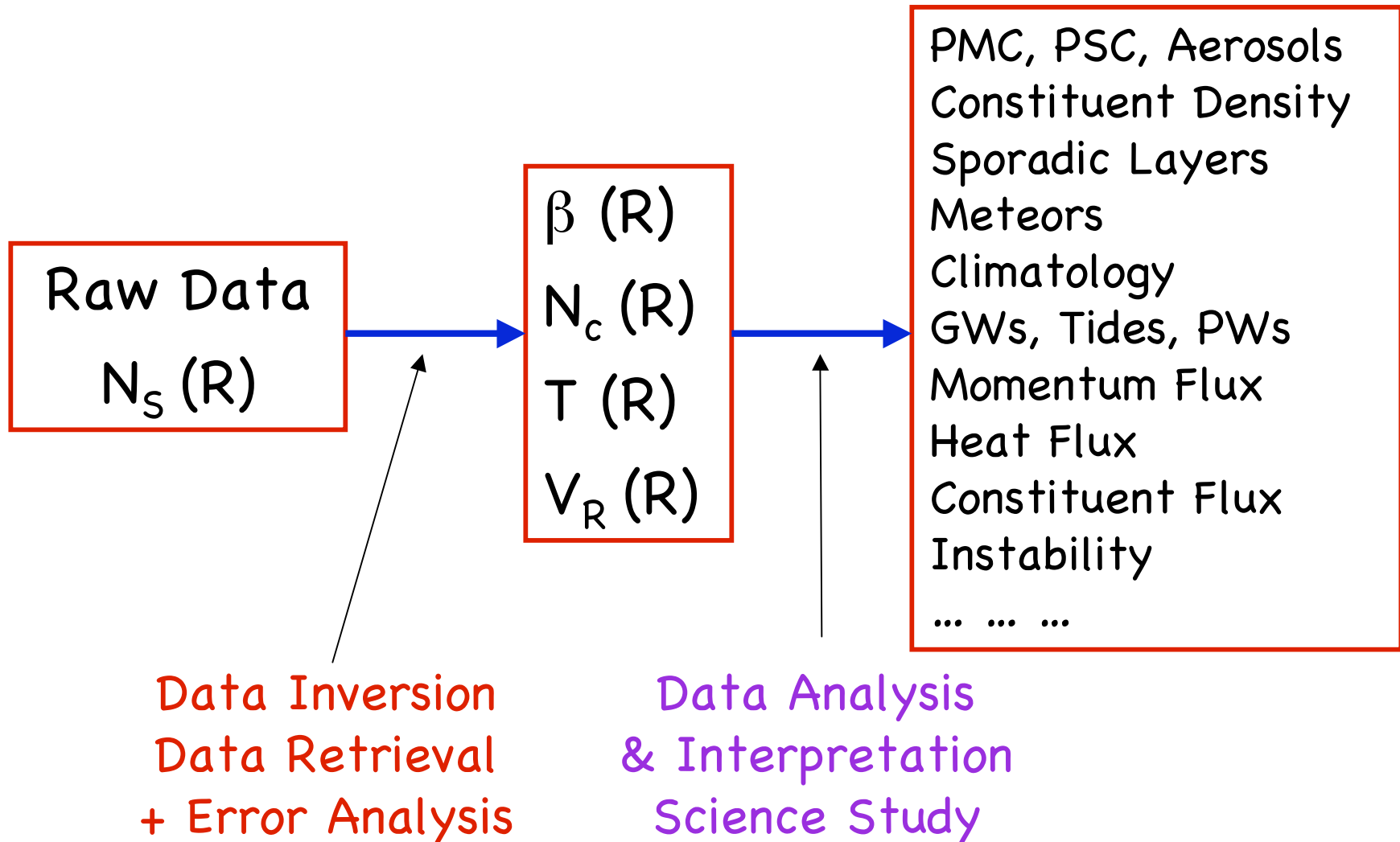


Lecture 12. Lidar Data Inversion

- ❑ Introduction
- ❑ From lidar equation solutions to data inversion
- ❑ Considerations in data inversion
- ❑ Preprocess procedure
- ❑ Process to retrieve β , n
- ❑ Process to retrieve T , V_R
- ❑ Summary

