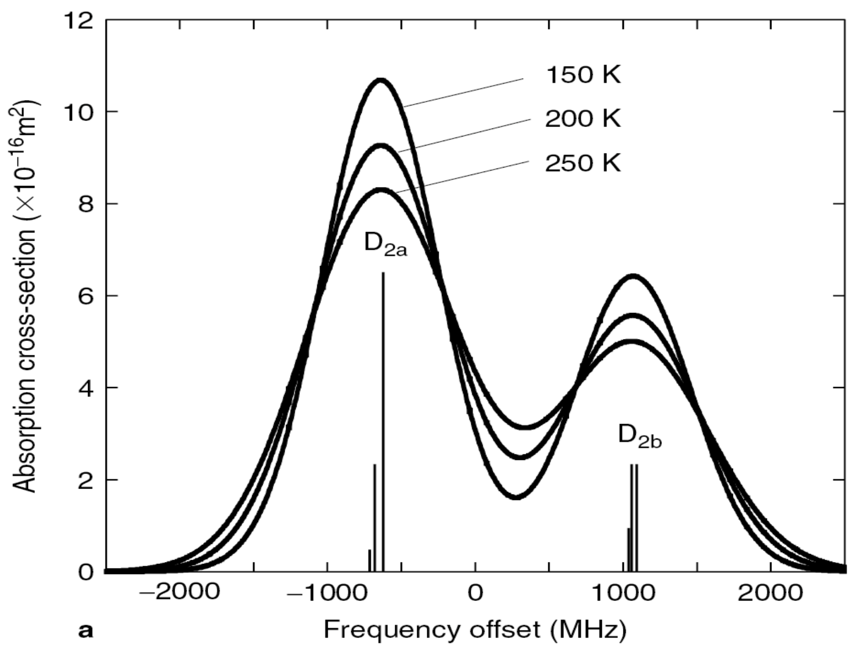
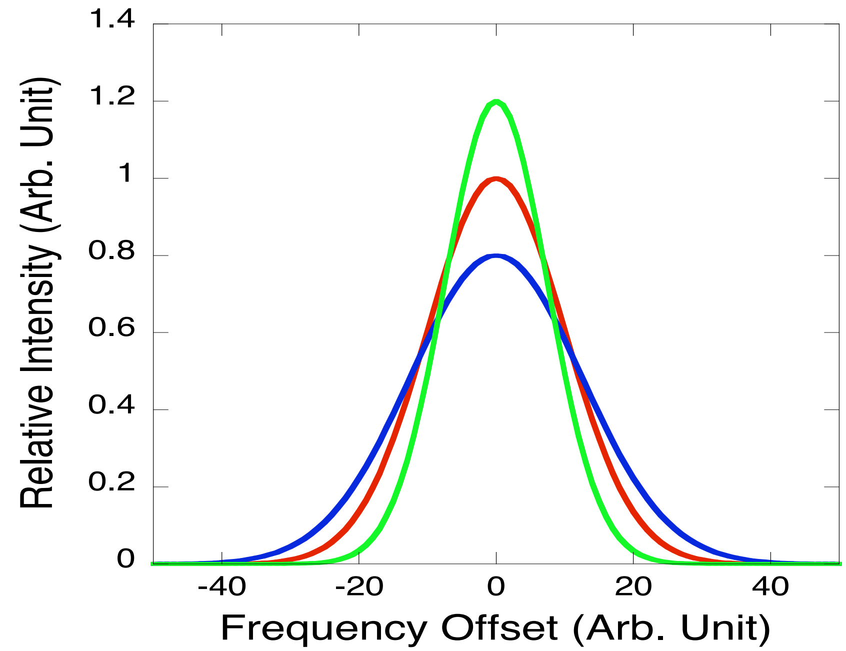
Lecture 18. Temperature Lidar (2)

- Review of Doppler Technique
- Procedure to Derive T/W from Na Lidar Data
- K Doppler Lidar Principle and Instrumentation
- Resonance Fluorescence Fe Boltzmann Lidar (Principle, Metrics, Calibration)
- Fe Boltzmann Lidar Instrumentation
- Sensitivity Analysis of Temperature Lidars
- Summary

Doppler Technique Review

- Doppler Technique – Doppler linewidth broadening and Doppler frequency shift are temperature-dependent and wind-dependent, respectively (applying to both Na, K, Fe resonance fluorescence and molecular scattering)





$$\sigma_{rms} = \frac{\omega_0}{c}\sqrt{\frac{k_B T}{M}} = \frac{1}{\lambda_0}\sqrt{\frac{k_B T}{M}}$$

$$\Delta\omega = \omega - \omega_0 = -\vec{k}\cdot\vec{v} = -\omega_0 \frac{v\cos\theta}{c}$$

Ratio Technique Review

- Lidar equation for resonance fluorescence (Na, K, or Fe)

$$N_S(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_{eff}(\lambda,z)n_c(z)R_B(\lambda) + \sigma_R(\pi,\lambda)n_R(z)\right]\Delta z\left(\frac{A}{4\pi z^2}\right) \times \left(T_a^2(\lambda)E^2(\lambda,z)\right)\left(\eta(\lambda)G(z)\right) + N_B$$

R_B = 1 for current Na Doppler lidar since return photons at all wavelengths are received by the broadband receiver, so no fluorescence is filtered off.

- Pure Na signal and pure Rayleigh signal in Na region are

$$N_{Na}(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_{eff}(\lambda,z)n_c(z)\right]\Delta z\left(\frac{A}{4\pi z^2}\right)\left(T_a^2(\lambda)E^2(\lambda,z)\right)\left(\eta(\lambda)G(z)\right)$$

$$N_R(\lambda,z) = \left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_R(\pi,\lambda)n_R(z)\right]\Delta z\left(\frac{A}{z^2}\right)\left(T_a^2(\lambda)E^2(\lambda,z)\right)\left(\eta(\lambda)G(z)\right)$$

- So we have

$$N_S(\lambda,z) = N_{Na}(\lambda,z) + N_R(\lambda,z) + N_B$$

Ratio Technique Review

- Lidar equation at pure molecular scattering region (35-55km)

$$N_S(\lambda,z_R)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_R(\pi,\lambda)n_R(z_R)\right]\Delta z\left(\frac{A}{z_R^{\ 2}}\right)T_a^{\ 2}(\lambda,z_R)\left(\eta(\lambda)G(z_R)\right)+N_B$$

- Pure Rayleigh signal in molecular scattering region is

$$N_R(\lambda,z_R)=\left(\frac{P_L(\lambda)\Delta t}{hc/\lambda}\right)\left[\sigma_R(\pi,\lambda)n_R(z_R)\right]\Delta z\left(\frac{A}{z_R^{\ 2}}\right)T_a^{\ 2}(\lambda,z_R)\left(\eta(\lambda)G(z_R)\right)$$

- So we have

$$N_S(\lambda,z_R)=N_R(\lambda,z_R)+N_B$$

- The ratio between Rayleigh signals at z and z_R is given by

$$\frac{N_R(\lambda,z)}{N_R(\lambda,z_R)}=\frac{\left[\sigma_R(\pi,\lambda)n_R(z)\right]T_a^{\ 2}(\lambda,z)E^2(\lambda,z)G(z)}{\left[\sigma_R(\pi,\lambda)n_R(z_R)\right]T_a^{\ 2}(\lambda,z_R)G(z_R)}\frac{z_R^{\ 2}}{z^2}=\frac{n_R(z)}{n_R(z_R)}\frac{z_R^{\ 2}}{z^2}E^2(\lambda,z)$$

Where n_R is the (total) atmospheric number density, usually obtained from atmospheric models like MSIS00.

Ratio Technique Review

- From above equations, we obtain

$$N_{Na}(\lambda,z) = N_S(\lambda,z) - N_B - N_R(\lambda,z)$$

$$N_R(\lambda,z_R) = N_S(\lambda,z_R) - N_B$$

- Normalized Na photon count is defined as

$$N_{Norm}(\lambda,z) = \frac{N_{Na}(\lambda,z)}{N_R(\lambda,z_R)E^2(\lambda,z)} \frac{z^2}{z_R^{\ 2}}$$

- So from physics point of view, we have

$$N_{Norm}(\lambda,z) = \frac{N_{Na}(\lambda,z)}{N_R(\lambda,z_R)E^2(\lambda,z)} = \frac{\sigma_{eff}(\lambda,z)n_c(z)}{\sigma_R(\pi,\lambda)n_R(z_R)} \frac{1}{4\pi}$$

- From actual photon counts, we have

$$N_{Norm}(\lambda,z) = \frac{N_{Na}(\lambda,z)}{N_R(\lambda,z_R)E^2(\lambda,z)} \frac{z^2}{z_R^{\ 2}} = \frac{N_S(\lambda,z) - N_B - N_R(\lambda,z)}{N_R(\lambda,z_R)E^2(\lambda,z)} \frac{z^2}{z_R^{\ 2}}$$

$$= \frac{N_S(\lambda,z) - N_B}{N_S(\lambda,z_R) - N_B} \frac{z^2}{z_R^{\ 2}} \frac{1}{E^2(\lambda,z)} - \frac{n_R(z)}{n_R(z_R)}$$

Ratio Technique Review

- From physics, the ratios of R_T and R_W are then given by

$$R_T = \frac{N_{Norm}(f_+,z) + N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)} = \frac{\dfrac{\sigma_{eff}(f_+,z)n_c(z)}{\sigma_R(\pi,f_+)n_R(z_R)} + \dfrac{\sigma_{eff}(f_-,z)n_c(z)}{\sigma_R(\pi,f_-)n_R(z_R)}}{\dfrac{\sigma_{eff}(f_a,z)n_c(z)}{\sigma_R(\pi,f_a)n_R(z_R)}} = \frac{\sigma_{eff}(f_+,z) + \sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}$$

$$R_W = \frac{N_{Norm}(f_-,z)}{N_{Norm}(f_+,z)} = \frac{\dfrac{\sigma_{eff}(f_-,z)n_c(z)}{\sigma_R(\pi,f_-)n_R(z_R)}}{\dfrac{\sigma_{eff}(f_+,z)n_c(z)}{\sigma_R(\pi,f_+)n_R(z_R)}} = \frac{\sigma_{eff}(f_-,z)}{\sigma_{eff}(f_+,z)}$$

Here, Rayleigh backscatter cross-section is regarded as the same for three frequencies, since the frequency difference is so small. Na number density is also the same for three frequency channels, and so is the atmosphere number density at Rayleigh normalization altitude.

