayleigh in the region free of aerosol and fluorescence (about 30-70 km).

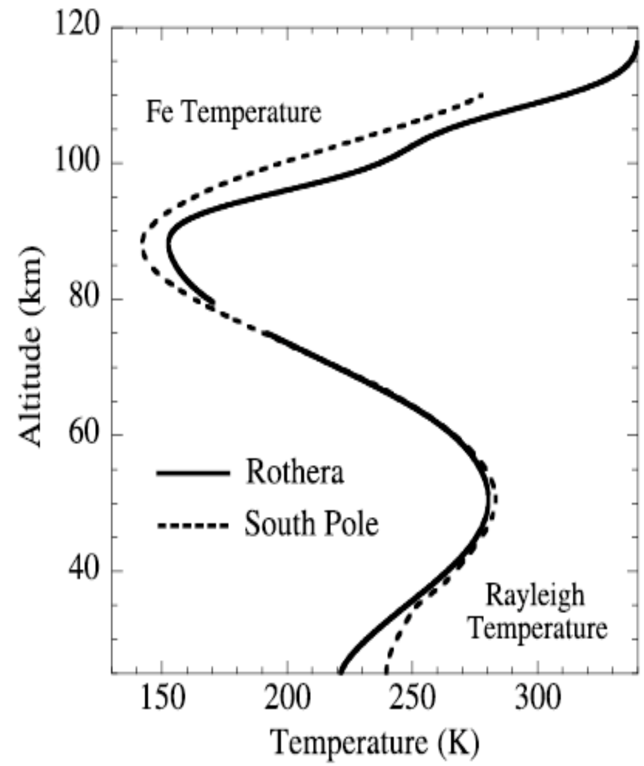
- Rayleigh scattering is inversely proportion to the 4th power of wavelength. So the shorter the wavelength, the stronger the Rayleigh scattering, as long as atmosphere absorption is not too strong.

- Operating in deep Fraunhofer lines will benefit daytime operation to reduce the solar background.

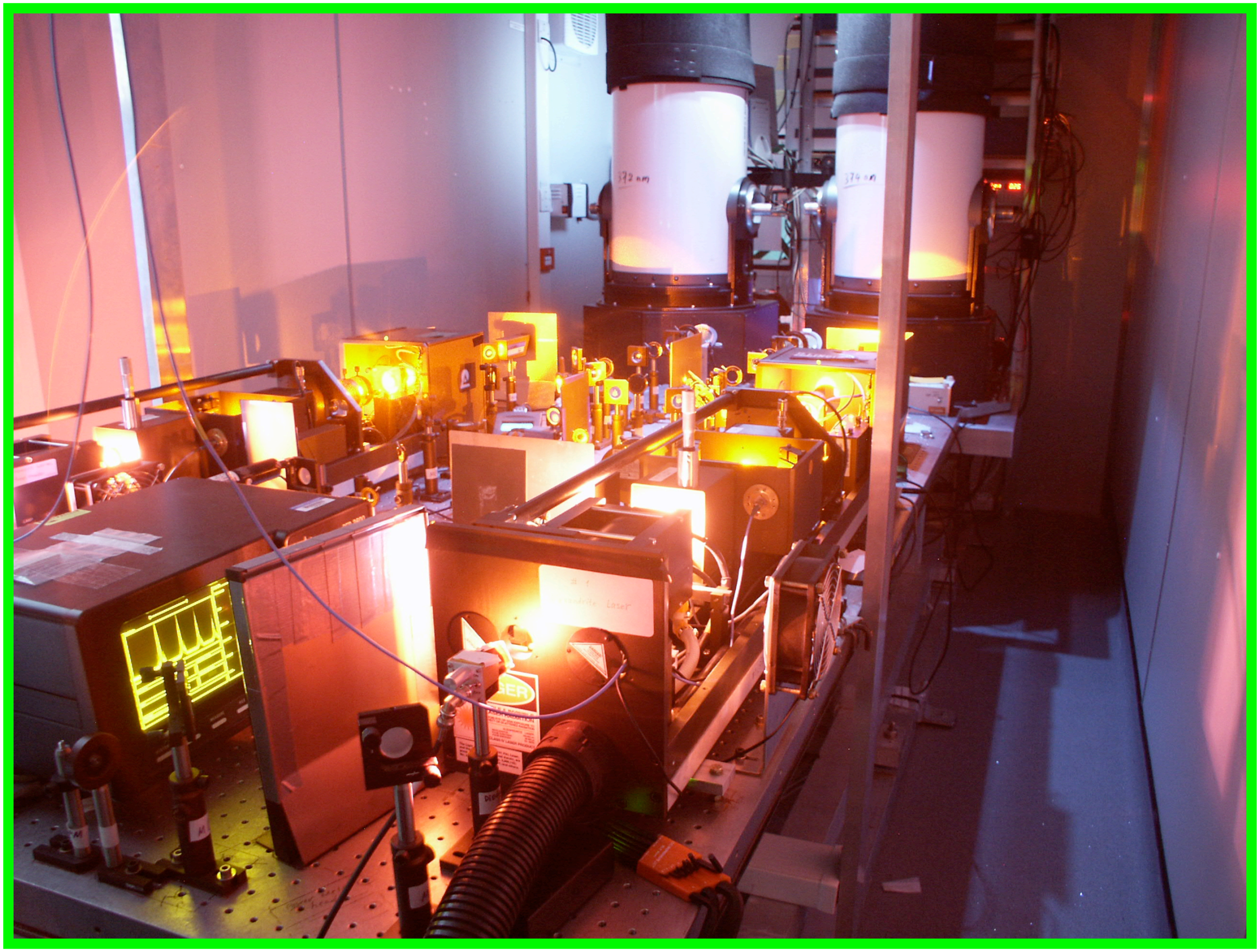
- Availability and robustness of laser systems are another consideration in lidar design.