Introduction

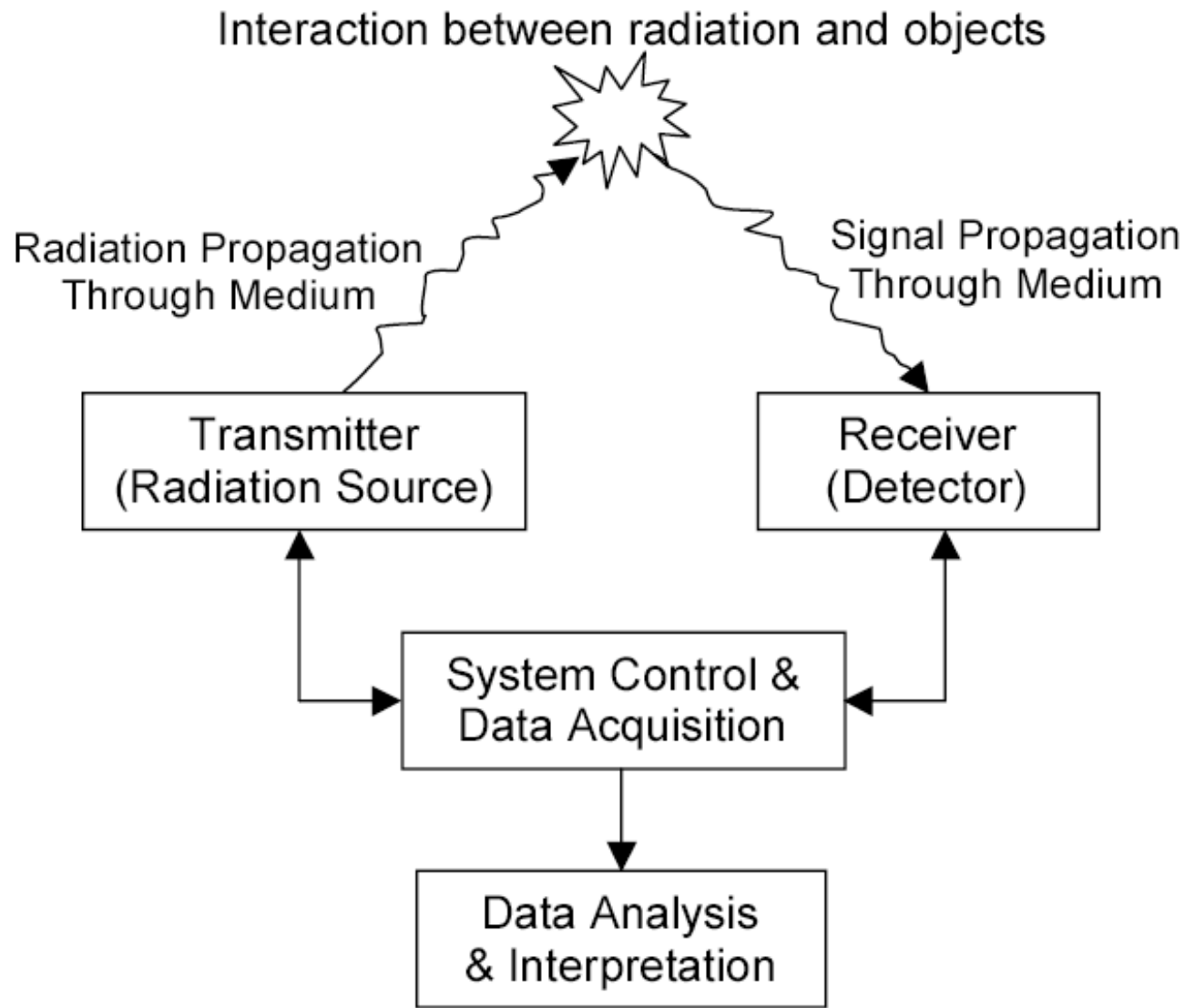
From Raw Data to Physical Parameters



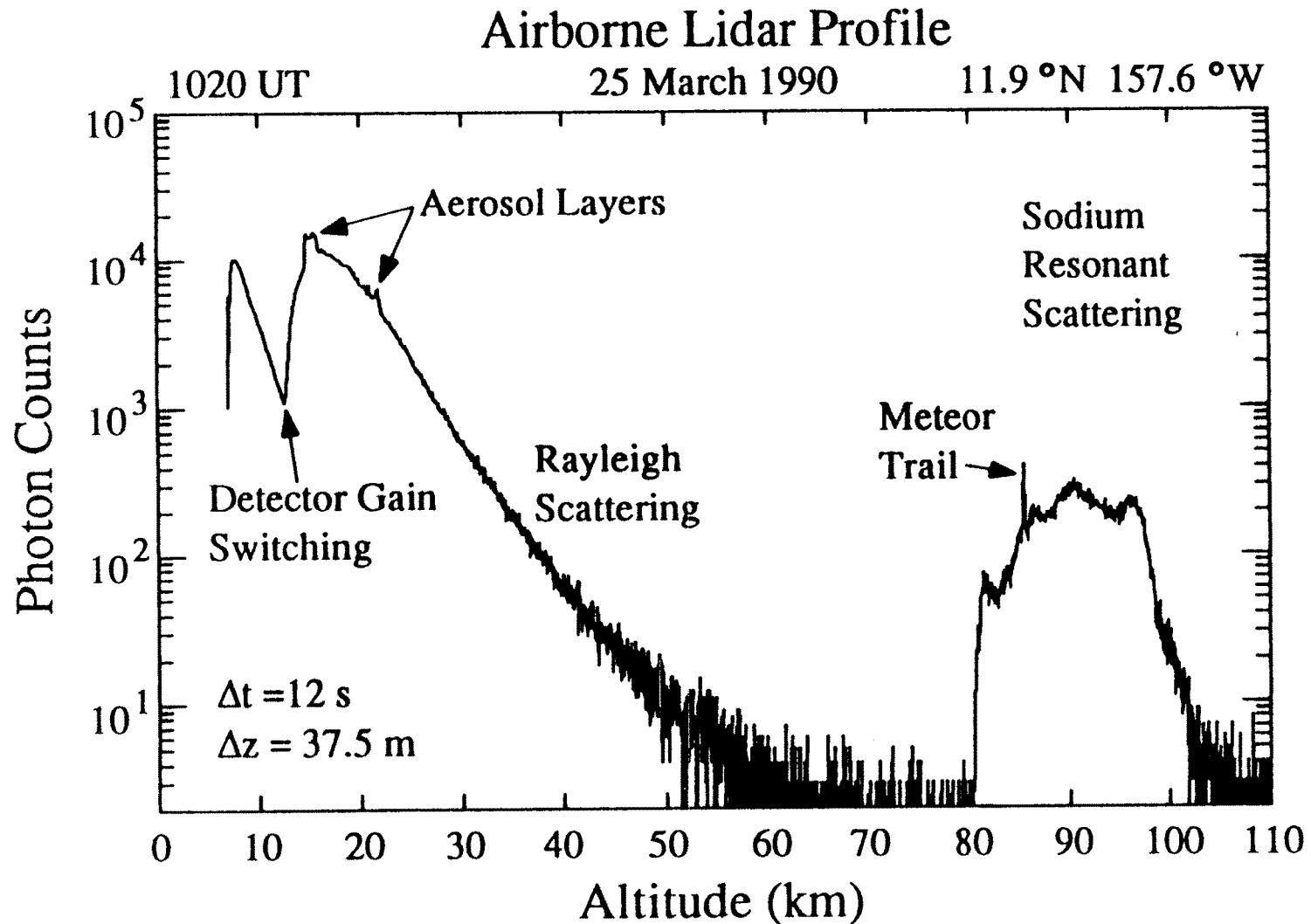
Introduction

- ❑ Lidar data inversion deals with the problems of how to derive meaningful physical parameters from raw photon counts.
- ❑ It is basically a reverse procedure to the development of lidar equation.
- ❑ It is necessary to understand the detailed physical procedure from light transmitting, to light propagation, to light interaction with objects, and to light detection, in order to conduct data inversion correctly.

Physical Picture in LIDAR Remote Sensing



Example Lidar Raw Signal



LIDAR Equation

□ General Form

$$N_S(\lambda, R) = \left(\frac{P_L(\lambda_L) \Delta t}{hc/\lambda_L} \right) \left(\beta(\lambda, \lambda_L, \theta, R) \Delta R \right) \left(\frac{A}{R^2} \right) \left(T(\lambda_L, R) T(\lambda, R) \right) \left(\eta(\lambda, \lambda_L) G(R) \right) + N_B$$

□ Lidar equation for aerosol (PMC) backscattering

$$N_S(\lambda, z) = \left(\frac{P_L(\lambda) \Delta t}{hc/\lambda} \right) \left[\beta_R(\lambda, z) + \beta_{aer}(\lambda, z) \right] \Delta z \left(\frac{A}{z^2} \right) T_a^2(\lambda, z) \left(\eta(\lambda) G(z) \right) + N_B$$

□ Lidar equation for pure Rayleigh backscattering

$$N_S(\lambda, z_R) = \left(\frac{P_L(\lambda) \Delta t}{hc/\lambda} \right) \left[\beta_R(\lambda, z_R) \right] \Delta z \left(\frac{A}{z_R^2} \right) T_a^2(\lambda, z_R) \left(\eta(\lambda) G(z_R) \right) + N_B$$

Solutions for Aerosol Backscatter

□ For lidar equations, we obtain

$$\frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} = \frac{\left(\frac{P_L(\lambda) \Delta t}{hc/\lambda} \right) [\beta_R(\lambda, z) + \beta_{aer}(\lambda, z)] \Delta z \left(\frac{A}{z^2} \right) T_a^2(\lambda, z) (\eta(\lambda) G(z))}{\left(\frac{P_L(\lambda) \Delta t}{hc/\lambda} \right) [\beta_R(\lambda, z_R)] \Delta z \left(\frac{A}{z_R^2} \right) T_a^2(\lambda, z_R) (\eta(\lambda) G(z_R))}$$

□ Therefore, the aerosol volume backscatter coefficient can be derived as

$$\beta_{aer}(\lambda, z) = \left[\frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} \cdot \frac{z^2}{z_R^2} - \frac{\beta_R(\lambda, z)}{\beta_R(\lambda, z_R)} \right] \beta_R(\lambda, z_R)$$

Solutions for Aerosol Backscatter

□ For well-mixed atmosphere below 100 km, Rayleigh volume backscatter coefficient can be simplified as

$$\beta(\lambda, z) = \sigma_R(z, \lambda) n_R(z)$$

□ Assuming $\sigma_R(z) = \sigma_R(z_R)$, we have

$$\beta_{aer}(\lambda, z) = \left[\frac{N_S(\lambda, z) - N_B}{N_S(\lambda, z_R) - N_B} \cdot \frac{z^2}{z_R^2} - \frac{n_R(z)}{n_R(z_R)} \right] \beta_R(\lambda, z_R)$$

where volume Rayleigh backscatter coefficient is given by an empirical formula for well-mixed atmosphere:

$$\beta_R(\lambda, z_R) = 2.938 \times 10^{-32} \frac{P(z_R)}{T(z_R)} \cdot \frac{1}{\lambda^{4.0117}}$$

LIDAR Equation

- Lidar equation for pure Rayleigh backscattering

$$N_S(\lambda, z_R) = \left(\frac{P_L(\lambda) \Delta t}{hc/\lambda} \right) \left[\sigma_R(\pi, \lambda) n_R(z_R) \right] \Delta z \left(\frac{A}{z_R^2} \right) T_a^2(\lambda, z_R) (\eta(\lambda) G(z_R)) + N_B$$

- Lidar equation for resonance fluorescence

$$N_S(\lambda, z) = \left(\frac{P_L(\lambda) \Delta t}{hc/\lambda} \right) \left[\sigma_{eff}(\lambda, z) n_c(z) R_B(\lambda) + \sigma_R(\pi, \lambda) n_R(z) \right] \Delta z \left(\frac{A}{4\pi z^2} \right) \times \left(T_a^2(\lambda) E^2(\lambda, z) \right) (\eta(\lambda) G(z)) + N_B$$

- Similar to the derivation for aerosol scattering

$$n_c(z) = \left[\frac{N_S(\lambda, z) - N_B}{N_R(\lambda, z_R) - N_B} \cdot \frac{z^2}{z_R^2} - \frac{n_R(z)}{n_R(z_R)} \right] \cdot \frac{4\pi\sigma_R(\pi, \lambda) n_R(z_R)}{\sigma_{eff}(\lambda) R_B(\lambda) E^2(\lambda, z)}$$

Solutions for Constituent

- Recall the empirical formula for Rayleigh scattering

$$\sigma_R(\pi, \lambda) n_R(z_R) = 2.938 \times 10^{-32} \frac{P(z_R)}{T(z_R)} \cdot \frac{1}{\lambda^{4.0117}}$$

- The extinction can be derived from

$$E(\lambda, z) = \exp\left(-\int_{z_{bottom}}^z \sigma_{eff}(\lambda, z) n_c(z) dz\right) = \exp\left(-\sum_{z_{bottom}}^z \sigma_{eff}(\lambda, z) n_c(z) \Delta z\right)$$

- The constituent density is given by

$$n_c(z) = \left[\frac{N_S(\lambda, z) - N_B}{N_R(\lambda, z_R) - N_B} \cdot \frac{z^2}{z_R^2} - \frac{n_R(z)}{n_R(z_R)} \right] \cdot \frac{4\pi \times 2.938 \times 10^{-32} \frac{P(z_R)}{T(z_R)} \cdot \frac{1}{\lambda^{4.0117}}}{\sigma_{eff}(\lambda) R_B(\lambda) E^2(\lambda, z)}$$

Indications From Solutions

- ☐ Background subtraction ($- N_B$)
 - ☐ Range-dependence removal ($\times z^2$)
 - ☐ Rayleigh normalization ($1/(N_S(z_R)-N_B)$)
 - ☐ Rayleigh subtraction ($- n_R(z)/n_R(z_R)$)
 - ☐ Rayleigh scattering coefficient calculation
(find $P(z_R)$ and $T(z_R)$ from model)
 - ☐ Effective cross-section calculation (σ_{eff})
 - ☐ Extinction calculation (E)
 - ☐ Start from layer bottom
- preprocess

Considerations in Data Inversion

□ How to obtain associated information like date, time, location, base altitude, operation conditions?

-- from data header and other info source

□ How to obtain range or altitude information?