Ratio Technique Review

- From actual photon counts, we have

$$R_T = \frac{N_{Norm}(f_+,z) + N_{Norm}(f_-,z)}{N_{Norm}(f_a,z)}$$

$$= \frac{\left(\frac{N_S(f_+,z) - N_B}{N_S(f_+,z_R) - N_B} \frac{z^2}{z_R^{\;2}} \frac{1}{E^2(f_+,z)} - \frac{n_R(z)}{n_R(z_R)}\right) + \left(\frac{N_S(f_-,z) - N_B}{N_S(f_-,z_R) - N_B} \frac{z^2}{z_R^{\;2}} \frac{1}{E^2(f_-,z)} - \frac{n_R(z)}{n_R(z_R)}\right)}{\frac{N_S(f_a,z) - N_B}{N_S(f_a,z_R) - N_B} \frac{z^2}{z_R^{\;2}} \frac{1}{E^2(f_a,z)} - \frac{n_R(z)}{n_R(z_R)}}$$

$$R_W = \frac{N_{Norm}(f_-,z)}{N_{Norm}(f_+,z)} = \frac{\frac{N_S(f_-,z) - N_B}{N_S(f_-,z_R) - N_B} \frac{z^2}{z_R^{\;2}} \frac{1}{E^2(f_-,z)} - \frac{n_R(z)}{n_R(z_R)}}{\frac{N_S(f_+,z) - N_B}{N_S(f_+,z_R) - N_B} \frac{z^2}{z_R^{\;2}} \frac{1}{E^2(f_+,z)} - \frac{n_R(z)}{n_R(z_R)}}$$

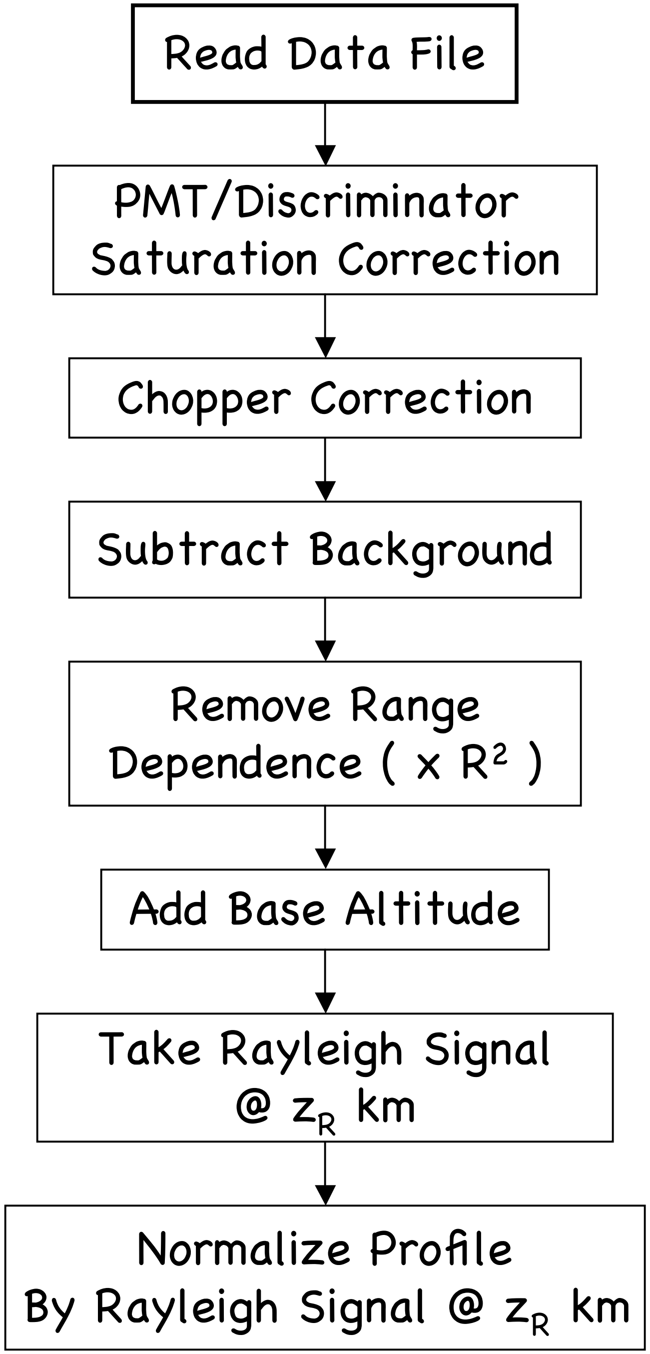
- Advantages of ratio technique

Main Ideas to Derive Na T and W

- Three frequencies give 3 lidar equations, and we want to derive 3 unknown parameters (temperature, radial wind, and Na number density) from these 3 equations.
- In the ratio technique, Na number density is cancelled out. So we have two ratios R_T and R_W that are independent of Na density but both dependent on T and W.
- The idea is to derive temperature and radial wind from these two ratios first, and then derive Na number density using computed temperature and wind at each altitude bin.
- To derive T and W from R_T and R_W, the basic idea is to use look-up table or iteration methods to derive them: (1) compute R_T and R_W from physics point-of-view to generate the table or calibration curves, (2) compute R_T and R_W from actual photon counts, (3) check the table or calibration curves to find the corresponding T and W. (4) If R_T and R_W are out of range, then set to nominal T = 200 K and W = 0 m/s.

Main Ideas to Derive Na T and W

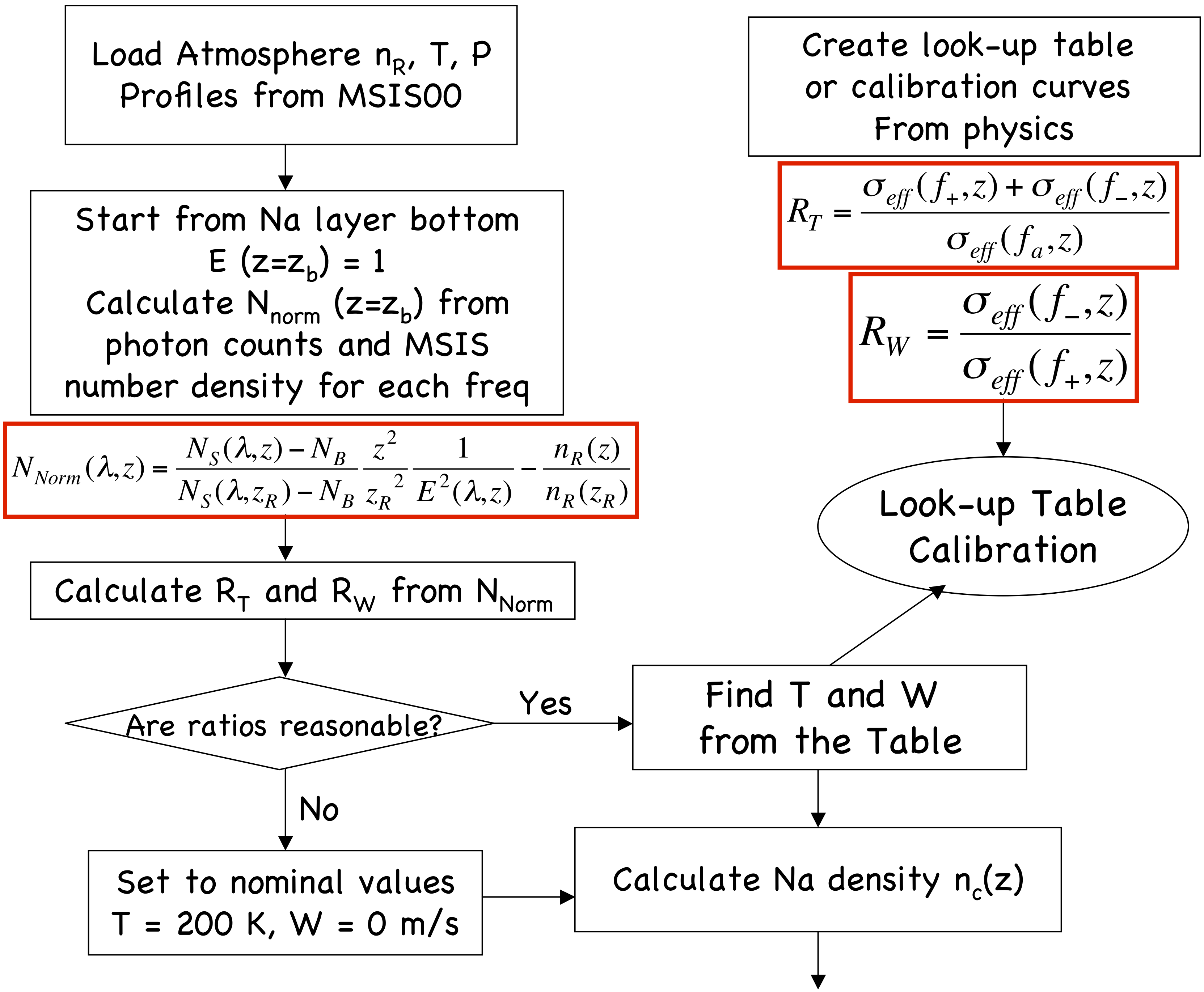
- However, because the Na extinction coefficient is involved, the upper bins are related to lower bins, and extinction coefficient is related to Na density and effective cross-section.
- Again, the solution is to start from the bottom of the Na layer.



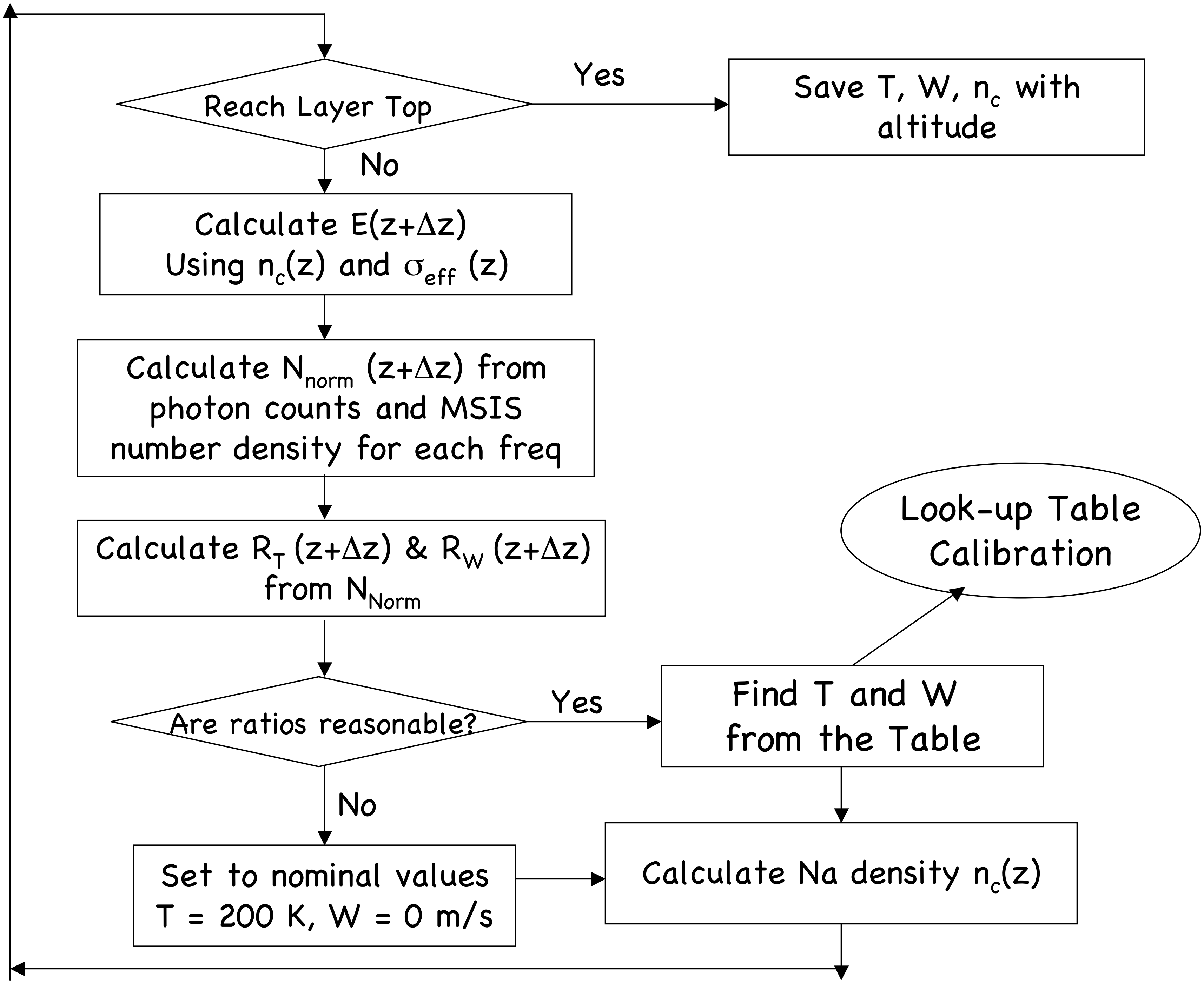
Procedure to Derive Na T and W

- Read data: for each set, and calculate T, W, and n for each set
- PMT/Discriminator saturation correction: see Lecture 12 notes
- Chopper correction: Lecture 12
- Range-dependence, not altitude
- Add base altitude: don't over-do it
- Take Rayleigh signal vs Rayleigh fit
- Normalization:

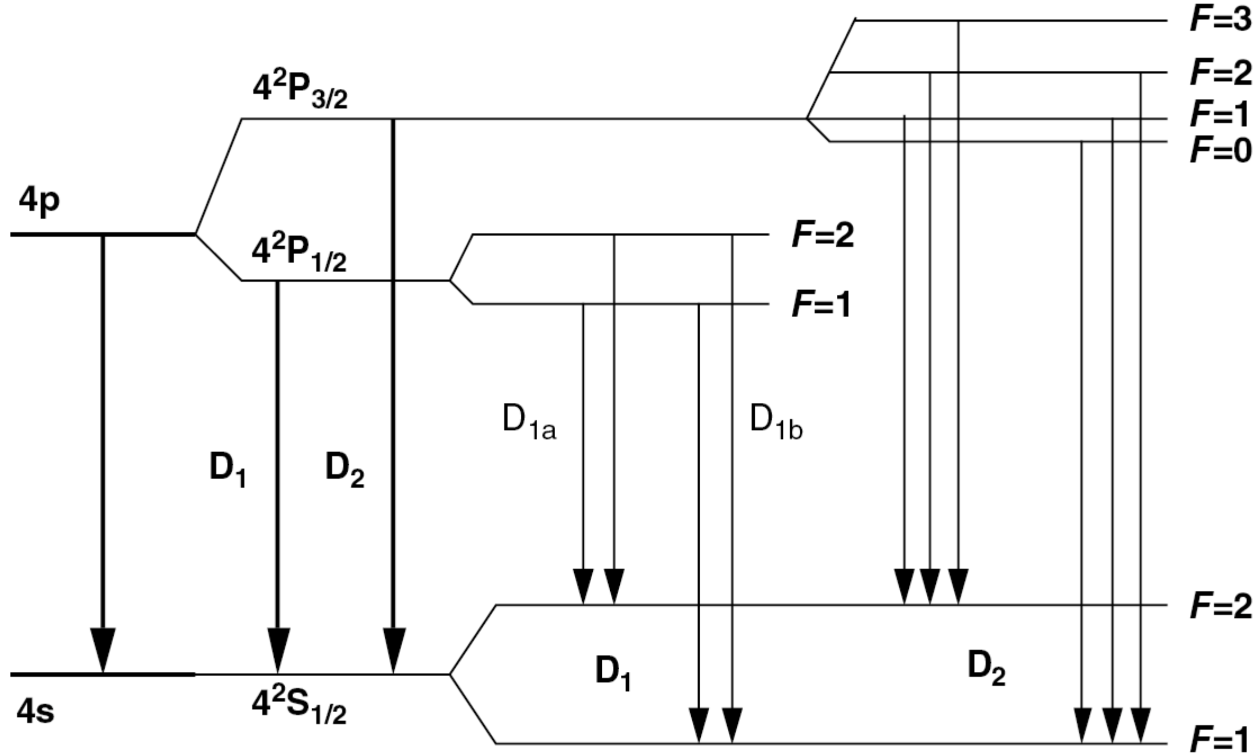
$$N_N(\lambda, z) = \frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} \frac{z^2}{z_R^{\ 2}}$$



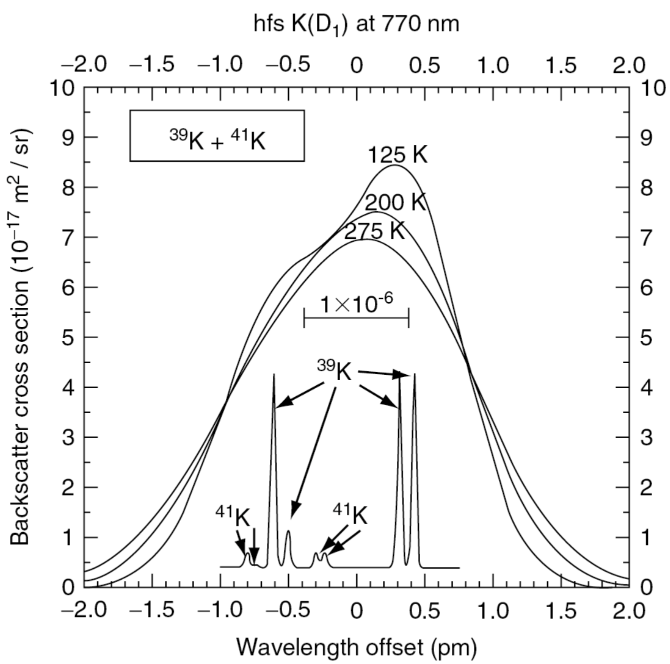
Load Atmosphere n_R, T, P Profiles from MSIS00
Start from Na layer bottom
$E(z=z_b) = 1$
Calculate $N_{norm}(z=z_b)$ from photon counts and MSIS number density for each freq
$$N_{Norm}(\lambda,z) = \frac{N_S(\lambda,z)-N_B}{N_S(\lambda,z_R)-N_B}\frac{z^2}{z_R^{\,2}}\frac{1}{E^2(\lambda,z)}\frac{n_R(z)}{n_R(z_R)}$$
Calculate R_T and R_W from N_{Norm}
Are ratios reasonable?
Yes
No
Set to nominal values
T = 200 K, W = 0 m/s
Create look-up table or calibration curves From physics
$$R_T = \frac{\sigma_{eff}(f_+,z)+\sigma_{eff}(f_-,z)}{\sigma_{eff}(f_a,z)}$$
$$R_W = \frac{\sigma_{eff}(f_-,z)}{\sigma_{eff}(f_+,z)}$$
Look-up Table Calibration
Find T and W from the Table
Calculate Na density $n_c(z)$


Reach Layer Top
Yes
Save T, W, n_c with altitude
No
Calculate $E(z+\Delta z)$ Using $n_c(z)$ and σ_{eff} (z)
Calculate N_{norm} $(z+\Delta z)$ from photon counts and MSIS number density for each freq
Calculate R_T $(z+\Delta z)$ & R_W $(z+\Delta z)$ from N_{Norm}
Look-up Table Calibration
Are ratios reasonable?
Yes
Find T and W from the Table
No
Set to nominal values T = 200 K, W = 0 m/s
Calculate Na density $n_c(z)$

K Atomic Energy Levels



K fine structure **K hyperfine structure**



Transition	K(D_1)	K(D_2)
Wavelength air [nm]	769.8974	766.4911
Wavelength vacuum [nm]	770.1093	766.7021
Rel. intensity	24	25
A_{ik} [$10^8 s^{-1}$]	0.382 (±10%)	0.387 (±10%)
f-value	0.340	0.682
Terms $^{2S+1}L_J$	$^2S_{1/2} - ^2P^o_{1/2}$	$^2S_{1/2} - ^2P^o_{3/2}$
$g_i - g_k$	2-2	2-4

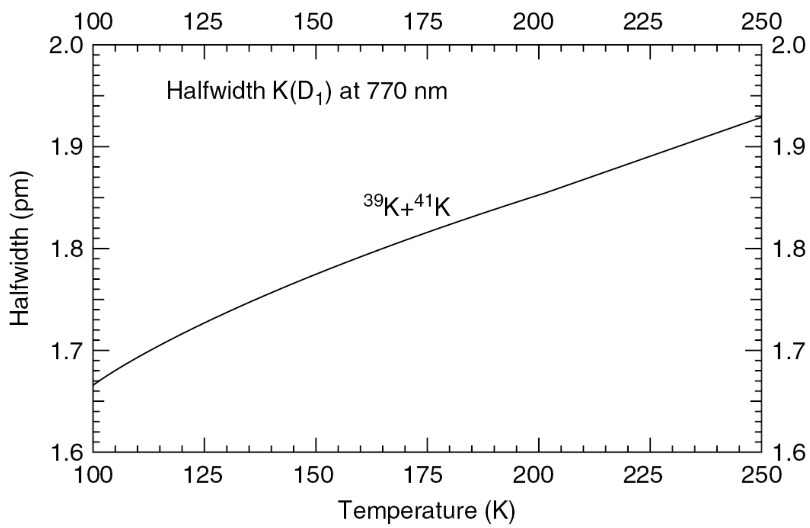
K Atomic Parameters

Isotope	Atomic mass	Abundance	Nuclear spin	$K(D_1)$ line shift
39	38.963 706 9(3)	0.932 581(44)	$I = 3/2$	0
40	39.963 998 67(29)	0.000 117(1)	$I = 4$	125.58 MHz
41	40.961 825 97(28)	0.067 302(44)	$I = 3/2$	235.28 MHz

Table 5.8 Quantum Numbers, Frequency Offsets, and Relative Line Strength for K (D_1) Hyperfine Structure Lines