Fe Boltzmann/Rayleigh Lidar



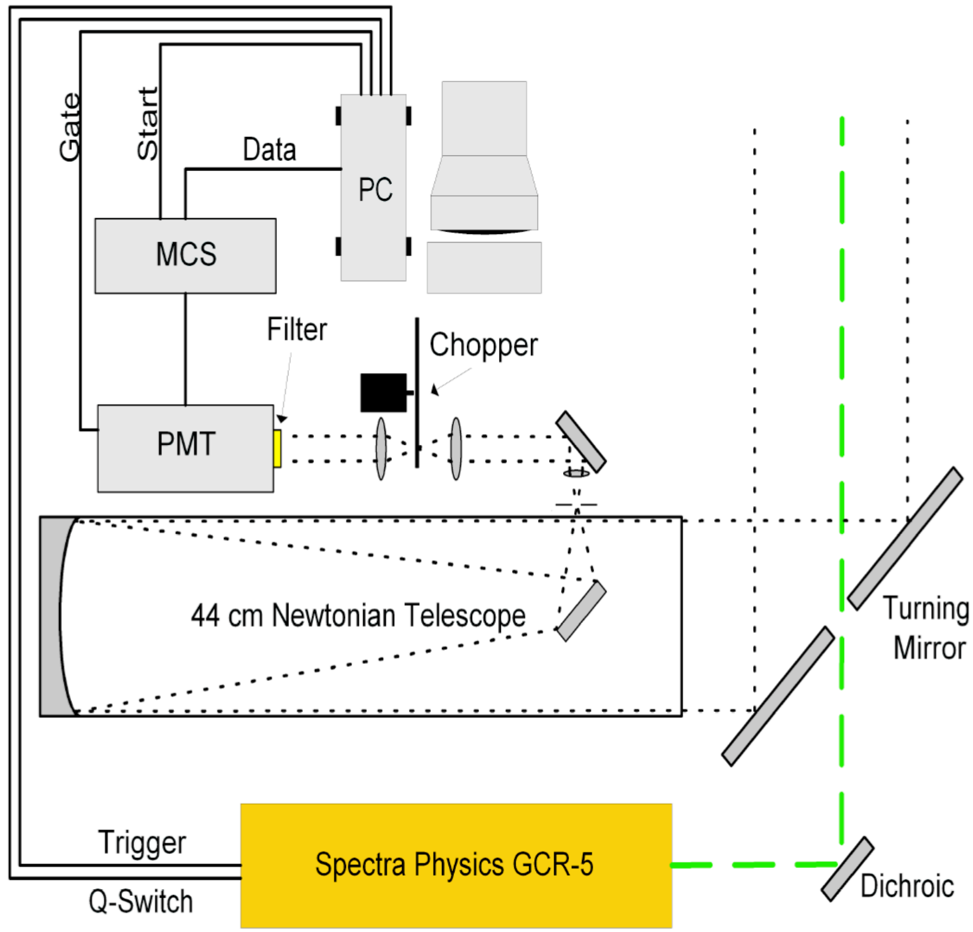


Fe Boltzmann/Rayleigh Lidar

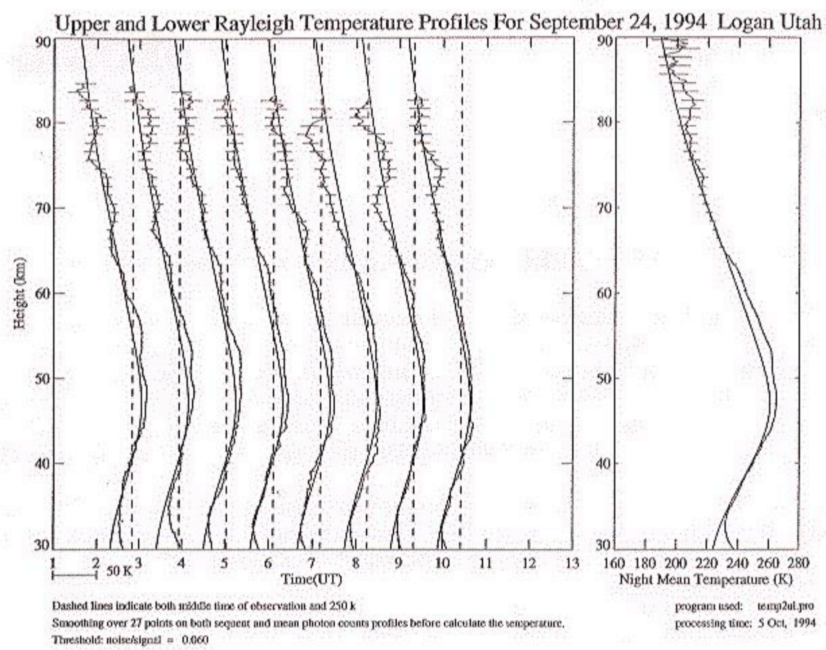