-- from bin number, data header and other source

$$R = n_{bin} \cdot t_{bin} \cdot c / 2$$

$$z = R \cdot \cos \theta + z_{base}$$

R is range, n_{bin} is bin number, t_{bin} is bin width in time, c is light speed, z is absolute altitude, θ is off-zenith angle, and z_{base} is the base altitude relative to sea-level.

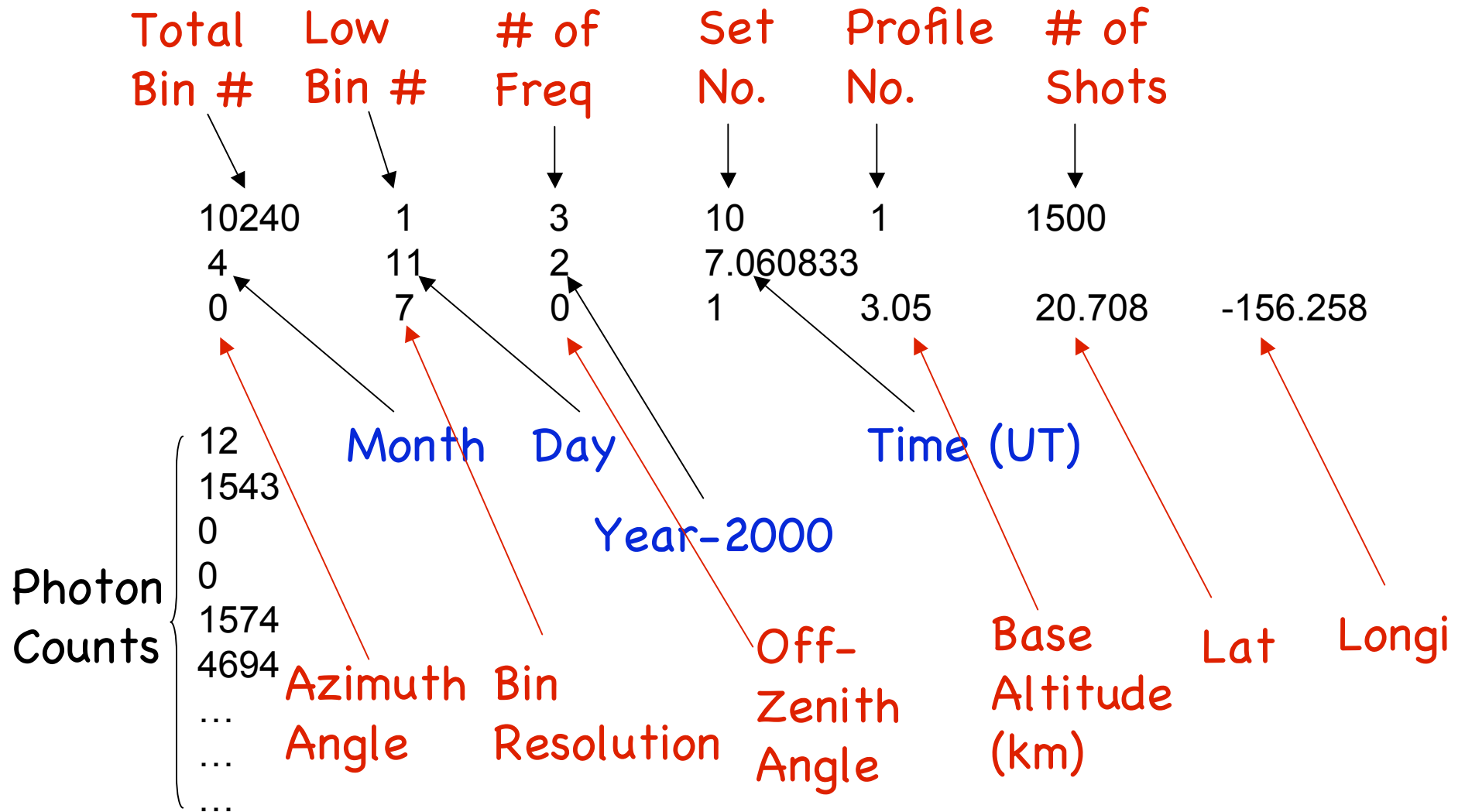
Considerations in Data Inversion

- ❑ Common raw data format:

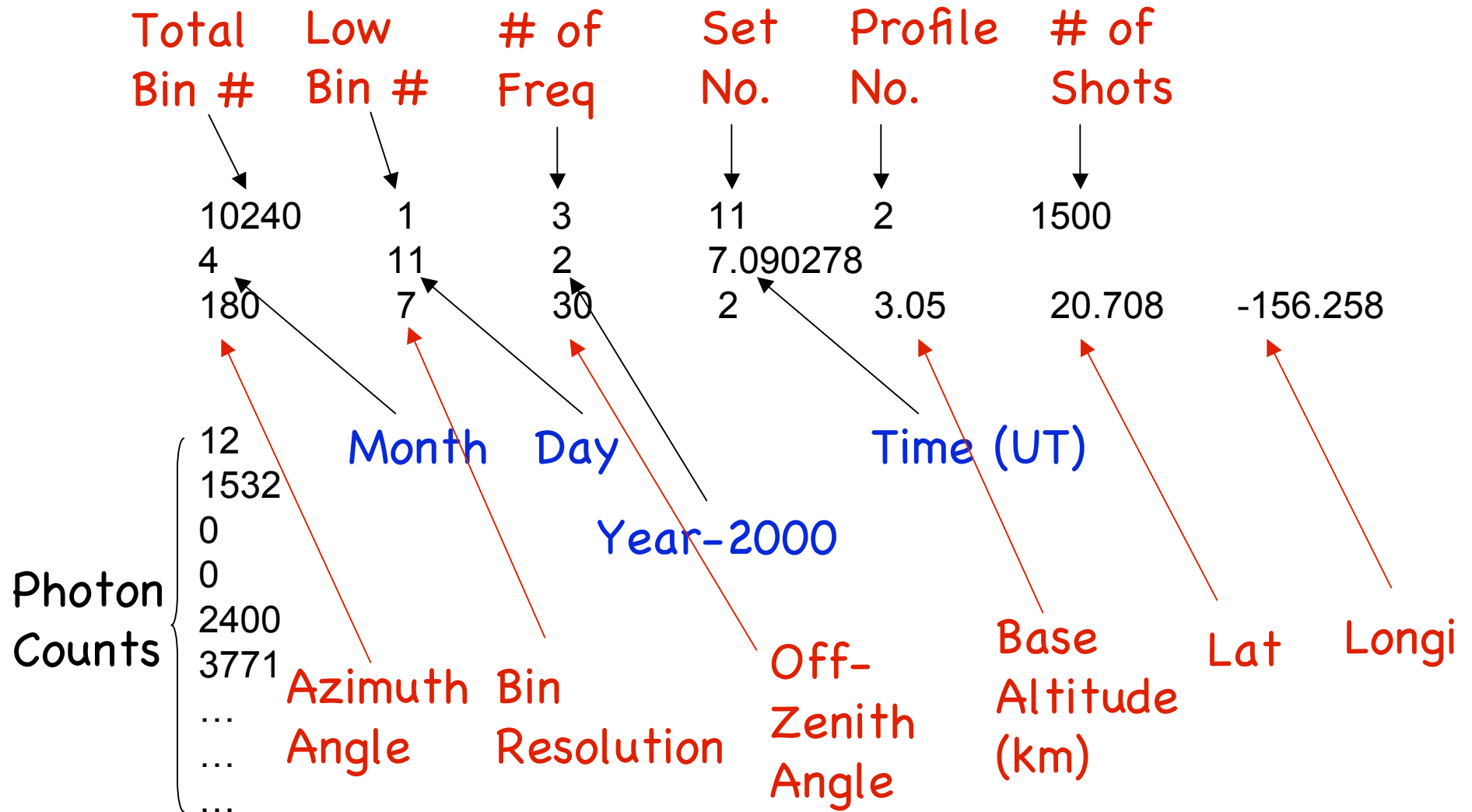
Header + One Column Photon Counts

- ❑ ASCII code - text file
- ❑ Binary code - special formats

Example of lidar raw data

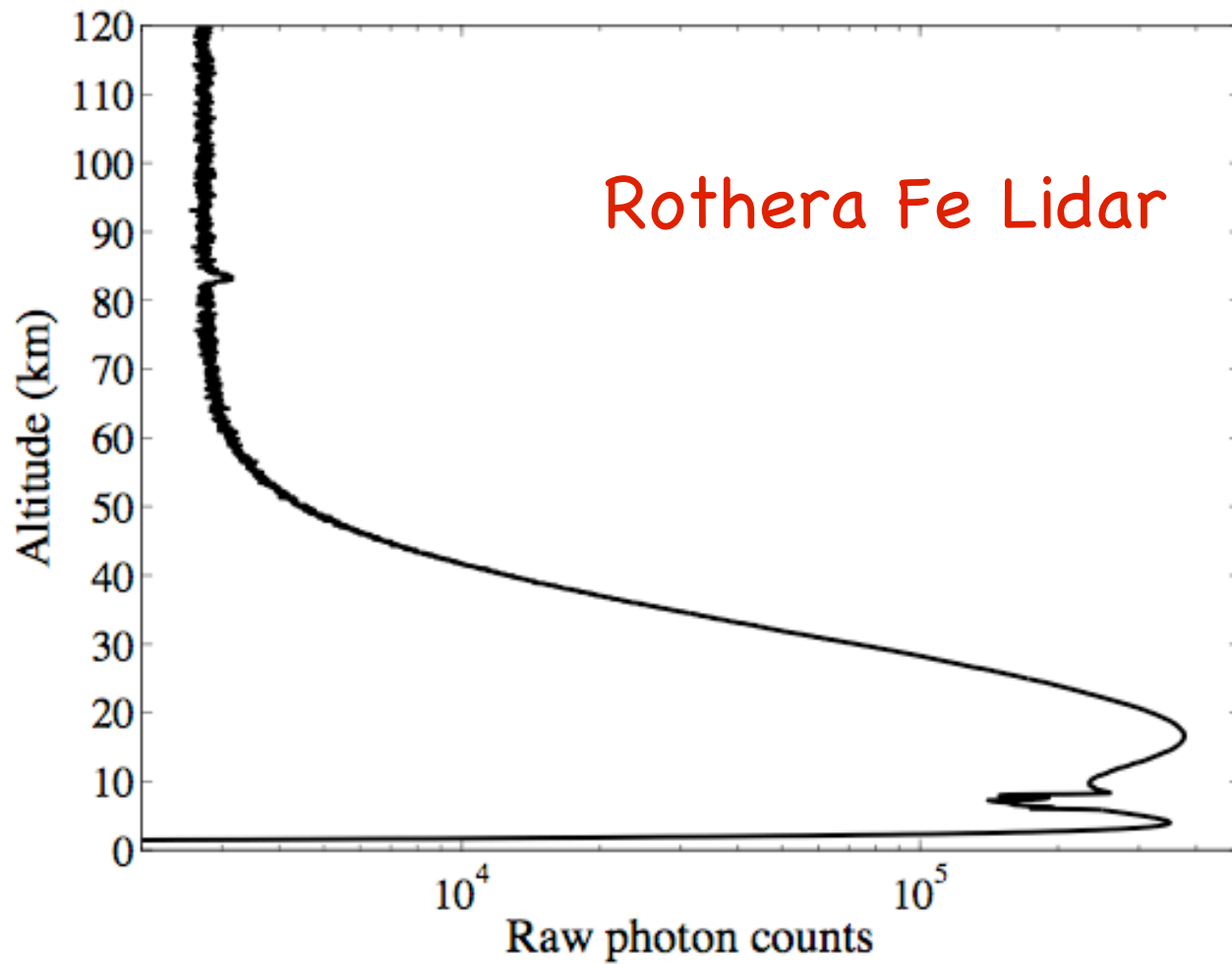


Example of lidar raw data

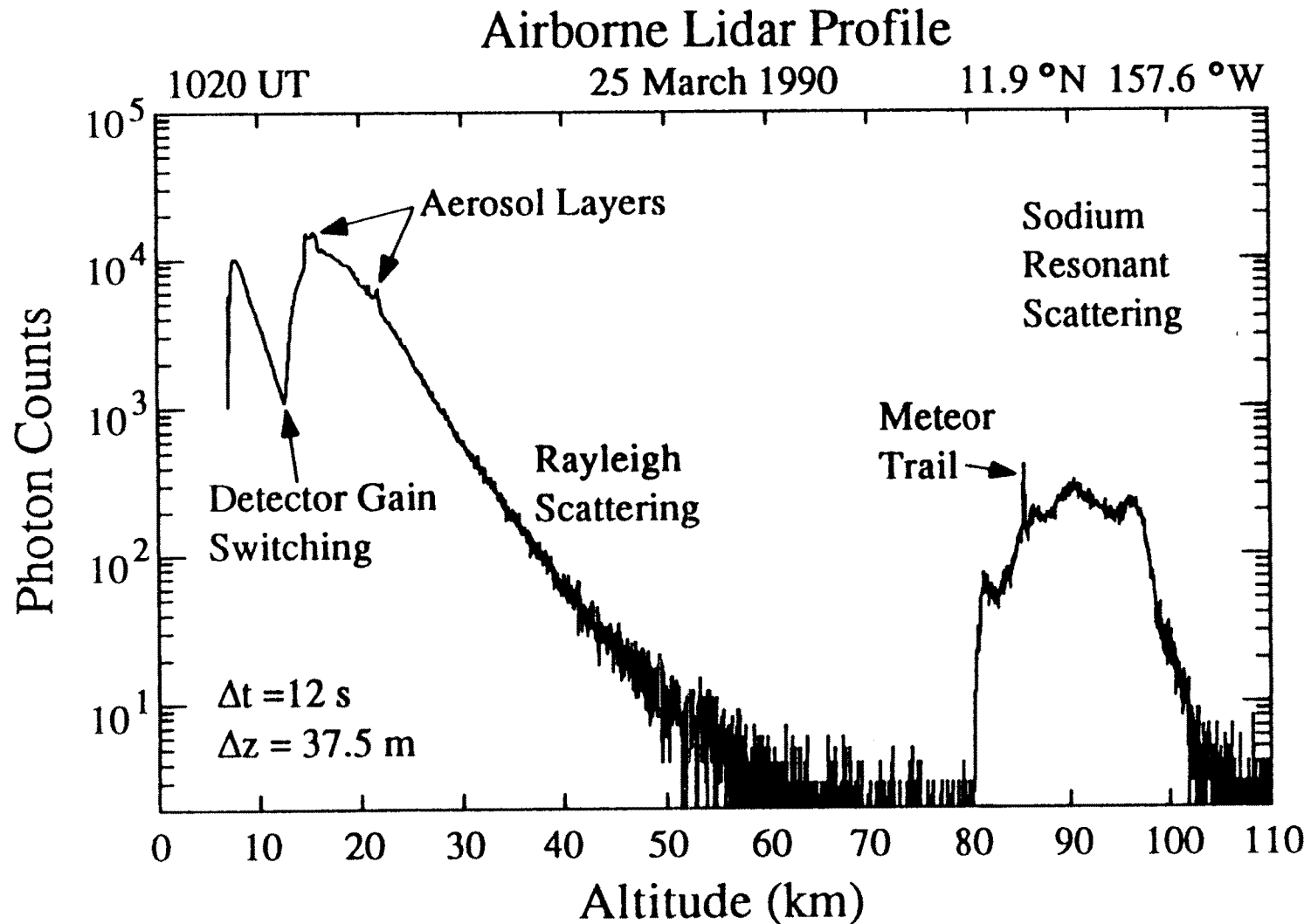


Lidar Raw Signal

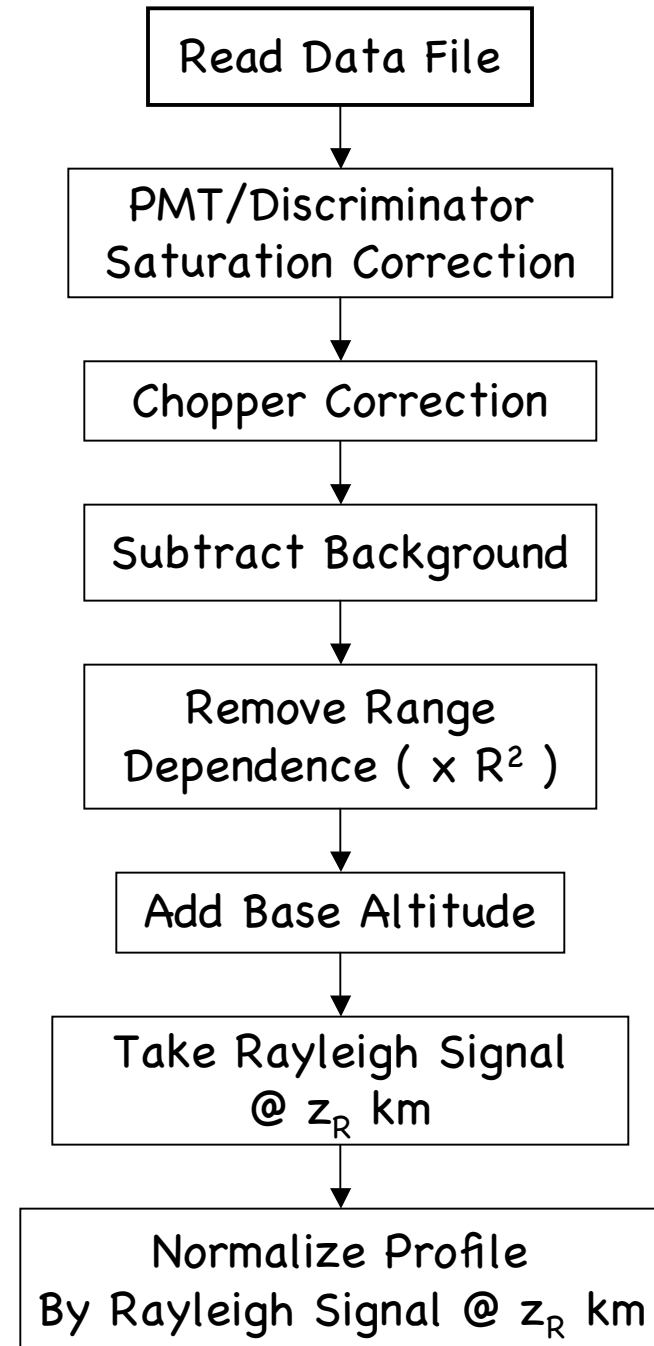
- ❑ Understand the lidar signal – where the signals originate from (Mie, Rayleigh, fluorescence)?