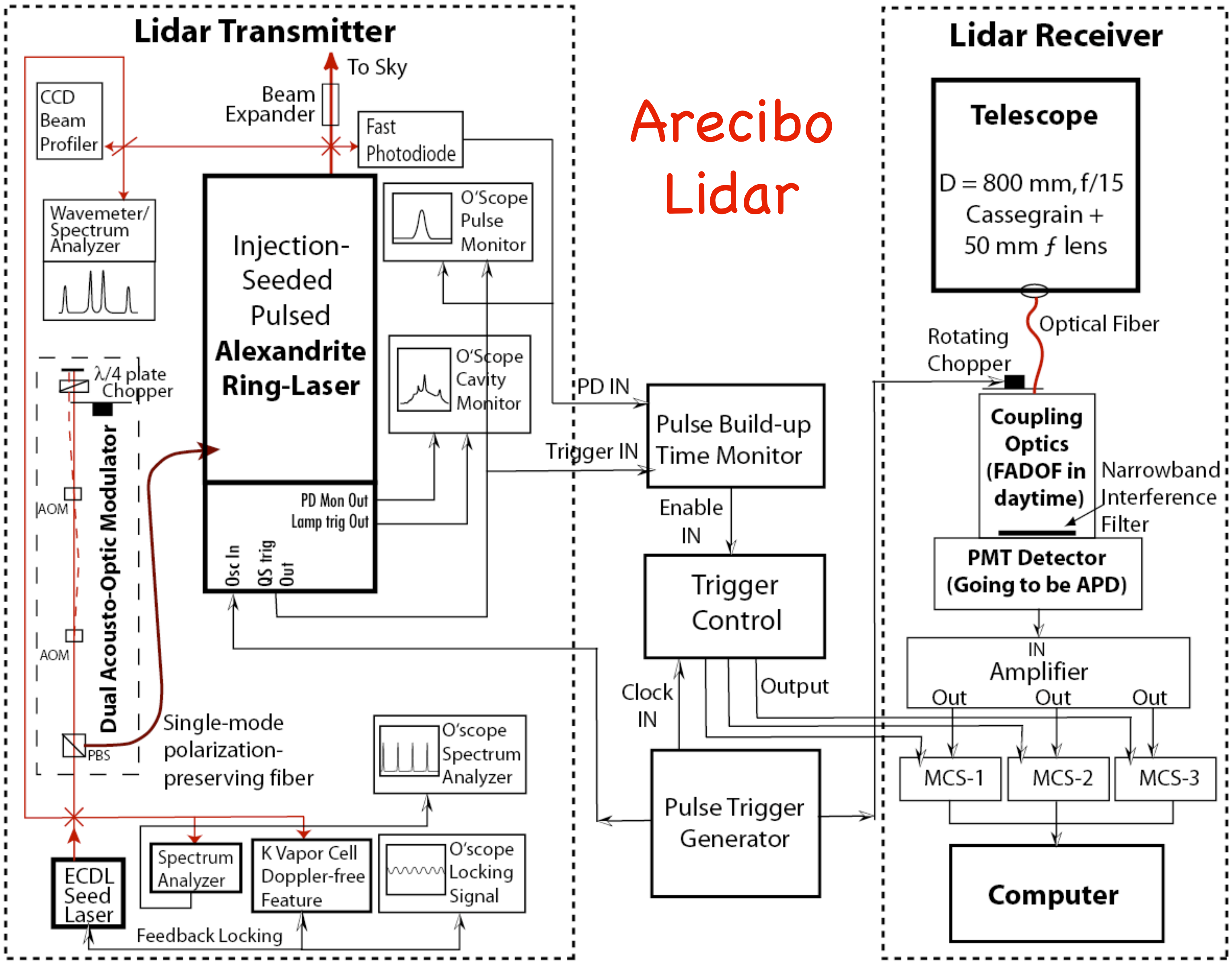
$^2S_{1/2}$	$^2P_{1/2}$	^{39}K (MHz)	^{41}K (MHz)	Relative Line Strength
$F = 1$	$F = 2$	310	405	5/16
	$F = 1$	254	375	1/16
$F = 2$	$F = 2$	−152	151	5/16
	$F = 1$	−208	121	5/16

K Doppler Lidar Principle & Metrics

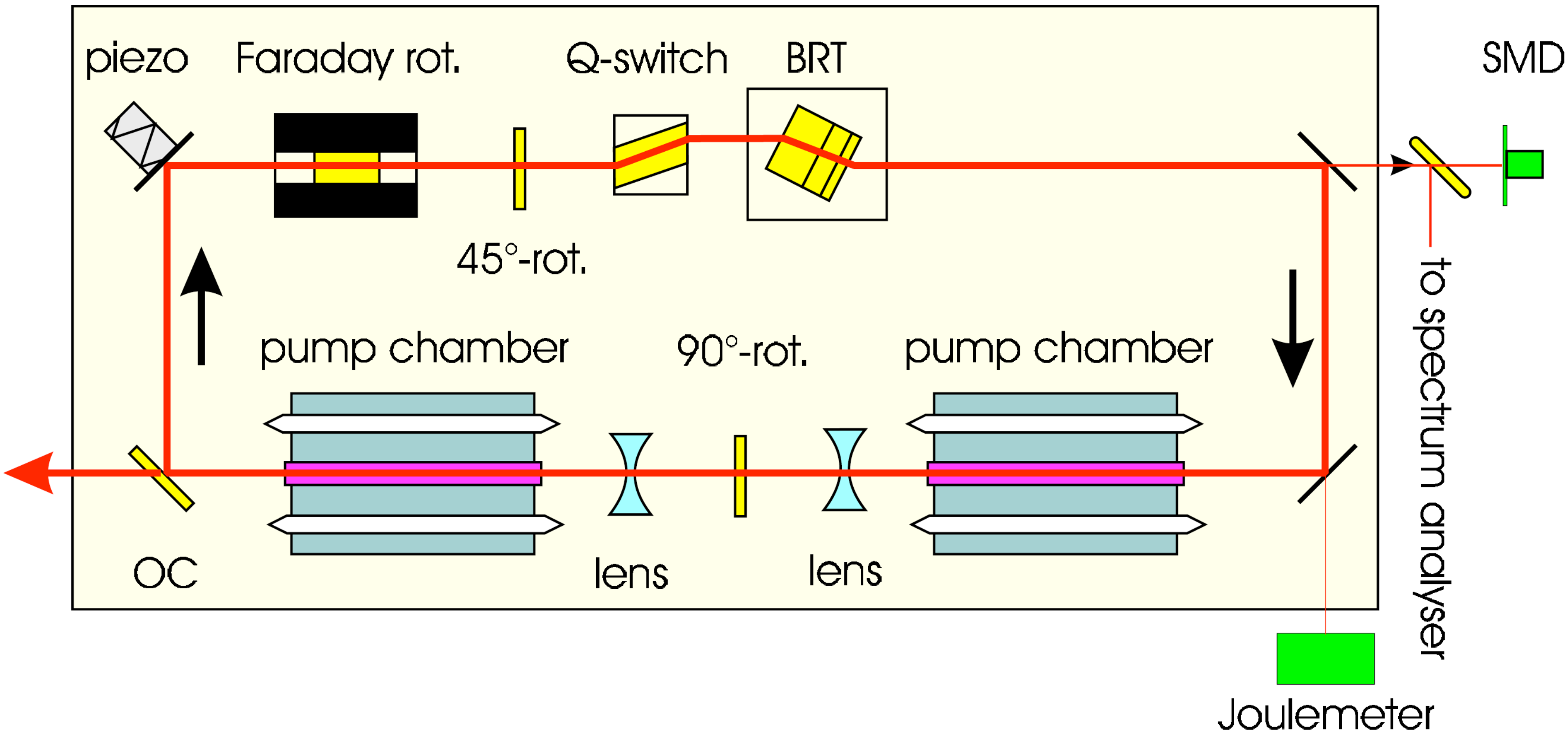


- Ratio technique versus scanning technique
- Scanning technique actually has its advantages on several aspects, depending on the laser system used – whether there is pedestal, background problems, etc.
- Let's use K Doppler lidar as an example