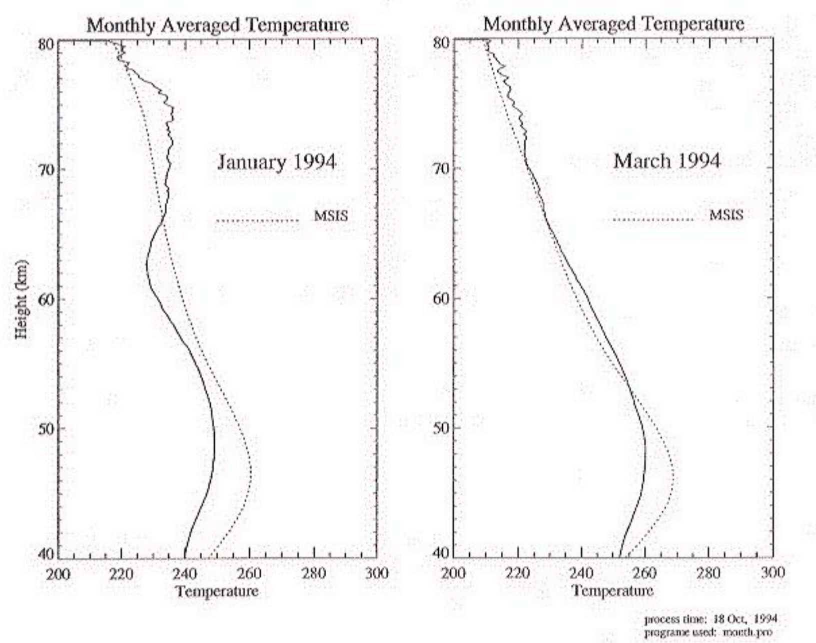
Utah State University Rayleigh Lidar

- Doubled Nd:YAG laser at 532 nm (630 mJ/pulse, 30 Hz)