Example Lidar Raw Signal

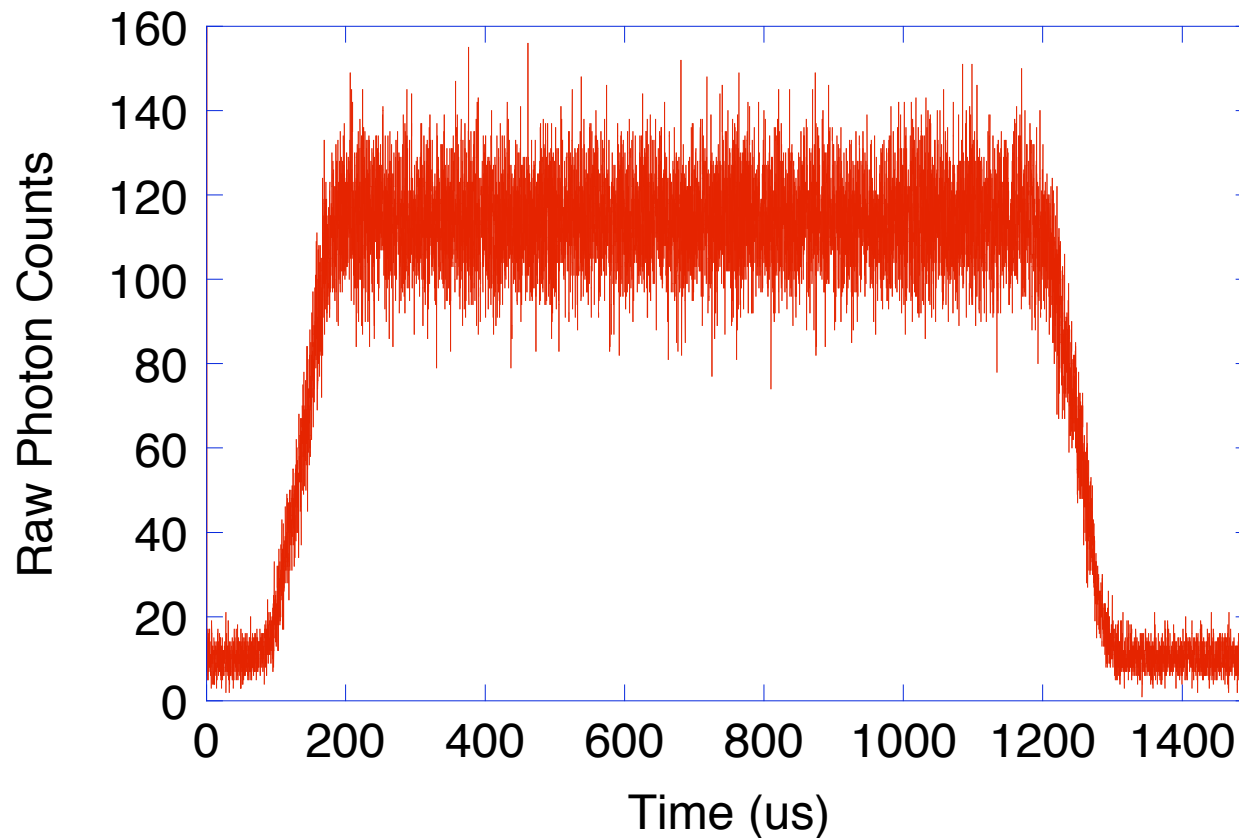


Preprocess Procedure Flow Chart

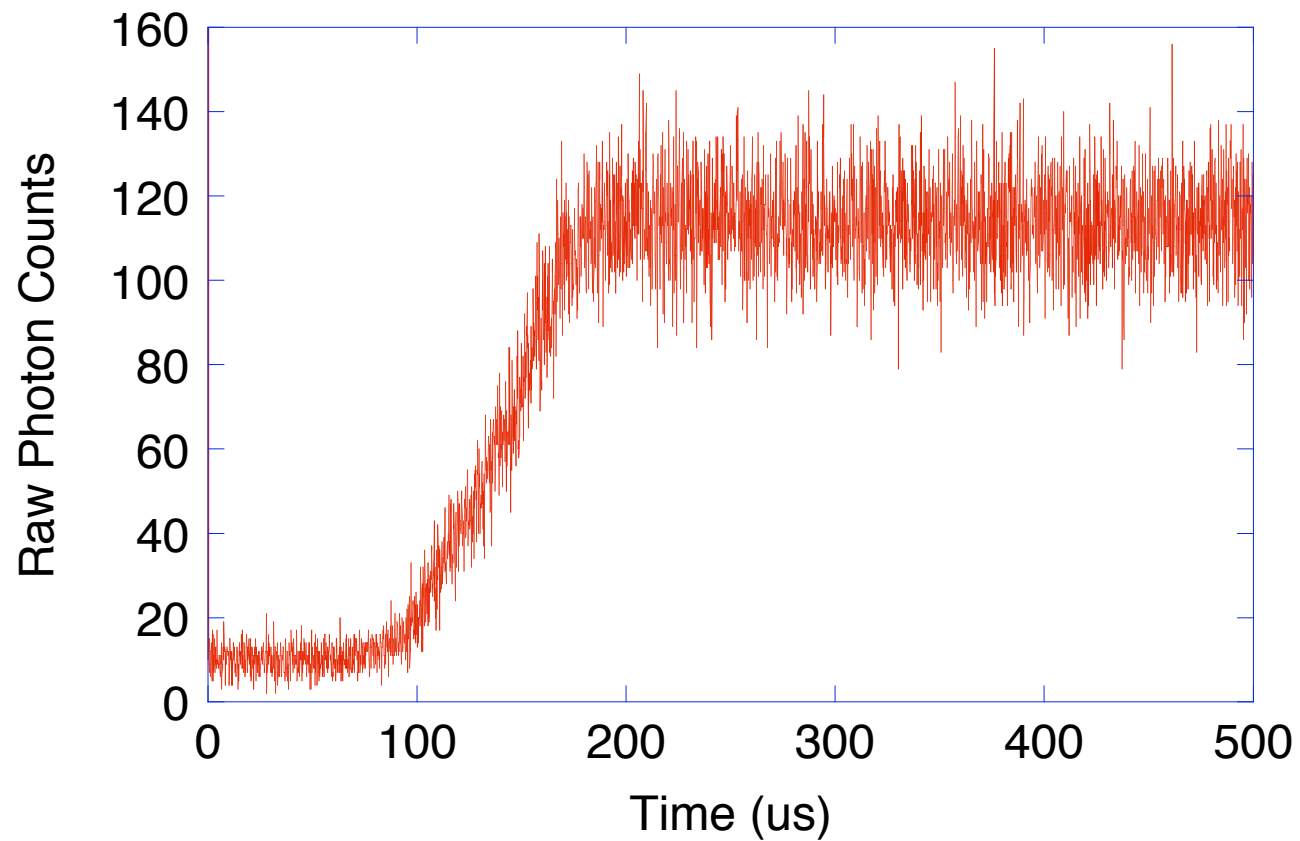


Considerations in Receiver Detection

- ❑ Chopper: how it works? – a real profile



Chopper Correction



Nonlinearity of PMT + Discriminator

For small input photon flux, PMT output photon counts are proportional to the input photon counts:

$$\lambda_{oP} = \lambda_S = \lambda_i \eta_{QE}$$

When the input photon flux is considerably large, the output photon counts are no longer linear with input photons. Nonlinearity of PMT occurs:

$$\lambda_{oP} = \lambda_S e^{-\lambda_S \tau_p}$$

A discriminator is used to judge real photon signals and also has a saturation effect, i.e., its output photon counts are smaller than input photon counts when input count rate is large:

$$\lambda_o = \frac{\lambda_{iD}}{1 + \lambda_{iD} \tau_d}$$

Nonlinearity of PMT + Discriminator

Since PMT output is the input of discriminator

$$\lambda_{iD} = \lambda_{oP}$$

we obtain

$$\lambda_o = \frac{\lambda_s e^{-\lambda_s \tau_p}}{1 + \lambda_s \tau_d e^{-\lambda_s \tau_p}} = \frac{\lambda_s e^{-\lambda_s \tau_p}}{1 + \lambda_s \tau_d e^{-\lambda_s \tau_p}}$$