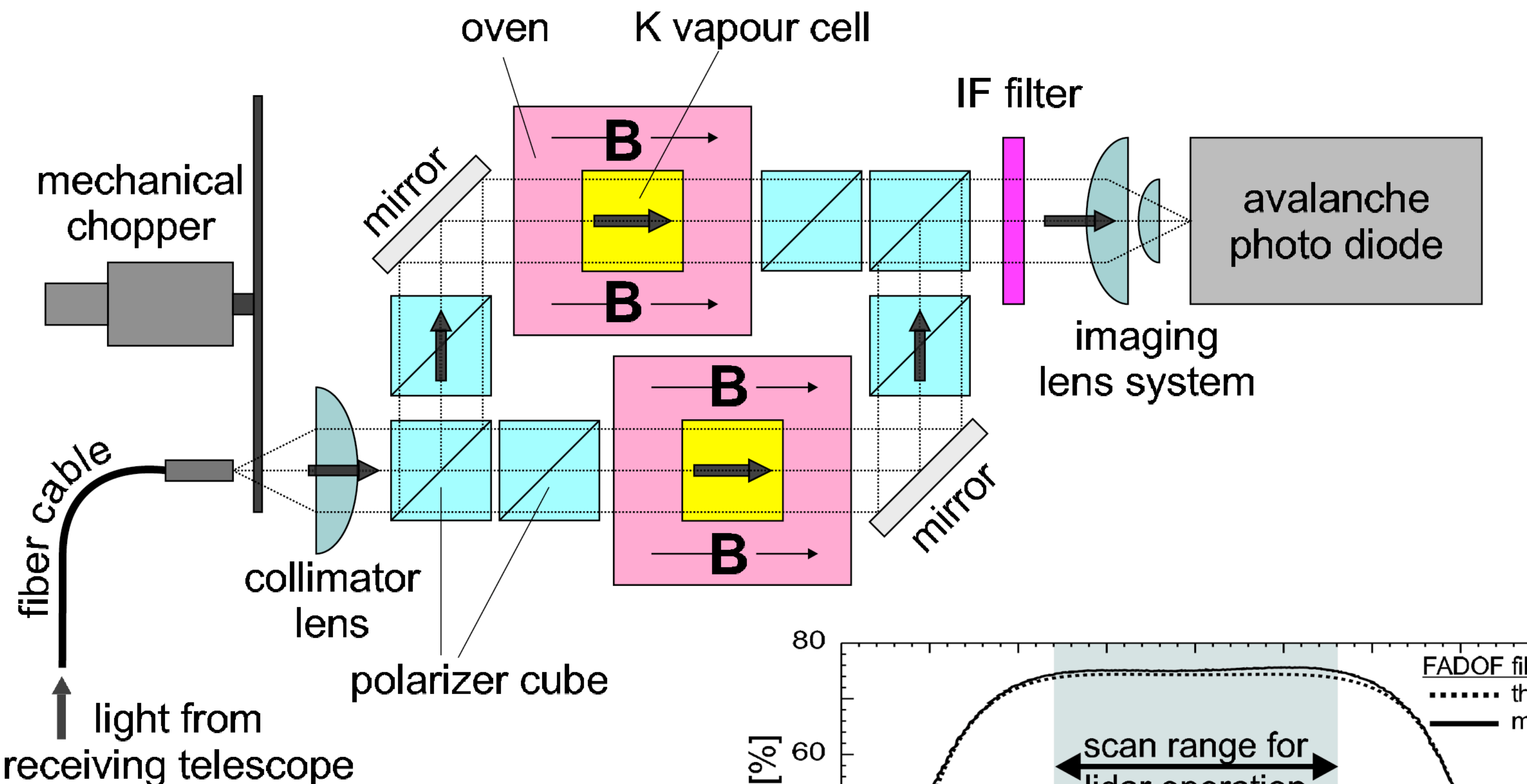
K Doppler Lidar Instrumentation

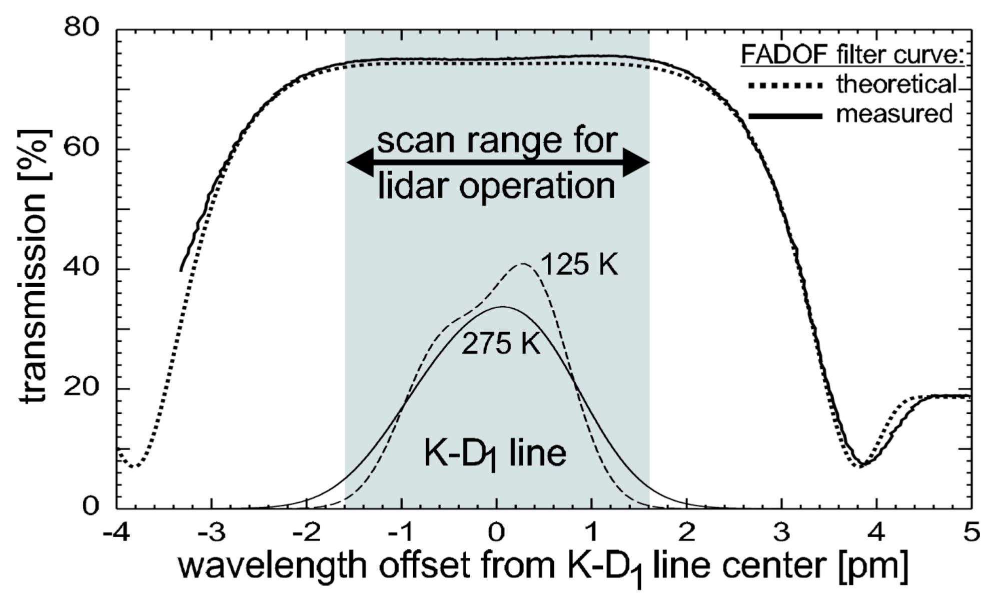


IAP Alexandrite Ring Laser



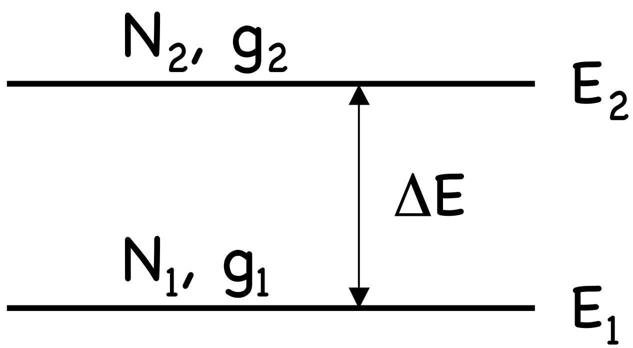
Dual K-Faraday Filter





Boltzmann Technique to Measure Temperature

- Boltzmann distribution is the law of particle population distribution according to energy levels



$$\frac{N_k}{N} = \frac{g_k \exp(-E_k / k_B T)}{\sum_i g_i \exp(-E_i / k_B T)}$$

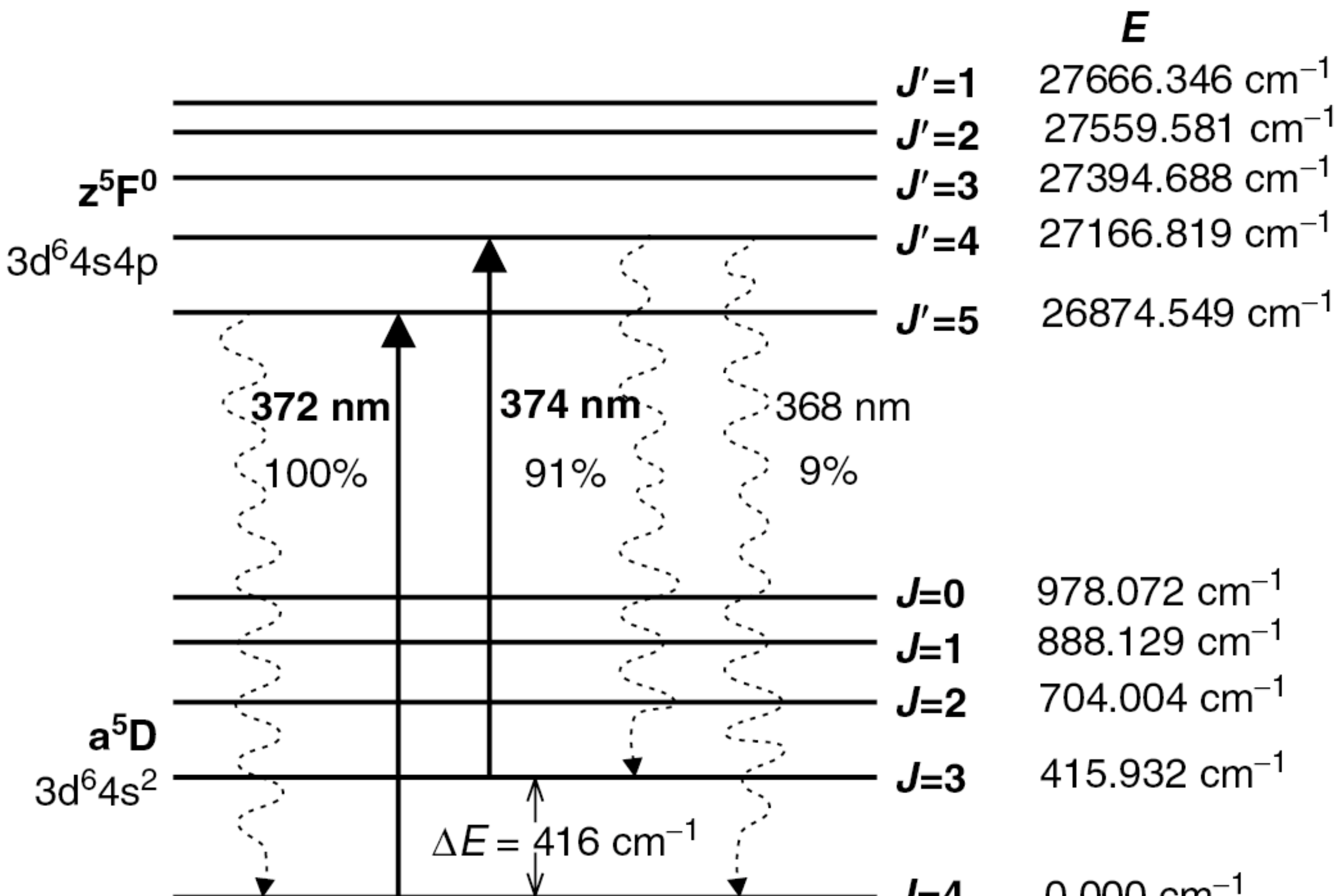
$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\left\{-\left(E_2 - E_1\right)/k_B T\right\}$$

$$T = \frac{\Delta E / k_B}{\ln\left(\frac{g_2}{g_1} \cdot \frac{N_1}{N_2}\right)}$$

N_1 and N_2 – particle populations on energy levels E_1 and E_2
g_1 and g_2 – degeneracy for energy levels E_1 and E_2, $\Delta E = E_2 - E_1$
k_B – Boltzmann constant, T – Temperature, N – total population

Population Ratio ⇨ Temperature

Fe Atomic Energy Levels



Fluorescence Intensity Ratio ⇨ Population Ratio ⇨ Temperature

Fe Atomic Parameters

Table 5.3 Isotopic Data of Fe Atoms

	^{54}Fe	^{56}Fe	^{57}Fe	^{58}Fe
Z	26	26	26	26
A	54	56	57	58
Nuclear spin	0	0	1/2	0
Natural abundance	5.845%	91.754%	2.119%	0.282%

Table 5.4 Fe Resonance Line Parameters

Transition wavelength λ	372.0993 nm	373.8194 nm
Degeneracy for ground state	$g_1 = 9$	$g_2 = 7$
Degeneracy for excited state	$g_1' = 11$	$g_2' = 9$
Radiative lifetime of excited state (ns)	61.0	63.6
Einstein coefficient A_{ki} (10^8 s^{-1})	0.163	0.142
Oscillator strength f_{ik}	0.0413	0.0382
Branching ratio R_B	0.9959	0.9079
σ_0 (10^{-17} m^2)	9.4	8.7

Fe Boltzmann Lidar Principle

$$N_{\mathrm{Fe}}(\lambda,z) = \left(\frac{P_{\mathrm{L}}\Delta t T_{\mathrm{a}}}{hc/\lambda}E(\lambda,z)\right)\sigma_{\mathrm{eff}}(\lambda,T,\sigma_{\mathrm{L}})R_{\mathrm{B}\lambda}\rho_{\mathrm{Fe}}(\lambda,z)$$
$$\times\Delta z\left(E(\lambda,z)T_{\mathrm{a}}\frac{A_{\mathrm{R}}}{4\pi z^2}\eta\right) \quad (5.92)$$

$$N_{\mathrm{norm}}(\lambda,z) = \frac{N_{\mathrm{Fe}}(\lambda,z) + N_{\mathrm{B}}(\lambda,z) - \hat{N}_{\mathrm{B}}(\lambda)}{N_{\mathrm{R}}(\lambda,z_{\mathrm{R}}) + N_{\mathrm{B}}(\lambda,z_{\mathrm{R}}) - \hat{N}_{\mathrm{B}}(\lambda)}$$
$$= \frac{z_{\mathrm{R}}^2 E^2(\lambda,z)R_{\mathrm{B}\lambda}\sigma_{\mathrm{eff}}(\lambda,T,\sigma_{\mathrm{L}})\rho_{\mathrm{Fe}}(\lambda,z)}{z^2\sigma_{\mathrm{R}}(\lambda)\rho_{\mathrm{atmos}}(z_{\mathrm{R}})}$$

Fe Boltzmann Lidar Principle

$$R_{\mathrm{T}}(z) = \frac{N_{\mathrm{norm}}(\lambda_{374}, z)}{N_{\mathrm{norm}}(\lambda_{372}, z)}$$

$$= \frac{g_2}{g_1} \frac{R_{\mathrm{B374}}}{R_{\mathrm{B372}}} \left(\frac{\lambda_{374}}{\lambda_{372}}\right)^{4.0117} \frac{E^2(\lambda_{374}, z)}{E^2(\lambda_{372}, z)}$$

$$\times \frac{\sigma_{\mathrm{eff}}(\lambda_{374}, T, \sigma_{\mathrm{L374}})}{\sigma_{\mathrm{eff}}(\lambda_{372}, T, \sigma_{\mathrm{L372}})} \exp(-\Delta E / k_{\mathrm{B}} T)$$