http://www.usu.edu/alo/aboutlidar.htm



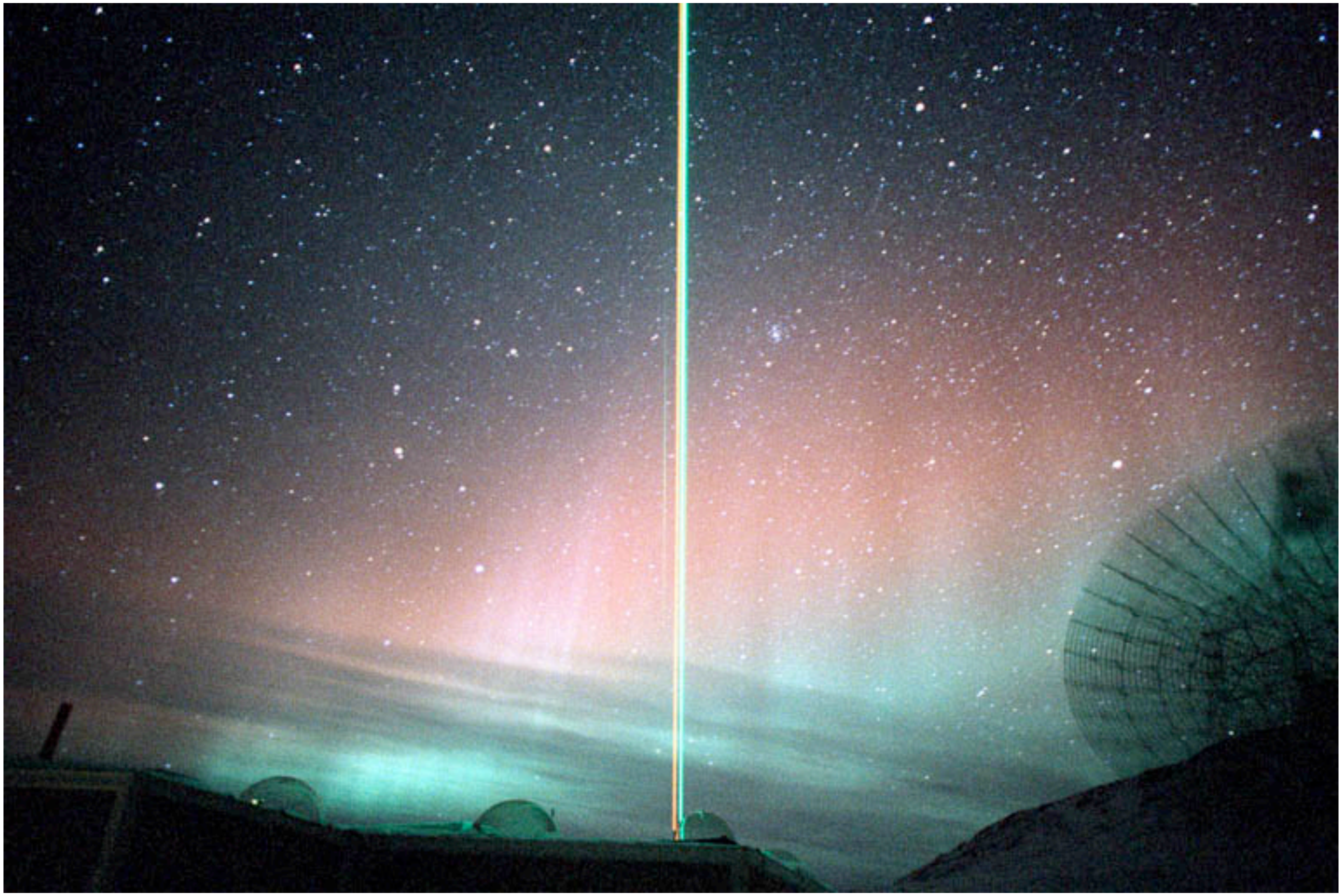


Sample Results from