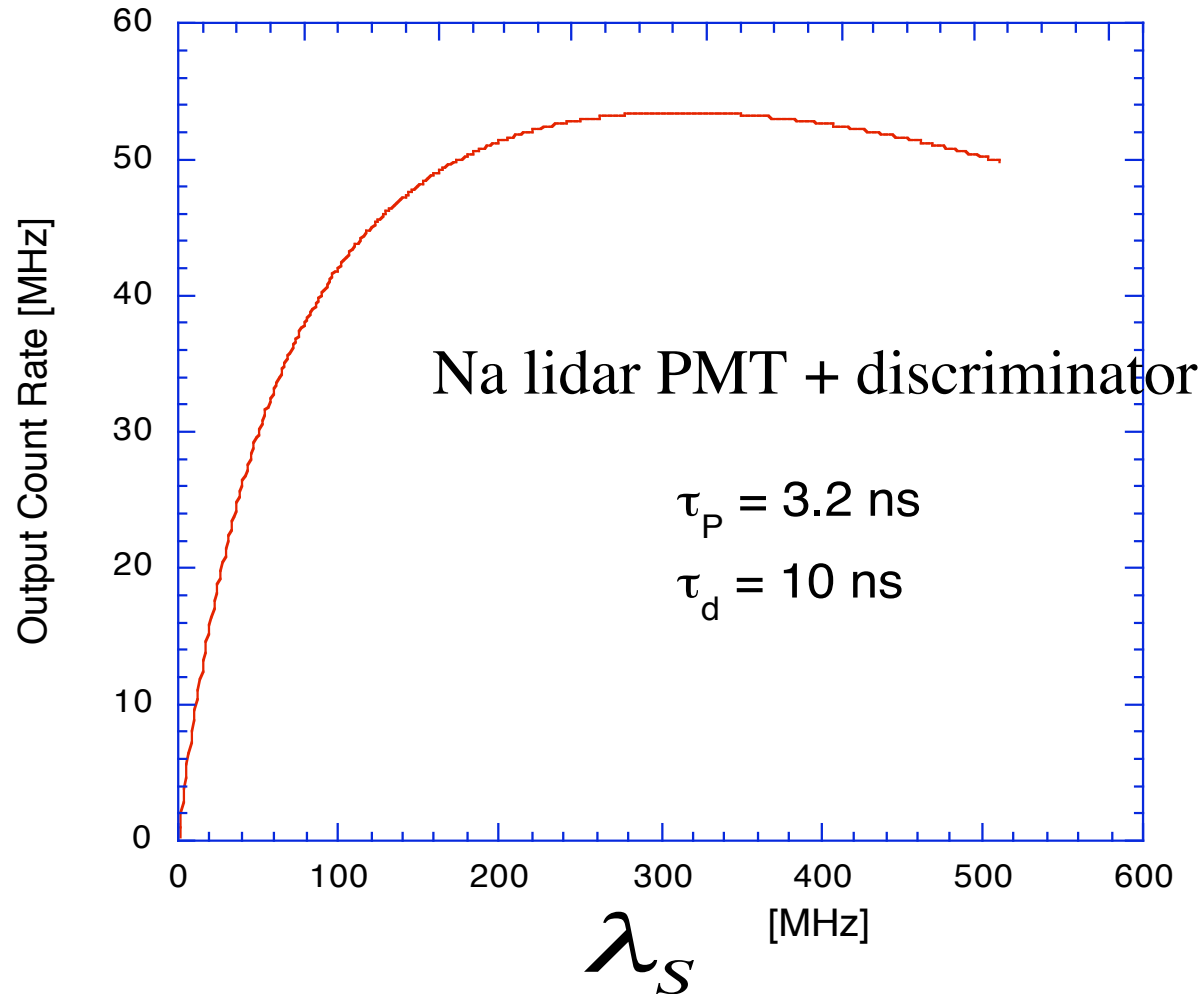
where

$$\lambda_s = \lambda_i \eta_{QE} \quad \eta_{QE} \text{ is the quantum efficiency of cathode}$$

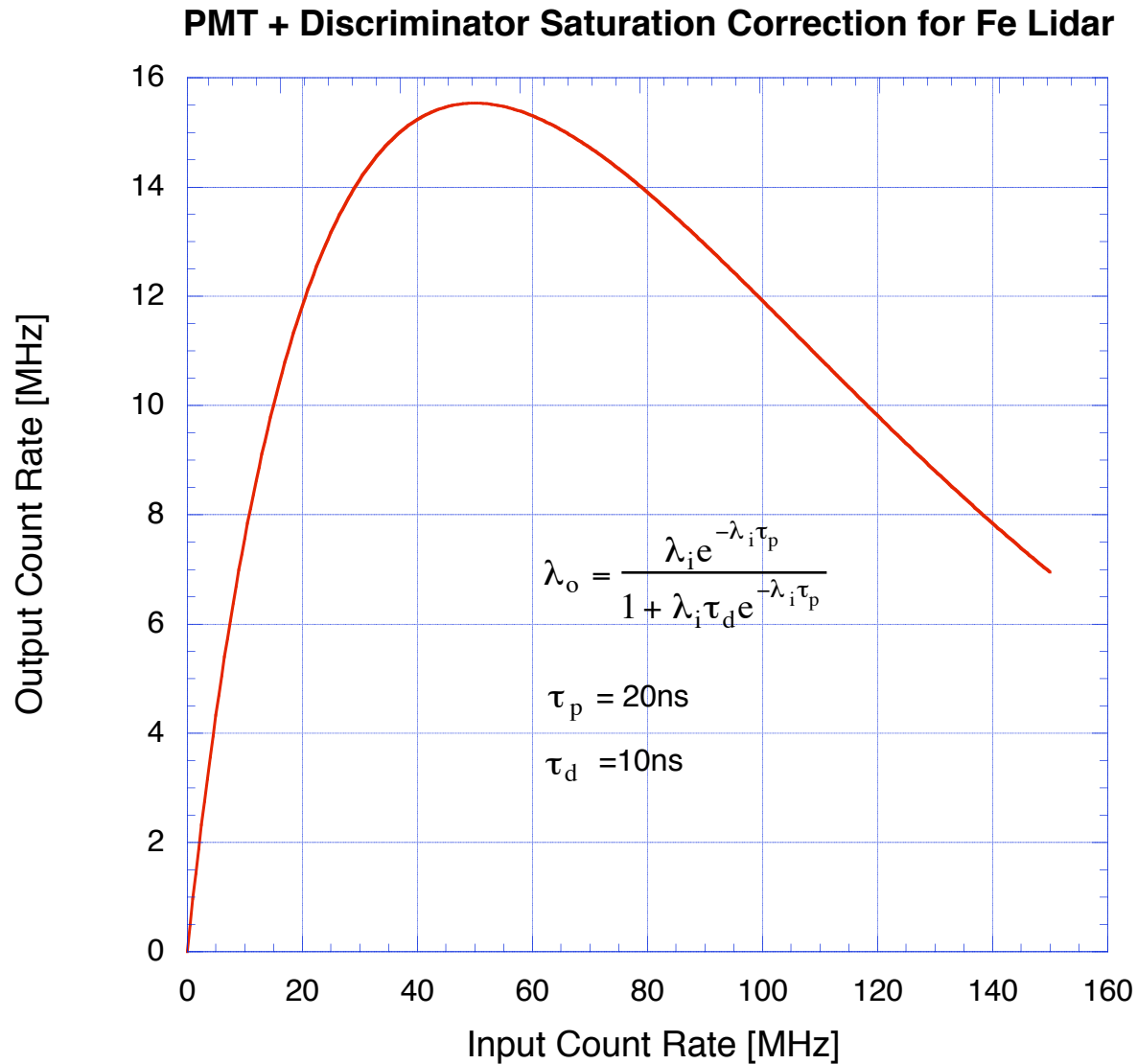
Maximum output count rate is reached when $\lambda_s = 1/\tau_p$

$$\lambda_{o \max} = \frac{1}{\tau_p e + \tau_d} \quad \longrightarrow \quad \tau_p = \frac{\frac{1}{\lambda_{o \max}} - \tau_d}{e}$$