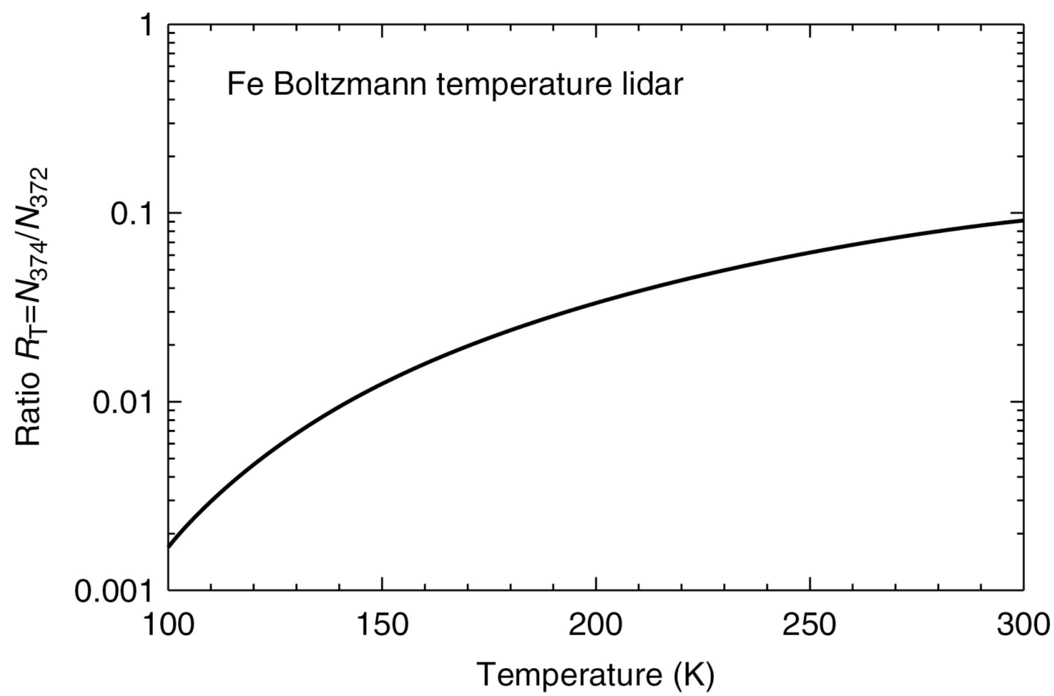
$$R_\sigma = \frac{\sigma_{\mathrm{eff}}(\lambda_{374}, T, \sigma_{\mathrm{L374}})}{\sigma_{\mathrm{eff}}(\lambda_{372}, T, \sigma_{\mathrm{L372}})}$$

$$R_E(z) = \frac{E(\lambda_{374}, z)}{E(\lambda_{372}, z)}$$

$$T(z) = \frac{\Delta E / k_{\mathrm{B}}}{\ln\left[\frac{g_2}{g_1} \frac{R_{\mathrm{B374}}}{R_{\mathrm{B372}}} \left(\frac{\lambda_{374}}{\lambda_{372}}\right)^{4.0117} \frac{R_E^2(z) R_\sigma}{R_{\mathrm{T}}(z)}\right]}$$

$$= \frac{598.44K}{\ln\left[\frac{0.7221 R_E^2(z) R_\sigma}{R_{\mathrm{T}}(z)}\right]}$$

Fe Boltzmann Lidar Calibration

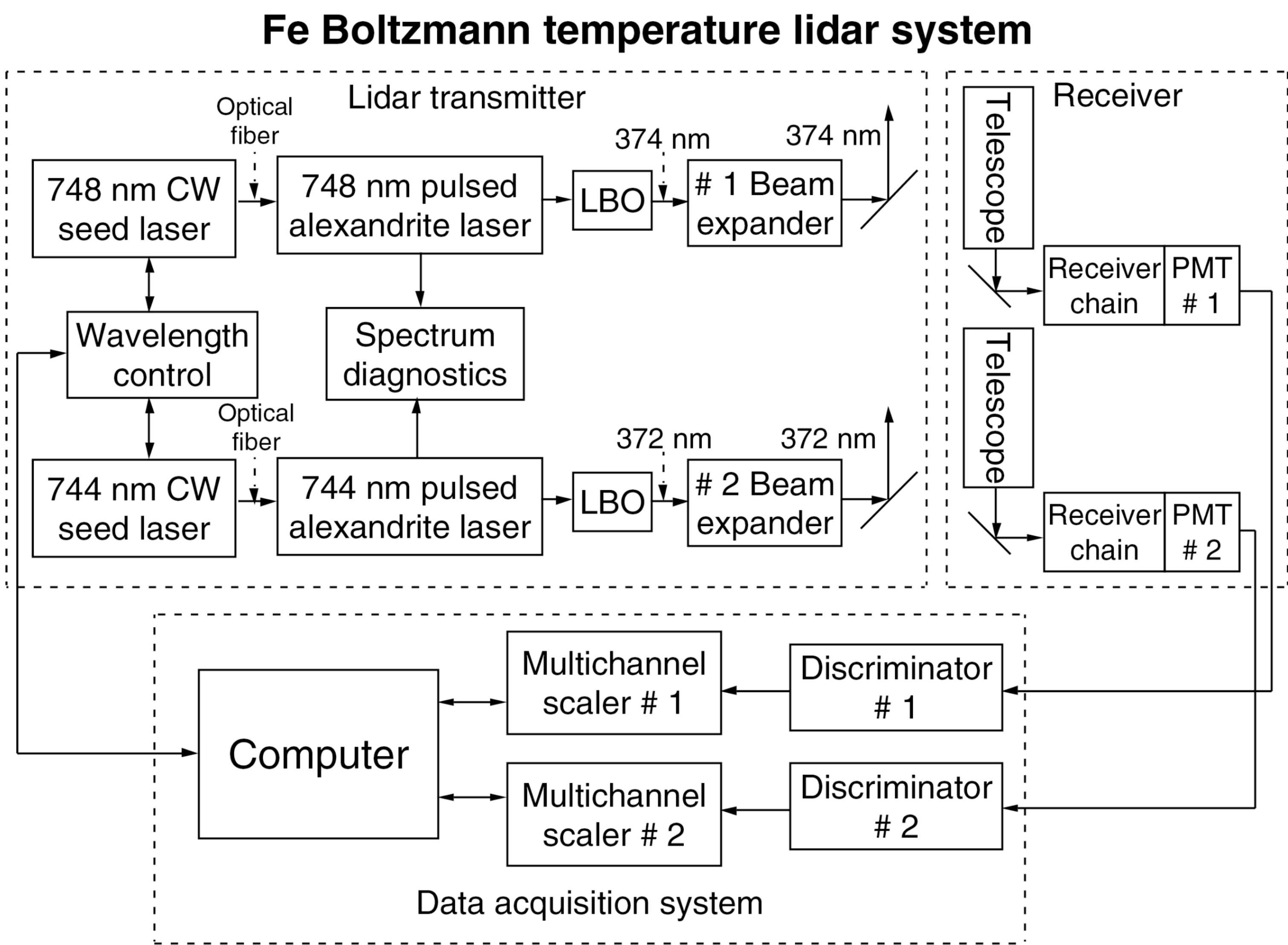


$$R_{\rm T}(z) = \frac{N_{\rm norm}(\lambda_{374}, z)}{N_{\rm norm}(\lambda_{372}, z)}$$

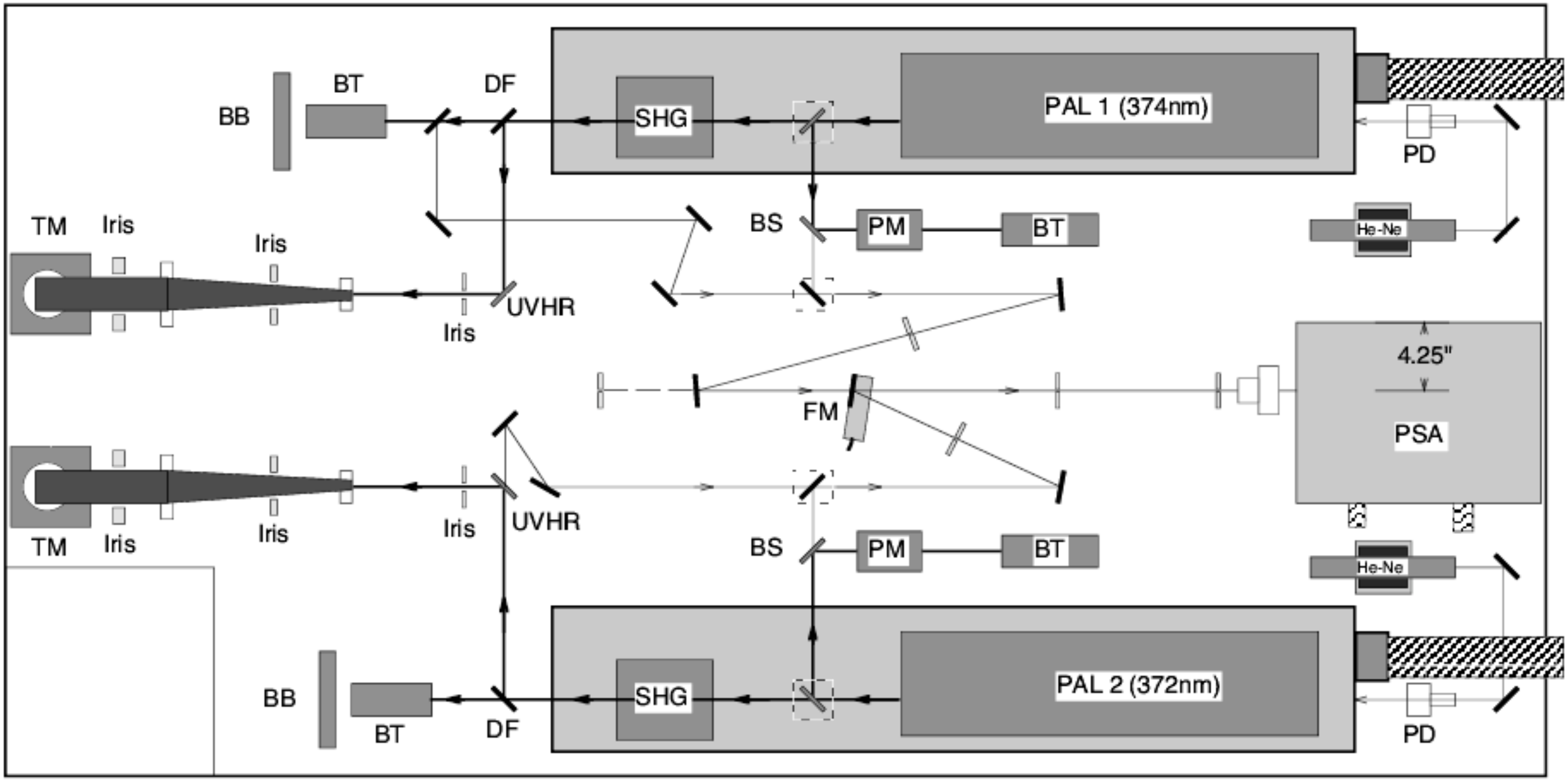
$$= \frac{g_2}{g_1} \frac{R_{\rm B374}}{R_{\rm B372}} \left(\frac{\lambda_{374}}{\lambda_{372}}\right)^{4.0117} \frac{E^2(\lambda_{374}, z)}{E^2(\lambda_{372}, z)}$$

$$\times \frac{\sigma_{\rm eff}(\lambda_{374}, T, \sigma_{\rm L374})}{\sigma_{\rm eff}(\lambda_{372}, T, \sigma_{\rm L372})} \exp(-\Delta E / k_{\rm B} T)$$