USU Rayleigh Lidar

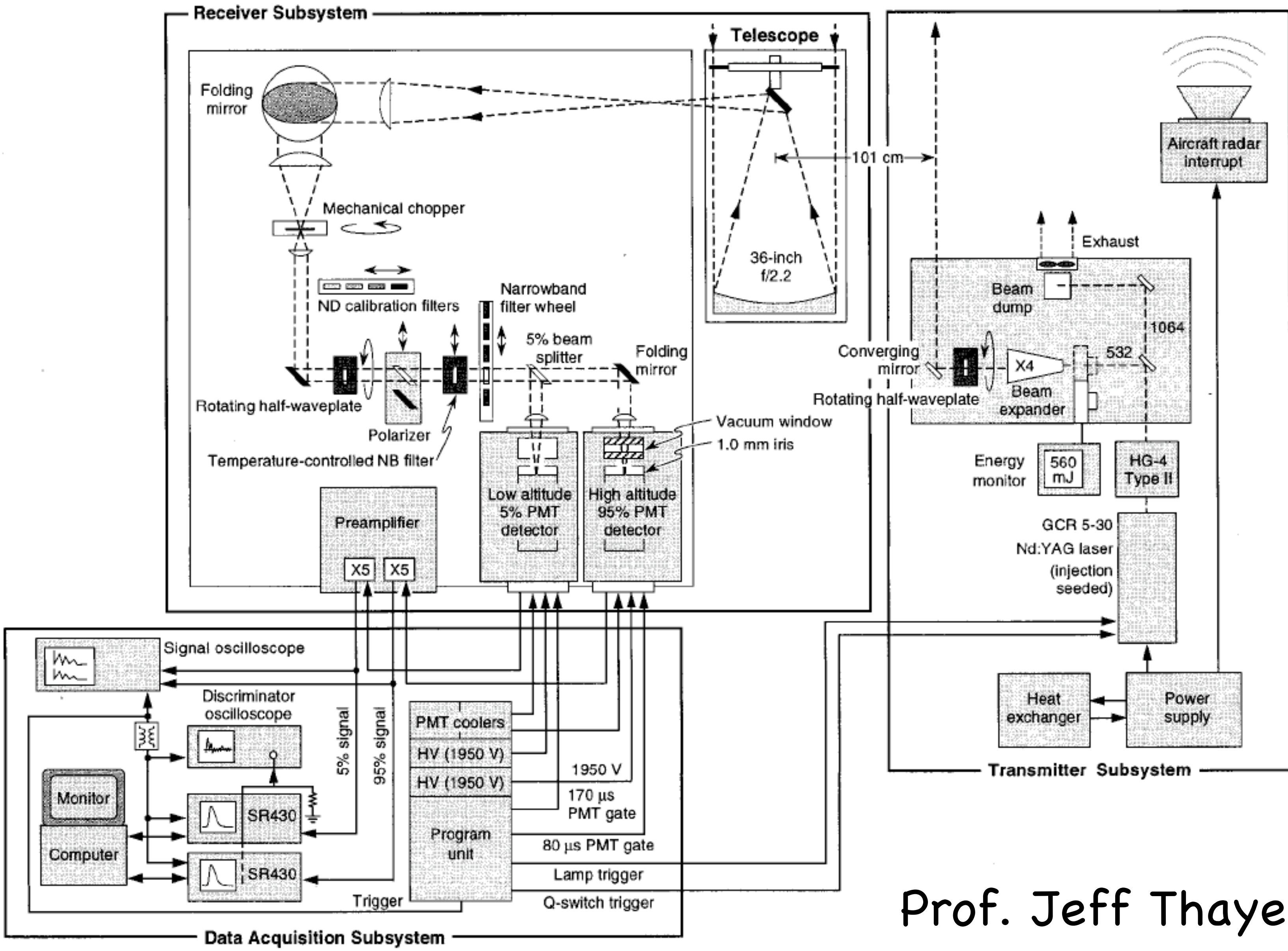
Prof. Vincent Wickwar
& Josh Herron @ USU