Nonlinearity of PMT + Discriminator



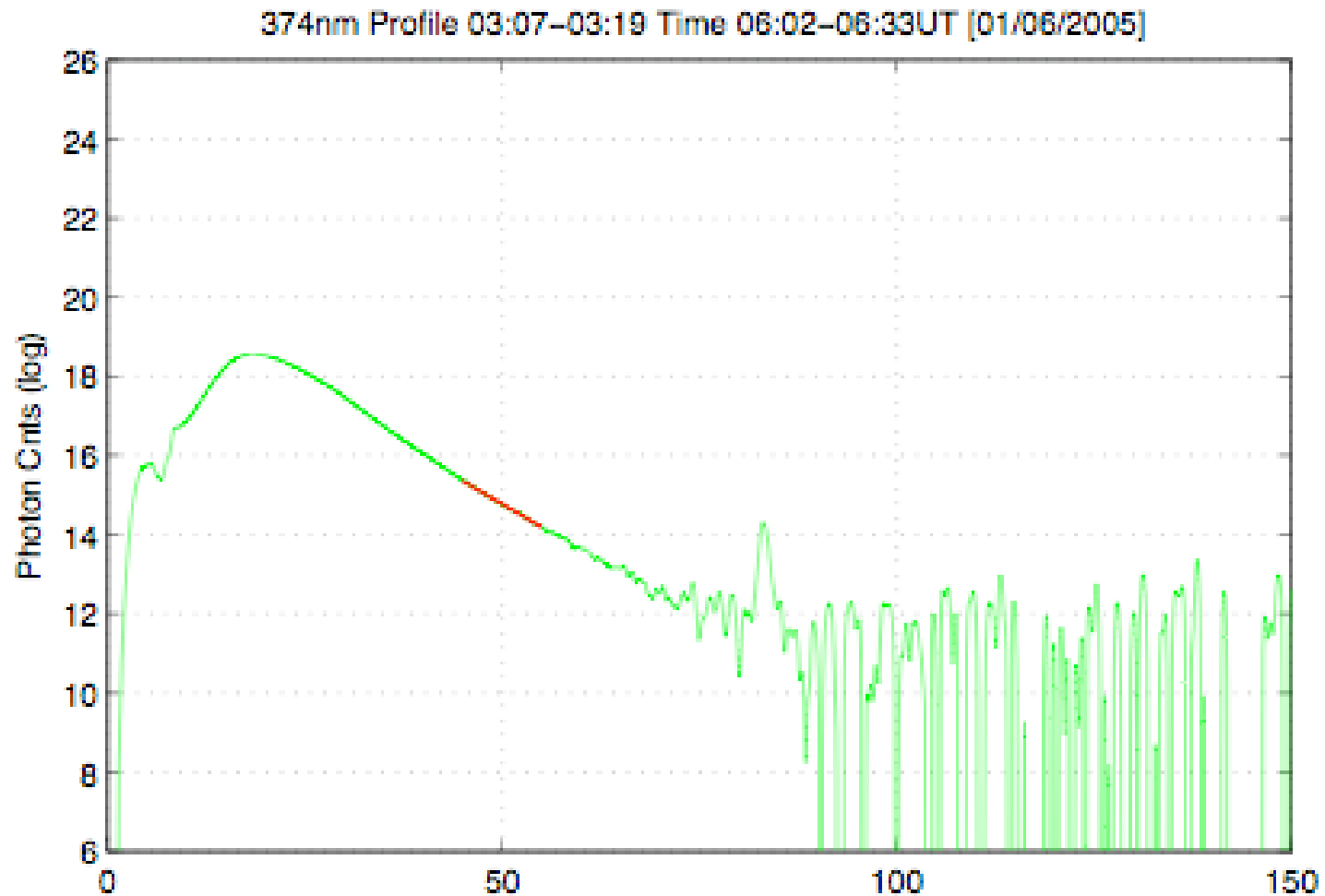
Nonlinearity of PMT + Discriminator



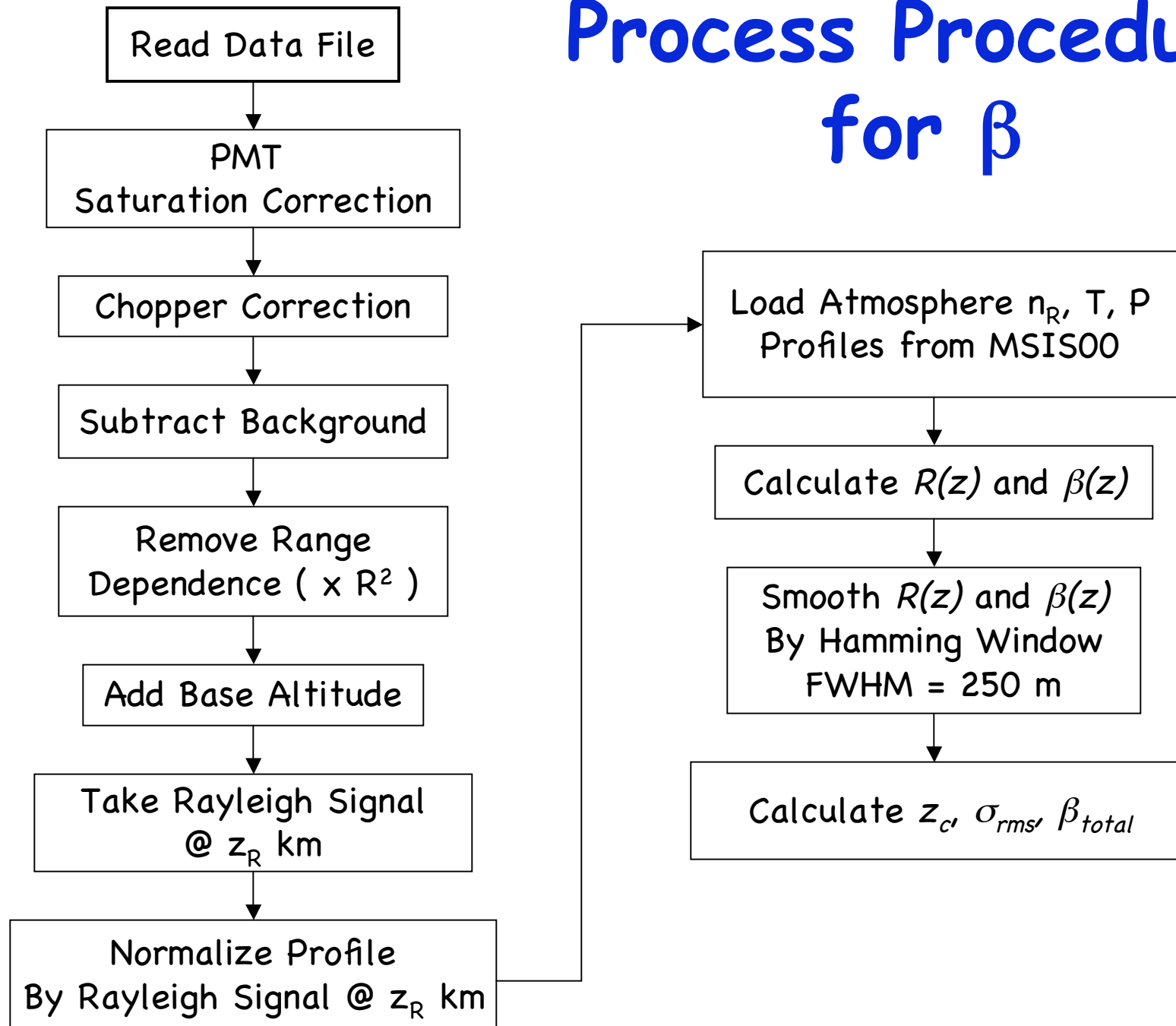
Preprocess Procedure

- ☐ Read raw data
- ☐ PMT/Discriminator saturation correction
- ☐ Chopper correction
- ☐ Background subtraction
- ☐ Range-dependence removal
- ☐ Base altitude adjustment
- ☐ Rayleigh normalization

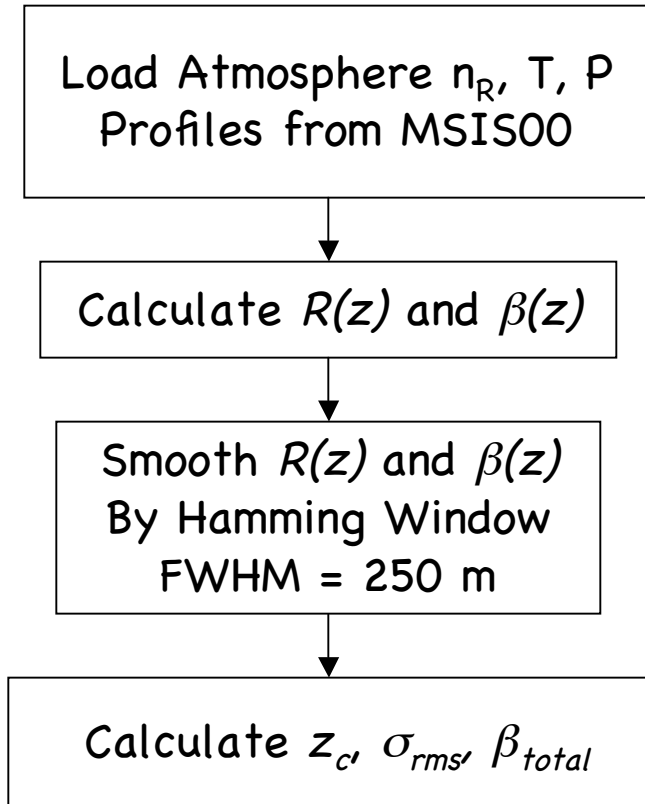
Example: Rayleigh Fit



Process Procedure for β



Process Procedure for β



$$R = \frac{[N_S(z) - N_B] \cdot z^2}{[N_S(z_{RN}) - N_B] \cdot z_{RN}^2} \cdot \frac{n_R(z_{RN})}{n_R(z)}$$

$$\beta_{PMC}(z) = \left[\frac{[N_S(z) - N_B] \cdot z^2}{[N_S(z_{RN}) - N_B] \cdot z_{RN}^2} - \frac{n_R(z)}{n_R(z_{RN})} \right] \cdot \beta_R(z_{RN})$$