Fe Boltzmann Lidar Instrumentation



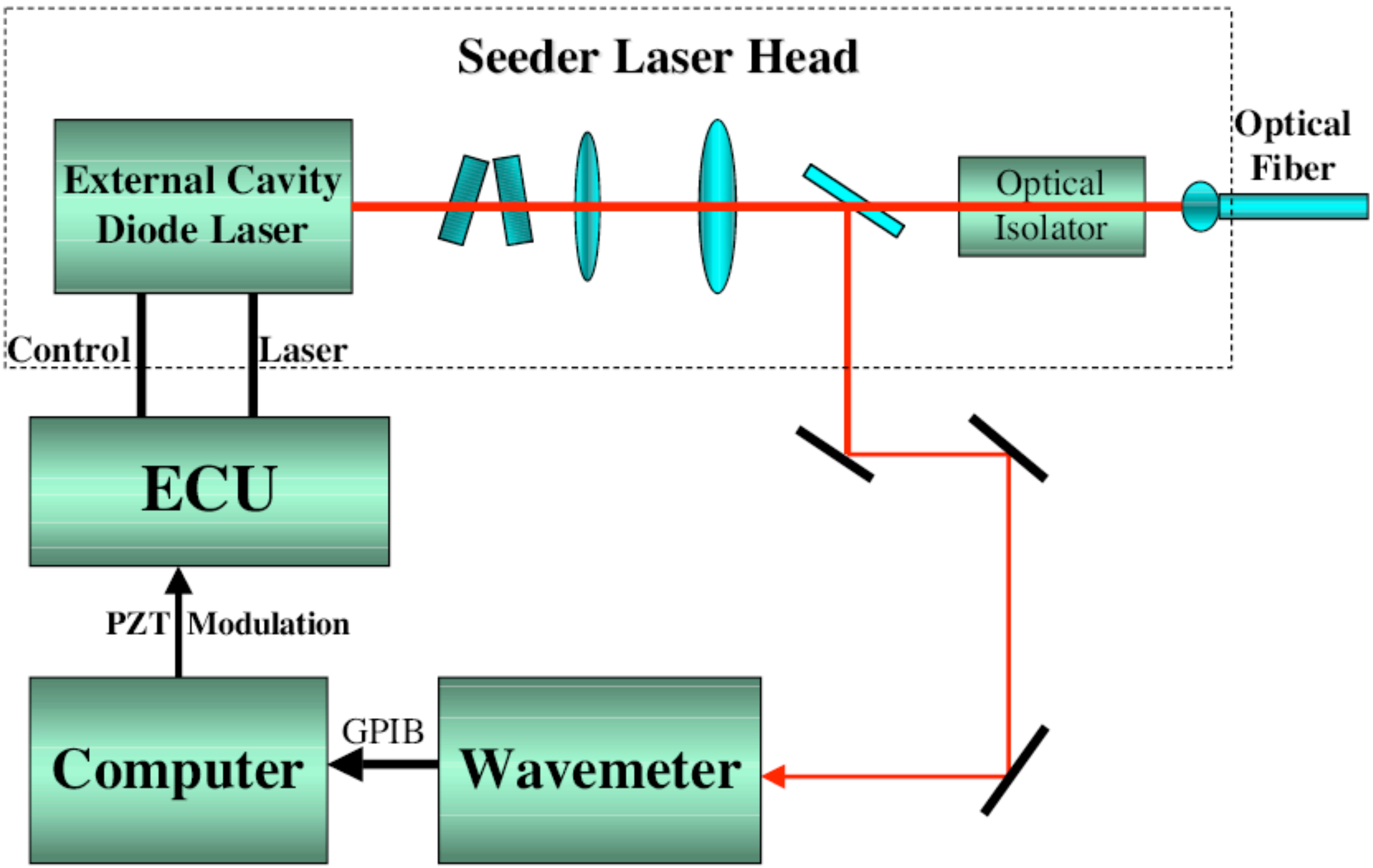
Fe Boltzmann Lidar Transmitter



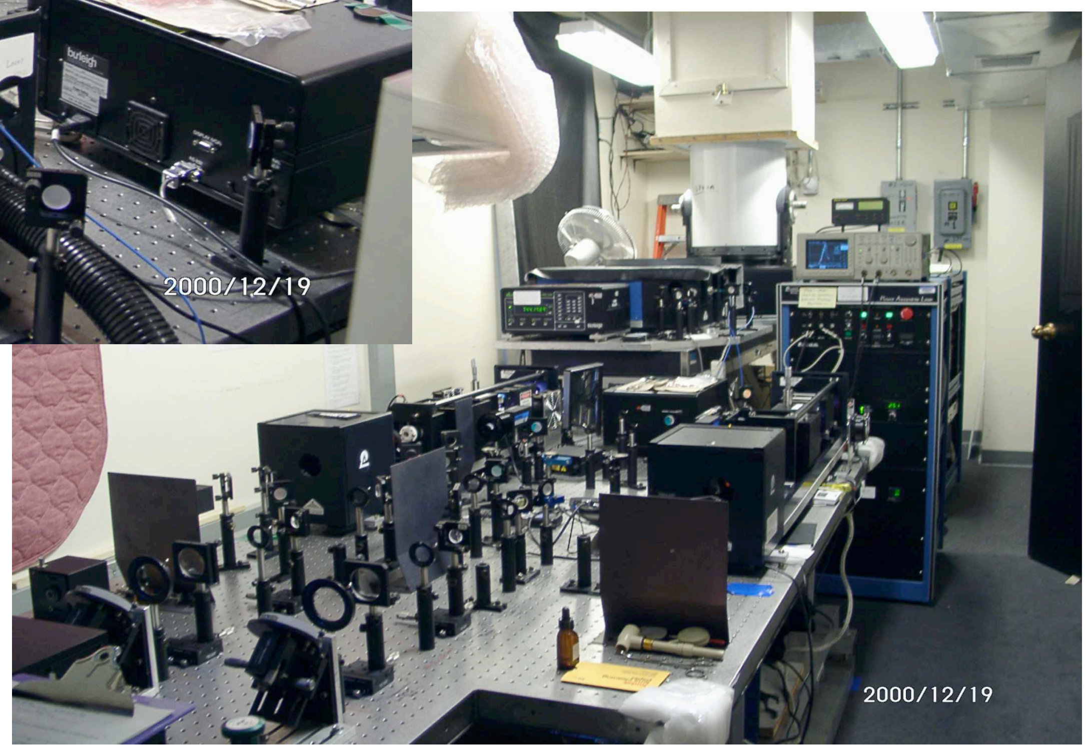
- Based on injection-seeded, frequency-doubled, pulse Alexandrite laser systems (372 and 374 nm output)

Fe Lidar Seeder Laser System

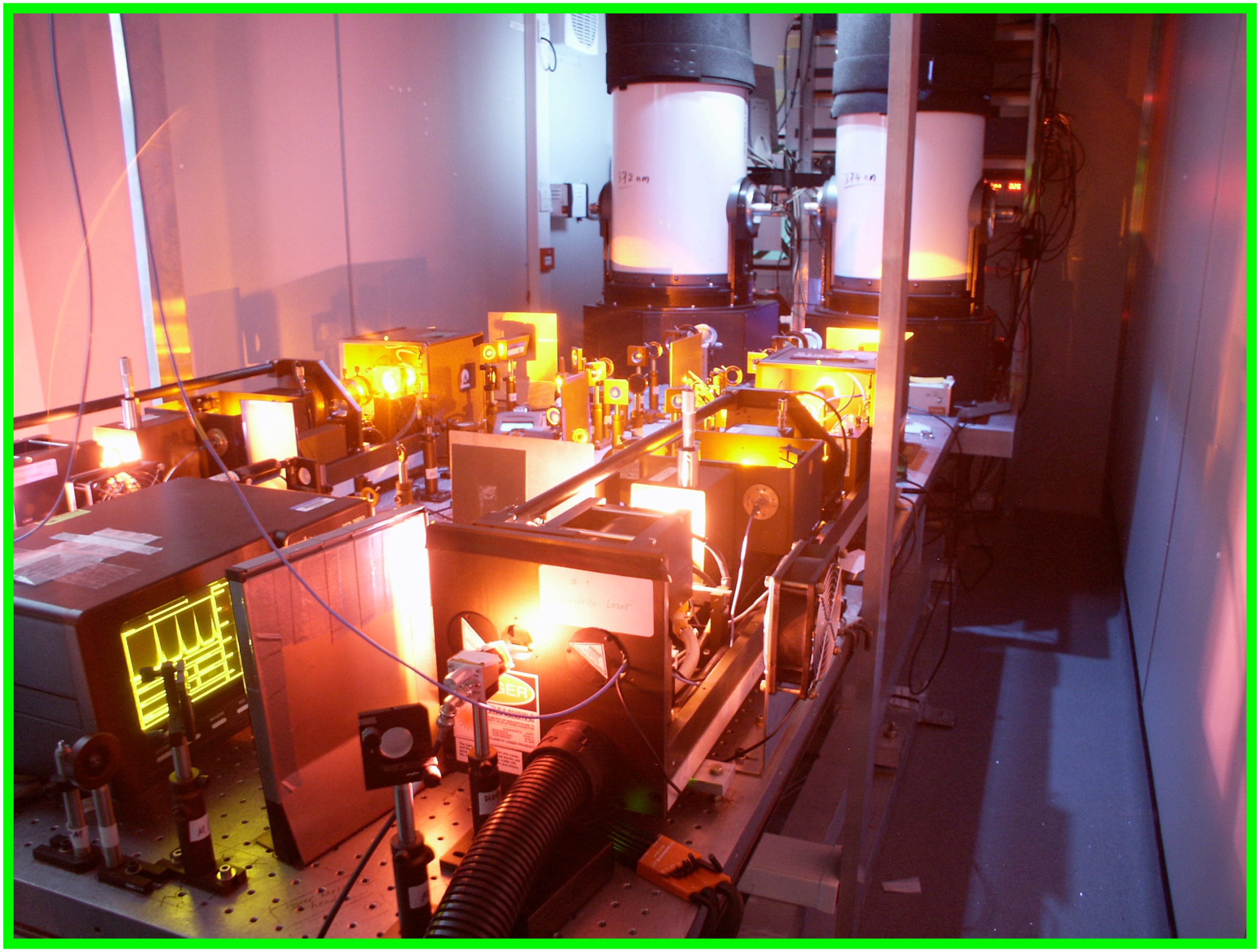
Seeder Laser Structure and Wavelength Control