http://www.usu.edu/alo/aboutlidar.htm

Greenland Rayleigh Lidar

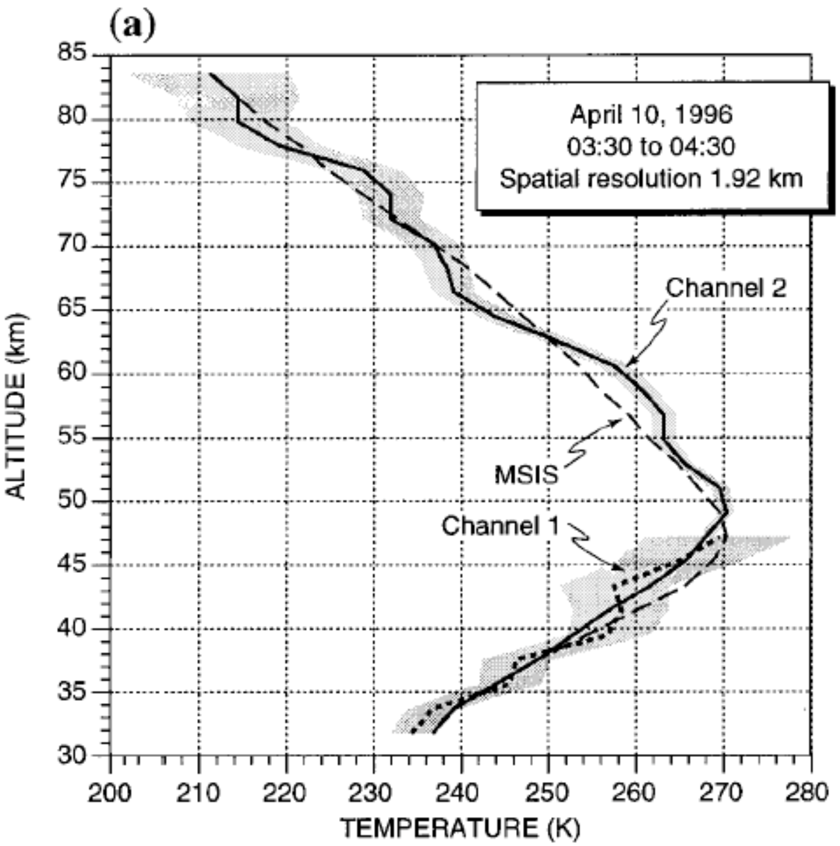


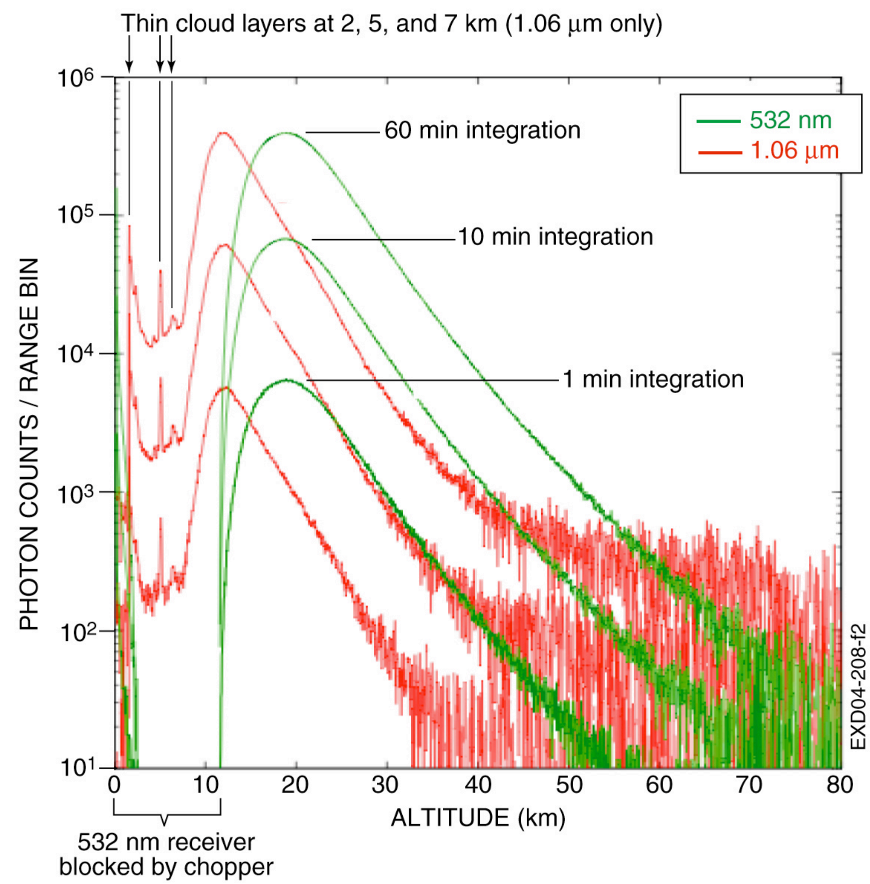
Greenland Rayleigh Lidar



Prof. Jeff Thayer

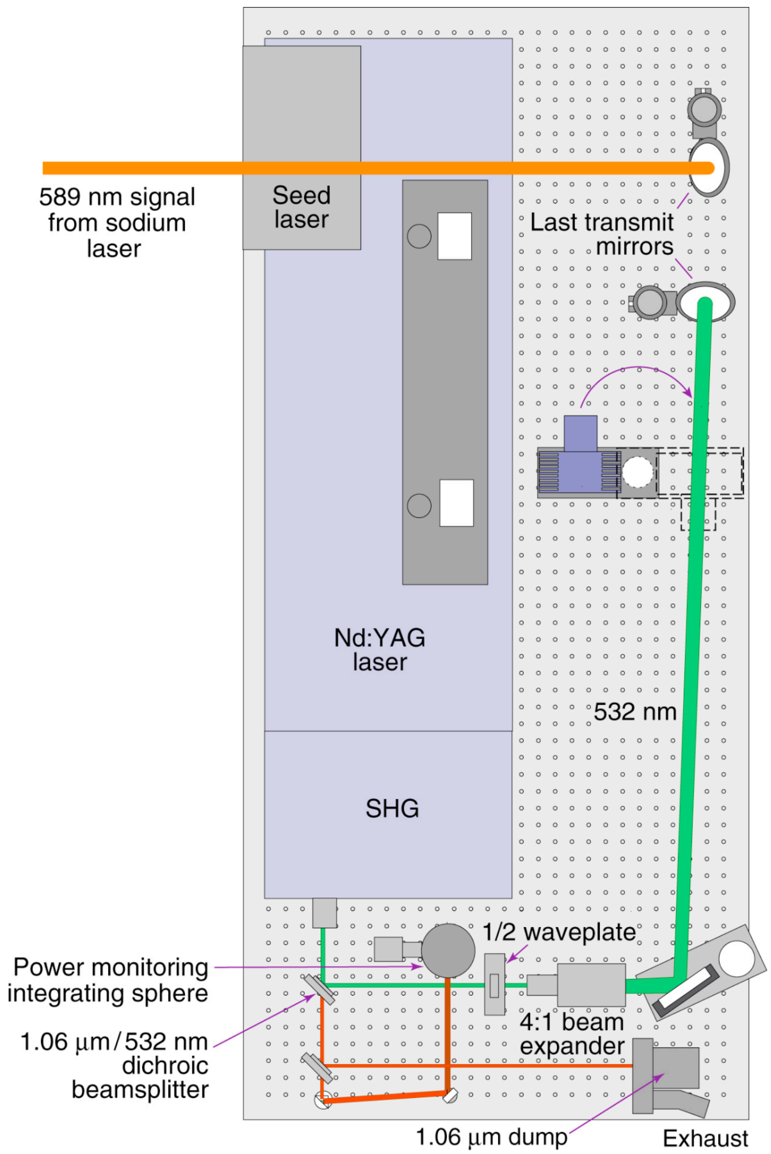
Sample Results from Greenland





Courtesy: Prof. Jeff Thayer @ CU

Further Upgrade



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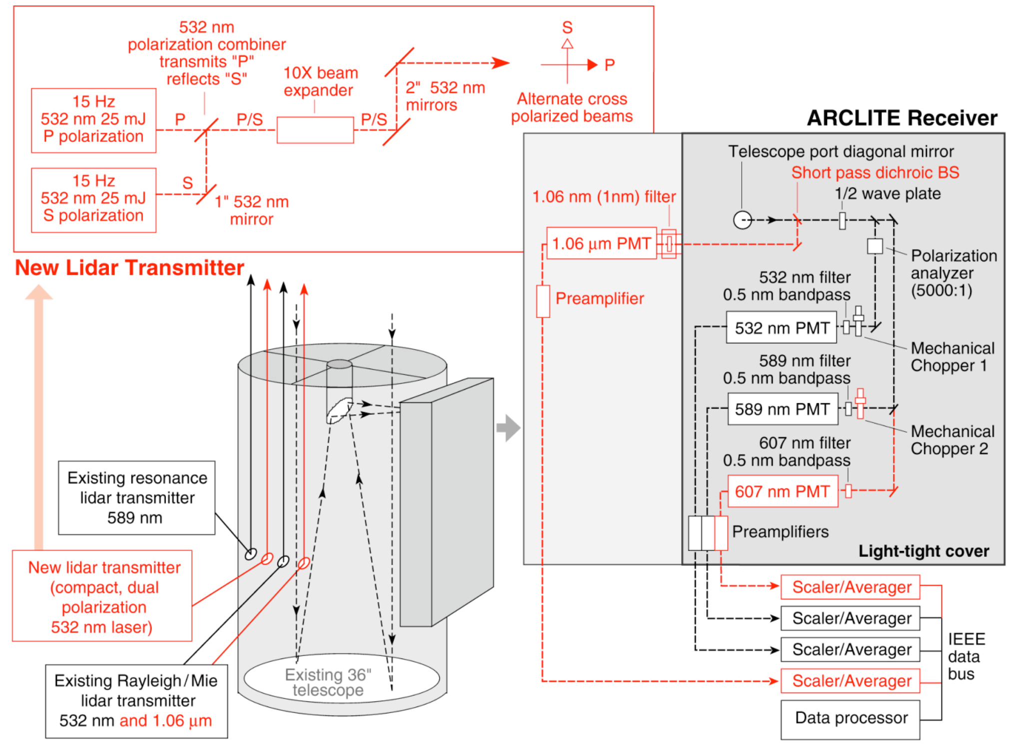

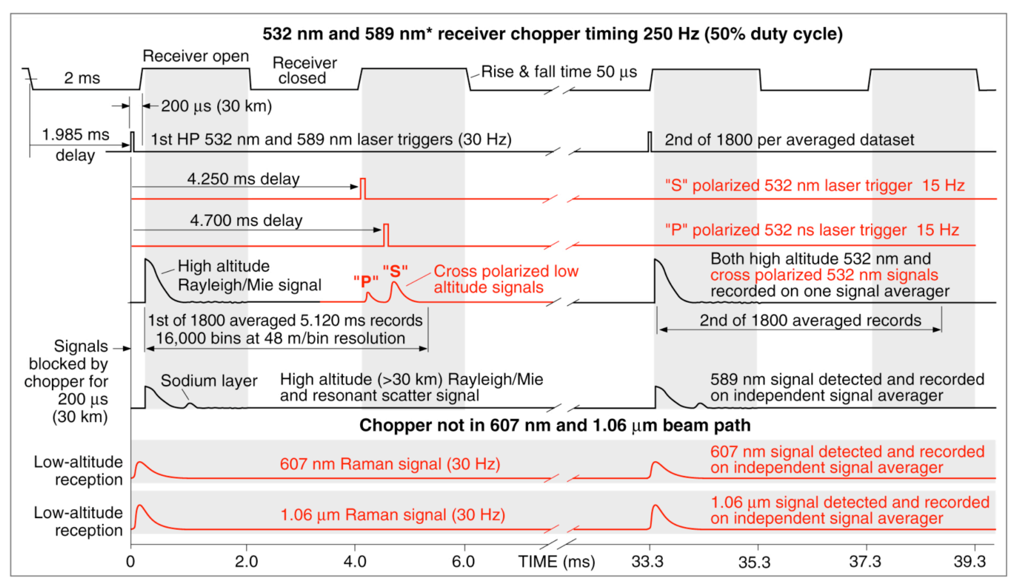

Laser and Data Acquisition Timing

Note: System additions/modifications in red

*589 nm timing and chopper functions similar but independent of 532 nm

Summary

- Accuracy and precision are two different concepts for lidar error analysis. Accuracy concerns about bias, usually determined by systematic errors. Precision concerns about uncertainty, mainly determined by random errors, and in lidar photon counting case, mainly by photon noise.
- Calculation of errors for ratio technique utilizes the differentiation of the metric ratios as described in text. It works for both systematic and random errors. Certainly, extra work is needed to identify the systematic errors and their sources. Photon noise obeys Poisson distribution.
- (Rayleigh) Integration technique relies on the assumptions of hydrostatic equilibrium and ideal gas law in the atmos interested. It involves integrating the relative density profile downward using a starting temperature at an upper altitude.

HW Project #2

- Add error analysis of temperature and wind to your code.
- Of course, you first need to derive the equations for the temperature and wind errors caused by photon noise, using the method described in our lecture, textbook, and papers.