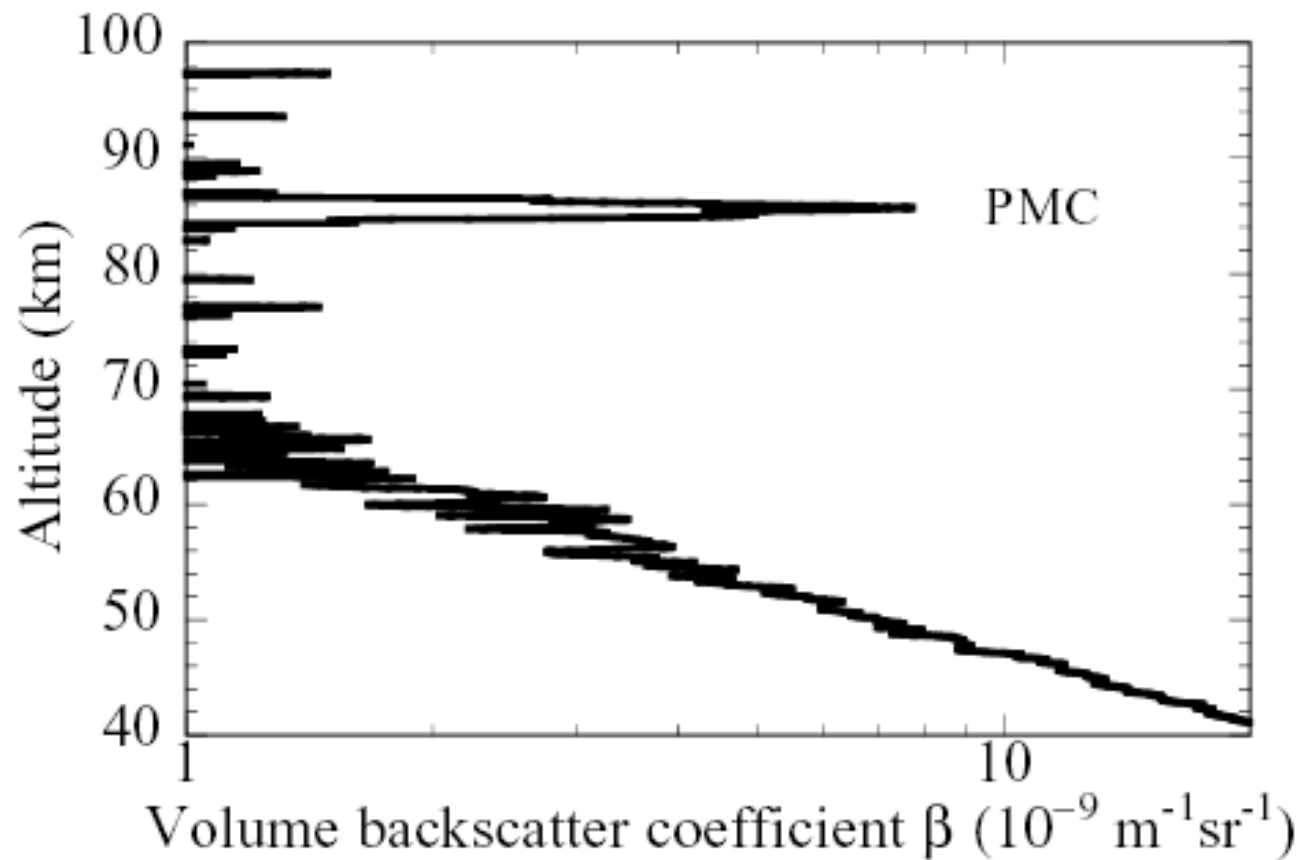
$$\beta_R(z_{RN}, \pi) = \frac{\beta}{4\pi} P(\pi) = 2.938 \times 10^{-32} \frac{P(z_{RN})}{T(z_{RN})} \cdot \frac{1}{\lambda^{4.0117}}$$

$$z_c = \frac{\sum_i \beta_{PMC}(z_i) \cdot z_i}{\sum_i \beta_{PMC}(z_i)}$$

$$\sigma_{rms} = \sqrt{\frac{\sum_i (z_i - z_c)^2 \beta_{PMC}(z_i)}{\sum_i \beta_{PMC}(z_i)}}$$

$$\beta_{total} = \int \beta_{PMC}(z) dz$$

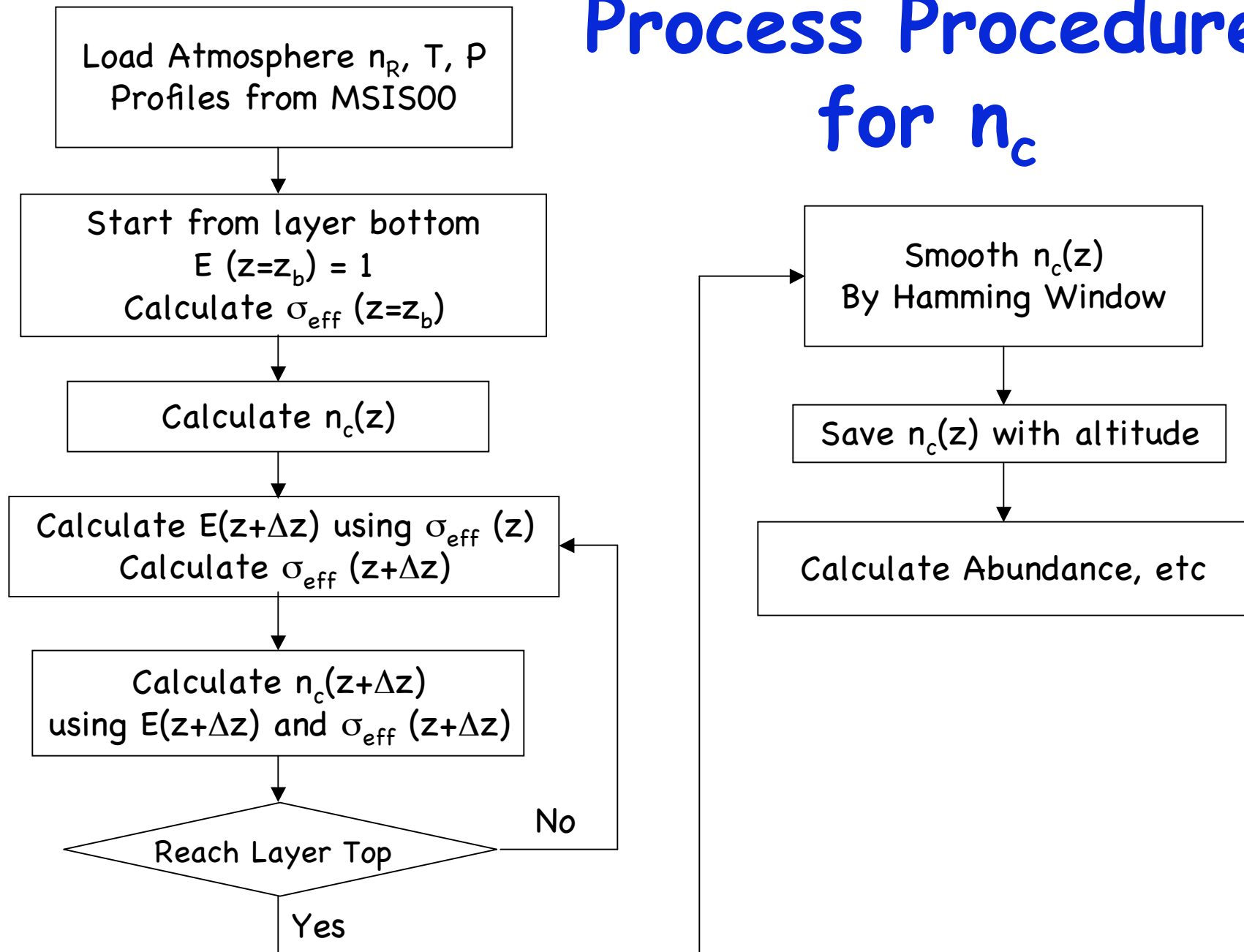
Example Result: South Pole PMC



To Improve SNR

- ❑ In order to improve signal-to-noise ratio (SNR), we have to sacrifice spatial and/or temporal resolutions.
- ❑ Spatial resolution
 - binning
 - smoothing
- ❑ temporal resolution
 - integration

Process Procedure for n_c



Process Procedure for n_c

- ❑ Computation of effective cross-section (concerning laser shape, assuming nominal T and W)
- ❑ Spatial resolution – binning or smoothing
- ❑ temporal resolution – integration
- in order to improve SNR
- ❑ Extinction coefficient
- ❑ Calculate density
- ❑ Calculate abundance, peak altitude, etc.
- ❑ Show Na lidar data as examples in class

Process Procedure for T and V_R

Rayleigh Temperature

Hydrostatic Equation

$$dP = -\rho g dz$$

+

Ideal Gas Law

$$P = \rho RT$$

$$T(z) = T(z_o) \frac{\rho(z_o)}{\rho(z)} + \frac{1}{R} \int_z^{z_o} g(r) dr \frac{\rho(r)}{\rho(z)}$$

Seeding
Temperature

Relative
Density

$T(z_0)$ - Seeding Temperature;

R - gas constant for dry air;

ρ - number density

g - gravitational acceleration

Lidar Backscatter Ratio \Rightarrow Relative Density \Rightarrow Temperature
(at different altitudes) (Rayleigh)

Process Procedure for T , V_R

- ☐ Fe T
- ☐ Iteration method or looking-up table method

Process Procedure for T , V_R

- Na T and V_R
- Iteration method or looking-up table method

Summary

- ❑ Lidar data inversion is to convert raw photon counts to meaningful physical parameters like volume backscatter coefficient, number density, temperature, and wind. It is a key step in the process of using lidar to study science.
- ❑ The basic procedure of data inversion originates from solutions of lidar equations, in combination with detailed considerations of hardware properties and limitations as well as detailed considerations of light propagation and interaction processes.
- ❑ The data inversion procedure consists of three main processes: (1) preprocess, (2) process of β and n , (3) process of T and V_R .

Summary

- ❑ The preprocess is to convert the raw photon counts to corrected and normalized photon counts in consideration of hardware properties and limitations.
- ❑ The process of β and n is to convert the normalized photon counts to meaningful β and n , in combination with some prior acquired knowledge or model knowledge of certain atmosphere information or atomic/molecular spectroscopy.
- ❑ The process of T and V_R is to convert the normalized photon counts to T and V_R through integration, iteration or looking-up table methods.
- ❑ These processes sometimes involve considerable binning, smoothing, or temporal integration in order to improve the signal-to-noise ratio (SNR) to result in meaningful results.