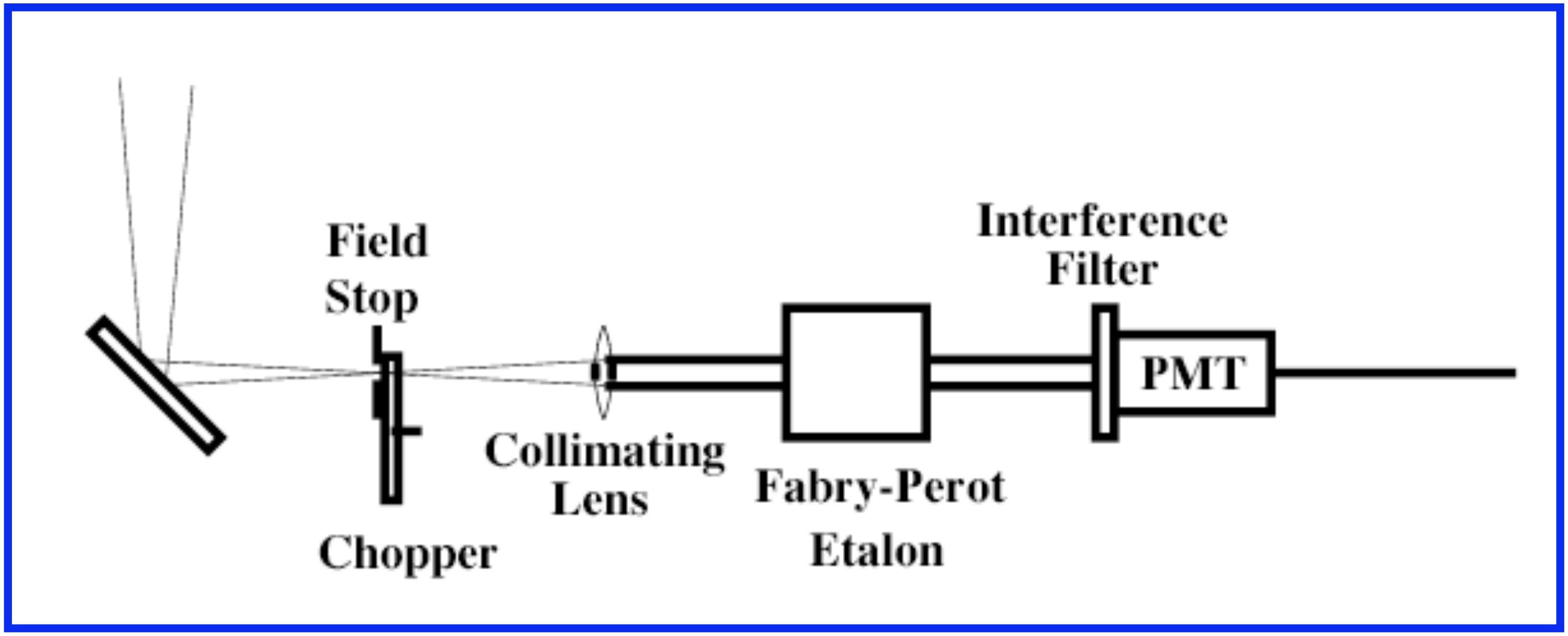
Fe Boltzmann Lidar @ South Pole

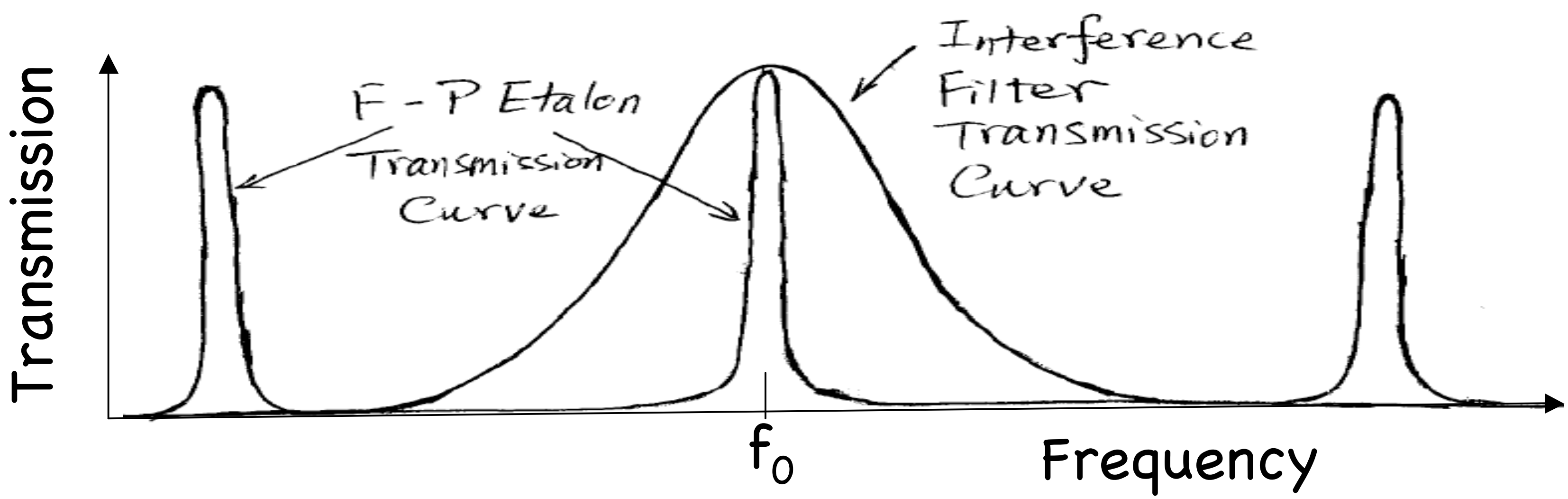


Fe Boltzmann Lidar @ Rothera

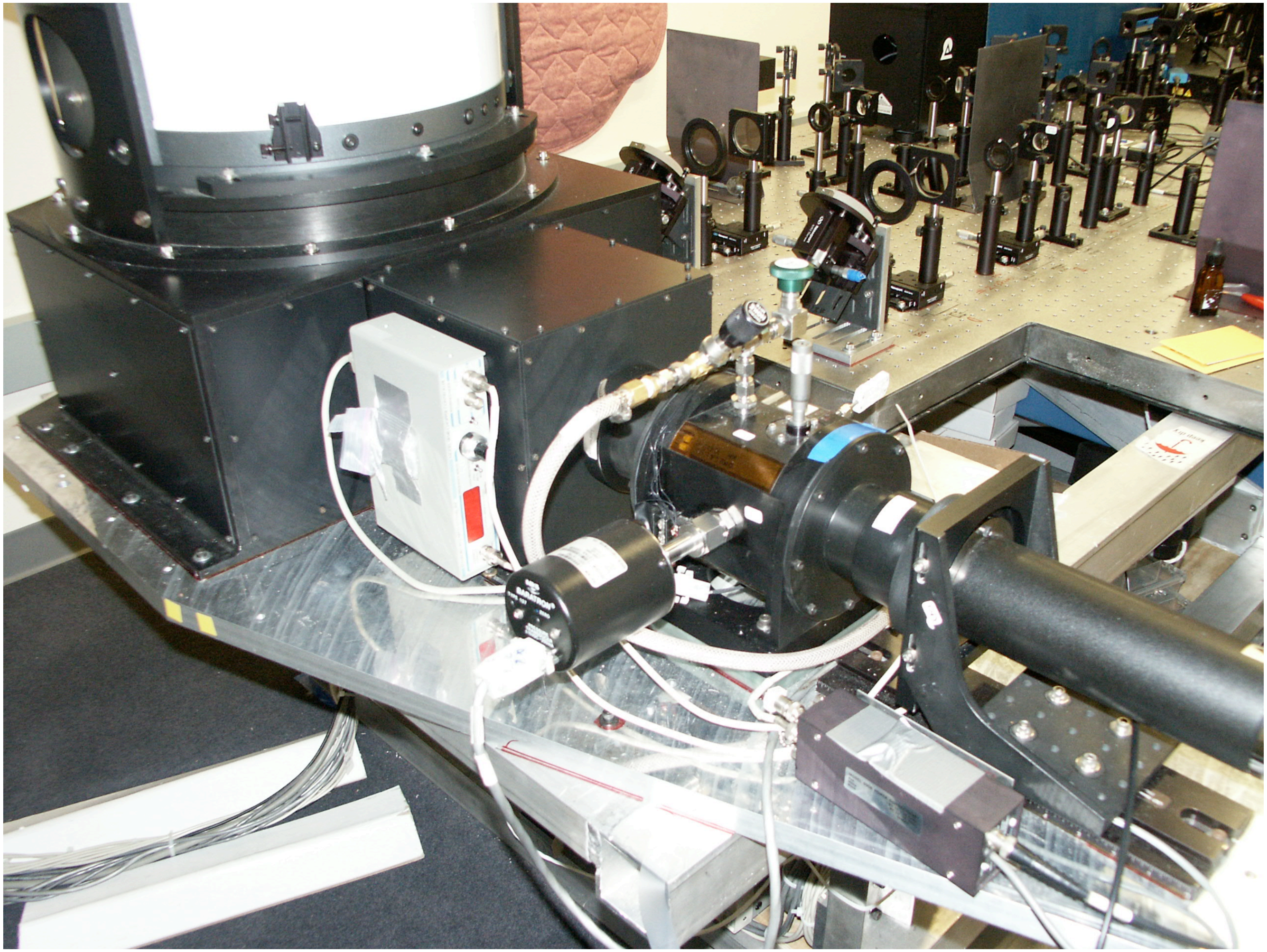


Fe Boltzmann Lidar Receiver



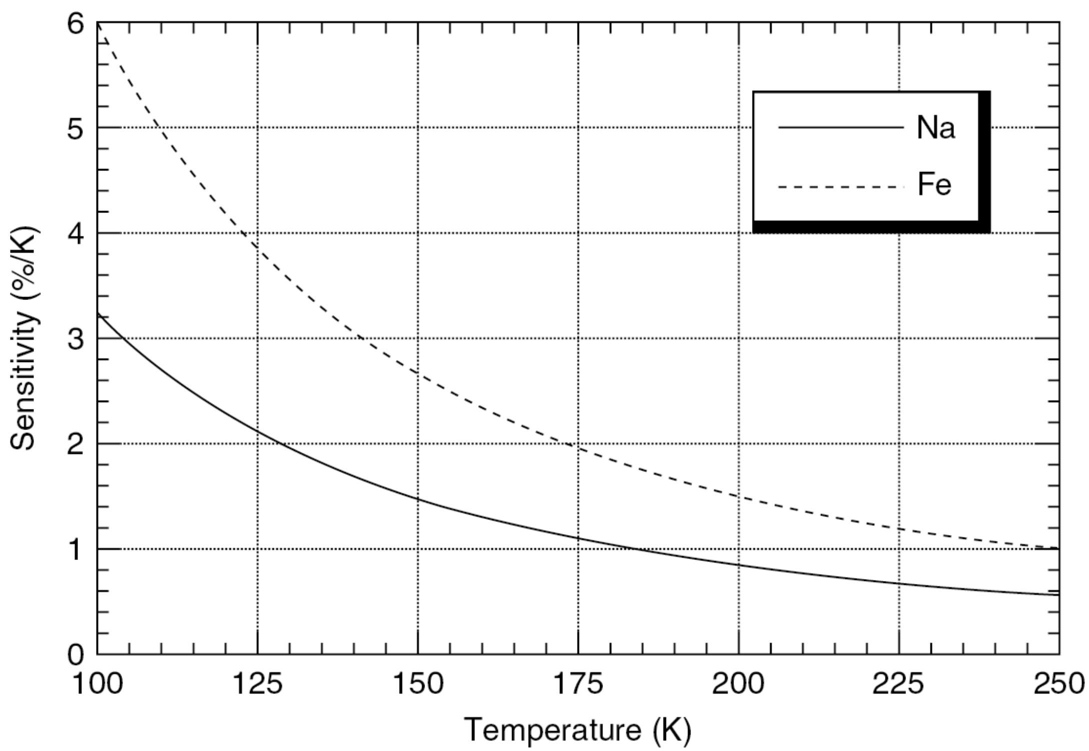


Fe Boltzmann Lidar Receiver



Sensitivity Analysis

$$S_{\mathrm{T}} = \frac{\partial R_{\mathrm{T}}/\partial T}{R_{\mathrm{T}}}$$



Summary

- Doppler technique and Boltzmann technique utilize Doppler effect and Boltzmann distribution that are directly temperature-dependent.
- The ratio technique has advantages to cancel lots of unknown parameters and leaves the ratio dependent on key parameters directly.
- Instrumentation for both techniques are sophisticated and complicated due to the high demands on frequency accuracy, linewidth, and power.

HW Project #2

- In addition to data retrieval, please derive sensitivities for the 2- and 3-frequency techniques of the Na Doppler lidar. Plot the sensitivities versus temperature from 100-300 K in one figure.